Inverse Scattering of a Buried Imperfect Conductor

Wei-Ting Chen and Chien-Ching Chiu
Electrical Engineering Department, Tamkang University
Tamsui, Taiwan, R.O.C.
Email:chiu@ee.tku.edu.tw

Abstract -The problem of determining the shape of an imperfectly conducting cylinder buried in a half-space by the genetic algorithm is investigated. Assume that a conducting cylinder of unknown shape and conductivity is buried in one half-space and scatters the field incident from another half-space where the scattered filed is measured. Based on the boundary condition and the measured scattered field, a set of nonlinear integral equations is derived and the imaging problem is reformulated into an optimization problem.

I. Introduction

The image problem of conducting objects has been a subject of considerable importance in noninvasive measurement, medical imaging, and biological application. In the past 20 years, many rigorous methods have been developed to solve the exact equation. However, inverse problem of this type are difficult to solve because they are ill-posed and nonlinear. As a result, many inverse problems are reformulated as optimization problems. General speaking, two main kinds of approaches have been developed. The first is based on gradient search approach such as the Newton-Kantorovitch method [1], the Levenberg-Marguart algorithm [2] and the successive-overrelaxation method [3]. This method is highly dependent on the initial guess and tends to get trapped in a local extreme. In contrast, the second approach is based on the genetic algorithm [4]. It usually converges to the global extreme of the problem, no matter what the initial estimate is.

II. Theoretical Formulation

Let us consider a imperfectly conducting cylinder buried in a lossy homogeneous half-space, as shown in Fig 1. Media in regions 1 and 2 are characterized by permittivity and conductivity $(\varepsilon_1, \sigma_1)$ and $(\varepsilon_2, \sigma_2)$ respectively. The metallic cylinder with cross section described in polar coordinates in xy plane by the equation $p = F(\theta)$ is illuminated by transverse magnetic (TM) waves. We assume that time dependence of the field is harmonic with the factor exp($j\omega t$). Let E^{inc} denote the incident field from region 1 with incident angle ϕ_1. A reflected wave (for $y \le -a$) and a transmitted wave (for $y > -a$).

$$E_i(x,y) = \begin{cases} E_1(x,y) = e^{-jk_1[x\sin\phi_1 + (y+a)\cos\phi_1]} + R_1 e^{-jk_1[x\sin\phi_1 - (y+a)\cos\phi_1]}, & y \le -a \\ E_2(x,y) = Te^{-jk_2[x\sin\phi_2 + (y+a)\cos\phi_2]} & , y > -a \end{cases}$$

$$R_1 = \frac{1-n}{1+n}, T = \frac{2}{1+n}, n = \frac{\cos\phi_2}{\cos\phi_1}\sqrt{\frac{\varepsilon_2 - j\sigma_2/\omega}{\varepsilon_1 - j\sigma_1/\omega}}$$

$$k_1 \sin\phi_1 = k_2 \sin\phi_2$$

$$k_i^2 = \omega^2 \varepsilon_i \mu_0 - j\omega\mu_0\sigma_i \,, i = 1,2 \qquad \mathrm{Im}(k_i) \le 0$$

For a TM incident wave, the scattered field can be expressed as

$$E_s(x,y) = -\int_0^{2\pi} G(x,y;F(\theta'),\theta')J(\theta')d\theta' \qquad (1)$$ where

$$J(\theta) = -j\omega\mu_0\sqrt{F^2(\theta)+F'^2(\theta)}J_s(\theta)$$

$$G(x,y;x',y') = \begin{cases} G_1(x,y;x',y') & ,y \le -a \\ G_2(x,y;x',y') = G_f(x,y;x',y') + G_s(x,y;x',y'), & y > -a \end{cases} \qquad (2)$$ where

$$G_1(x,y;x',y') = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{j}{\gamma_1+\gamma_2} e^{j\gamma_1(y+a)} e^{-j\gamma_2(y'+a)} e^{-j\alpha(x-x')} d\alpha \qquad (2a)$$

$$G_f(x,y;x',y') = \frac{j}{4} H_0^{(2)}[k_2\sqrt{(x-x')^2+(y-y')^2}] \qquad (2b)$$

$$G_s(x,y;x',y') = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{j}{2\gamma_2}\left(\frac{\gamma_2-\gamma_1}{\gamma_2+\gamma_1}\right) e^{-j\gamma_2(y+2a+y')} e^{-j\alpha(x-x')} d\alpha \qquad (2c)$$

$$\gamma_i^2 = k_i^2 - \alpha^2 \,, i = 1,2 \,, \mathrm{Im}(\gamma_i) \le 0 \,, y' > a$$

