AN ADAPTIVE GRIDDING FDTD METHOD FOR SOLVING ELECTROMAGNETIC SCATTERING PROBLEMS

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1. Introduction

To solving electromagnetic scattering problems, the finite difference time domain (FDTD) method is one of the potential approaches. Since it is a simple and versatile modeling technique, it is easier to solve electromagnetic scattering problems, which have the complicating effects of corners, apertures, and dielectric loading of structures. However, it suffers from a limitation that large computational resources are required. It is because the cell size in the solution domain is proportional to the wavelength of the signal under simulation. Therefore, huge number of grid points are required when a medium or large structure with a high frequency source is simulated. These grid points require large number of memory to store and lead to long computational time when the traditional Yee's FDTD method [1] is used.

In this paper, a wavelet based adaptive gridding [2], which depends on the variation of magnetic field, is applied to analyze the scattering of a TM wave on a perfectly conducting rectangular cylinder. Since most of the simulations use a Gaussian pulse or a modulated Gaussian pulse as the excitation, the magnetic field in most regions of the computational domain are smooth. These regions are over resolved if a uniform grid is used. Moreover, the regions, which do not contain high frequency components, can be modeled by a coarse grid. Therefore, the use of adaptive gridding, which according to the variation of the magnetic field, can reduce the required computational resources.

The wavelet analysis [3] is a powerful method to analyze localized areas of a signal. It can detect the variation of a signal at any location and scale. According to the information detected by the wavelet analysis, we can determine the cell size at every location of the domain. The conventional finite difference is then applied to the non-uniform grid. Moreover, the non-uniform grid is constructed again after several time steps as the variation of the magnetic field change. Therefore, the fields inside the solution domain can be modeled by a suitable resolution throughout the whole simulation.

2. Wavelet based adaptive gridding FDTD method

To construct the non-uniform grid, the magnetic field inside the solution domain is decomposed into wavelet coefficients first. For example, a magnetic field $H_x(x, t_0)$ is decomposed into two sets of wavelet coefficients, approximation (s^k_j) and detail (d^k_j), where j is the location parameter and k is the scale parameter.

$$s_j^k = \int_{-\infty}^{\infty} H_x(x,t_o)\phi_j^k(x)dx \qquad (1)$$

$$d_j^k = \int_{-\infty}^{\infty} H_x(x,t_o)\psi_j^k(x)dx \qquad (2)$$

$\phi^k_j(x)$ is the scaling function and $\psi^k_j(x)$ is the wavelet function of the mother wavelet $\phi(x)$ and $\psi(x)$, respectively.

The approximation coefficients are simply the average value of the signal over corresponding intervals (low frequency components). The detail coefficients are the high frequency components of the signal of corresponding intervals. The decomposition process can be iterated, with successive approximation being decomposed, so that a signal is broken down into many lower resolutions components to form a wavelet tree. A two level wavelet decomposition tree of an magnetic signal with 8 sampling point is shown below.

$$\begin{bmatrix} H_{x,1} \\ H_{x,2} \\ H_{x,3} \\ H_{x,4} \\ H_{x,5} \\ H_{x,6} \\ H_{x,7} \\ H_{x,8} \end{bmatrix} \rightarrow \begin{bmatrix} s^1_1 \\ s^1_2 \\ s^1_3 \\ s^1_4 \\ d^1_1 \\ d^1_2 \\ d^1_3 \\ d^1_4 \end{bmatrix} \rightarrow \begin{bmatrix} s^2_1 \\ s^2_2 \\ d^2_1 \\ d^2_2 \\ d^1_1 \\ d^1_2 \\ d^1_3 \\ d^1_4 \end{bmatrix} \qquad (3)$$

The scale parameter (k) reprensents the length of interval in the order of 2^k. For example, a coefficient with k = 1 and j = 1 contains the information from the first two samples, and if k = 2 and j = 1, the coefficeint contains the information from the first four samples. These coefficients are the necessary information to refine the cell size locally to different scale, Δx, $2\Delta x$ and $4\Delta x$.

To construct a non-uniform grid, these coefficients are first compared with a prescribed threshold. If the detail coefficient is smaller than the threshold, the grid density corresponds to that scale can be applied to that location. For example, if d^2_1 is smaller than the threshold, a $4\Delta x$ cell can be applied to location from $E_{x,1}$ to $E_{x,4}$. Otherwise, d^1_1 and d^1_2 should be considered. If d^1_1 is smaller than the threshold, a $2\Delta x$ cell is applied to the location from $E_{x,1}$ to $E_{x,2}$.

After several time steps, the non-uniform grid is reconstructed because the variation of the magnetic field is changed. Therefore, the field insides the solution domain can be modeled by a suitable resolution throughout the whole simulation.

3. Numerical results

A wavelet based adaptive gridding FDTD method is applied to analyze an electromagnetic scattering problem. The surface-electric current distribution and near-scattered electric field [4] are found for the case of a rectangular metal cylinder subject to a TM-polarized excitation (Figure. 1).