Here $J_s(\theta)$ is the induced surface current density which is proportional to the normal derivative of electric field on the conductor surface. $H_0^{(2)}$ is the Hankel function of the second kind of order zero. The boundary condition at the surface of the scatterer given by [5] then yield an integral equation for $J(\theta)$:

$$E_i(F(\theta),\theta) = \int_0^{2\pi} G_2(F(\theta),\theta;F(\theta'),\theta')J(\theta')d\theta' + j\sqrt{\frac{j}{\omega\mu_0\sigma}}\frac{J(\theta)}{\sqrt{F^2(\theta)+F'^2(\theta)}} \qquad (4)$$

For the direct scattering problem, the scattered field, E_s, is calculated by assuming that the shape and the conductivity of the object are known. This can be achieved by first solving J in (4) and calculating E_s in (2). The shape $F(\theta)$ function can be expanded as:

$$F(\theta) = \sum_0^{N/2} B_n \cos(n\theta) + \sum_{n=1}^{N/2} C_n \sin(n\theta) \qquad (5)$$

where B_n and C_n are real coefficient to be determined, and N+1 is the number of unknowns for shape function. The genetic algorithm is used to minimize the cost function:

$$CF = \{\frac{1}{M_t}\sum_{m=1}^{M_t}\left|E_s^{\exp}(\vec{r}_m) - E_s^{cal}(\vec{r}_m)\right|^2 / \left|E_s^{\exp}(\vec{r}_m)\right|^2 + \alpha\left|F'(\theta)\right|^2\}^{1/2} \tag{6}$$

where M_t is the total number of measured points. $E_s^{\exp}(\vec{r})$ and $E_s^{cal}(\vec{r})$ are the measured scattered field and the calculated scattered field respectively. The minimization of $\alpha\left|F'(\theta)\right|^2$ can be interpreted as the smoothness requirement for the boundary of $F(\theta)$. The basic GA for which a flowchart is shown in Fig. 2.

III. Numerical Results

Let us consider an imperfectly conducting cylinder buried in a half-space ($\sigma_1 = \sigma_2 = 0$). The permittivity in region 1 and region 2 is characterized by $\varepsilon_1 = \varepsilon_0$ and $\varepsilon_2 = 2.55\varepsilon_0$ respectively. The frequency of the incident wave is chosen to be 3 GHz. The object is buried at a depth 0.1 m. In the example, the shape function is chosen to be $F(\theta) = 0.12 + 0.048\cos 2\theta$ m with material $\sigma = 3.54\times 10^7$ s/m. The reconstructed shape function for the best population member is plotted in Fig. 3(a) with the error shown in Fig. 3(b), where DR and DSIG are called shape function and conductivity discrepancies respectively. In the second example, $F(\theta) = 0.02 + 0.004\sin 2\theta + 0.008\sin 3\theta$ m with material $\sigma = 5.8\times 10^7$ s/m. Good results are obtained in Fig. 4(a) and Fig. 4(b).

IV. Conclusions

We have presented a study of applying the genetic algorithm to reconstruct the shape and conductivity of a metallic object through knowledge of scattered field. Based on the boundary condition and measured scattered field, we have derived a set of nonlinear integral equations and reformulated the imaging problem into an optimization problem. By using the genetic algorithm, the shape and conductivity of the object can be reconstructed from the scattered fields. Numerical results also illustrate that the conductivity is more sensitive to noise than the shape function is.

V. References

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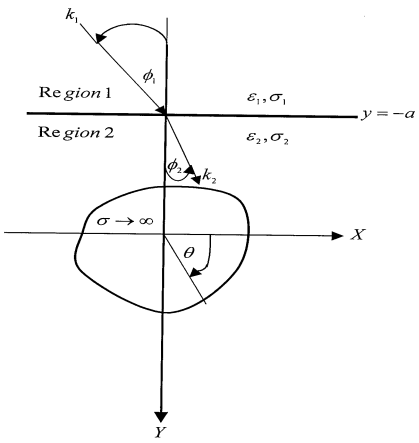

Fig. 1 Geometry of the problem in (x,y) plane

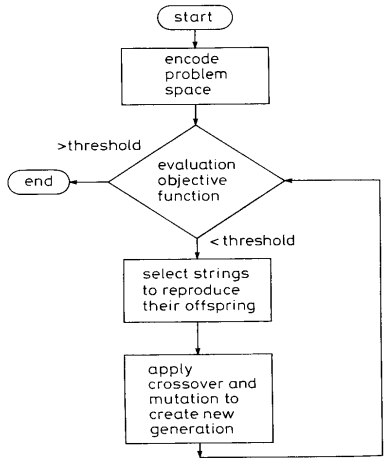

Fig. 2 Flow chart for the genetic algorithm

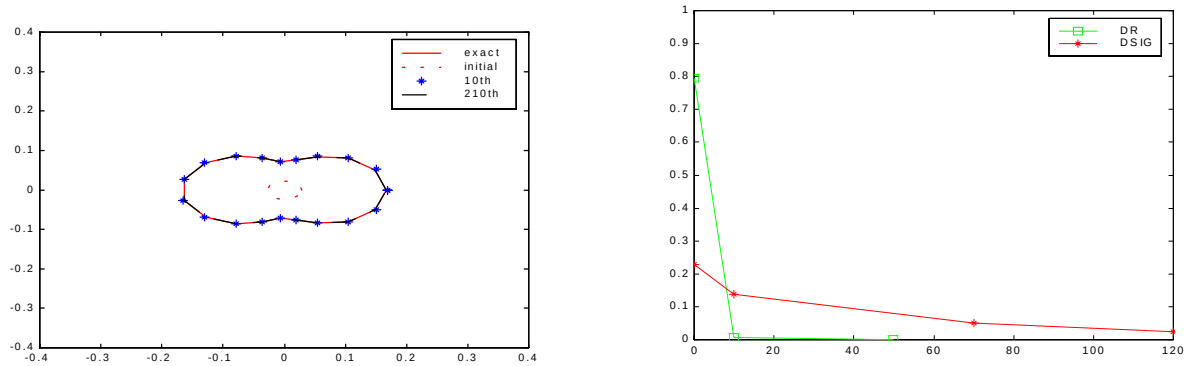

Fig. 3(a) Shape function. The solid curve represents the exact shape, while the dashed curves are calculated shape in iteration process. (b)Shape function error and conductivity error.

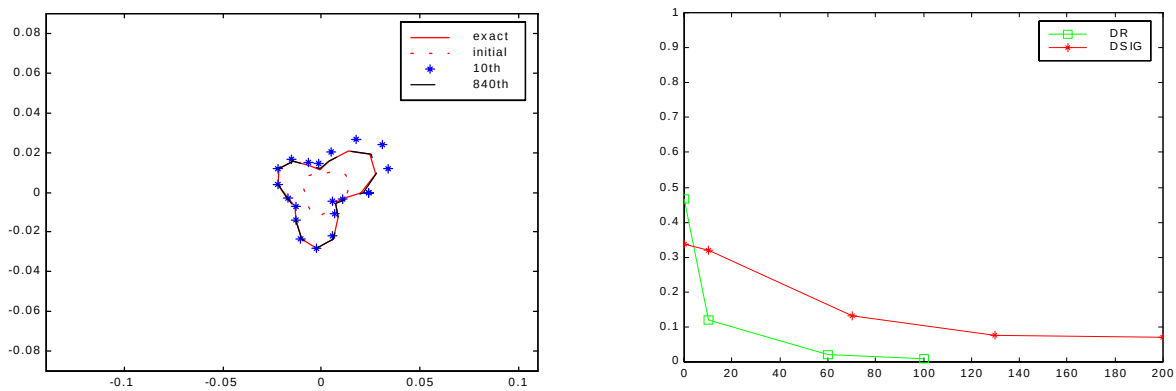

Fig. 4(a) Shape function. (b) Shape function error and conductivity error.