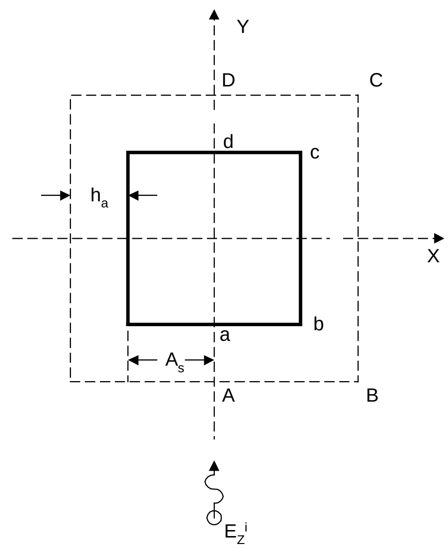
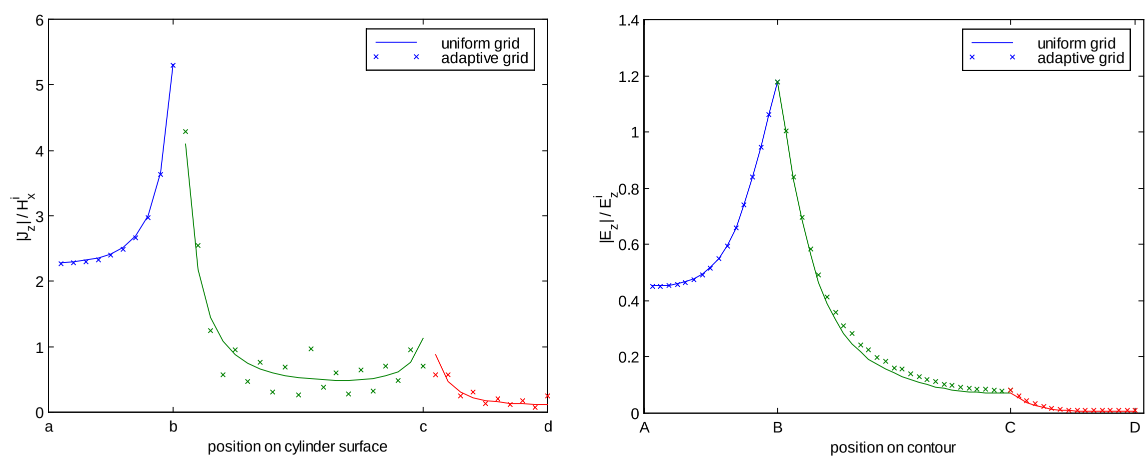


Figure.1 Scattering of a plane wave by a square conducting cylinder

The cylinder has the electrical size k_0A_s=1, where A_s is the half-width of the side of the cylinder. The plane wave excitation is TM-polarized, with field components E_z^i, and propagates in the +y direction, so that it is at normal incidence to one side of the cylinder.

Figure 2(a) shows the comparative result for the uniform Yee's and adaptive gridding scheme analyses of the magnitude of the cylinder surface electric-current distribution for this example. The surface tangential electric current is taken as $\mathbf{n} \times H_{tan}$, where **n** is the unit normal vector at the cylinder surface. The H_{tan} is the magnetic field parallel to the cylinder surface. Figure 2(b) shows the comparison of the magnitude of the near-scattered electric field computed by two schemes. The electric field is tangential to cylinder surface and located at a uniform distance (h_a = 5 cells) from the surface. The difference between uniform Yee's scheme and adaptive gridding scheme results for surface electric current is 2% and for near-scattered electric field is 3%. Table.1 shows the comparison of the computational resources required of two schemes.



(a) (b)

Figure 2. Comparison of uniform gridding and adaptive gridding results

	Average no. of grid points used per time step	Computational time (s)
Uniform gridding	36864	14109
Adaptive gridding	22274	9127

Table 1. Comparison of computational resources required

Figure 3 shows the electric field (E_z), when the Gaussian pulse contacted the square-conducting cylinder, and the grid used at that moment. It can show that the dense grid are followed both propagated and reflected pulse and remains coarse for other location.

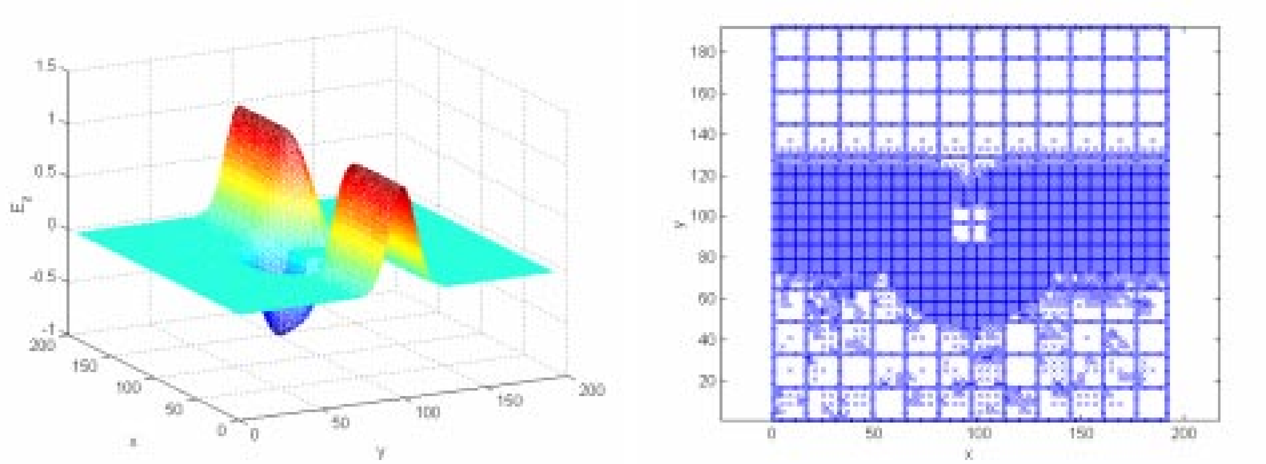


Figure 3 Electric field (E_z) and grid used at t=300Δt

4. Conclusion

The wavelet based adaptive gridding FDTD scheme has been proposed and applied to analyze an electromagnetic scattering problem. The new scheme can offer about 40 % reduction in grid points required and 35 % in computational time. The new scheme will be extended to three-dimension problems and the effect of the reduction in computational resources will be more obvious.

Acknowledgement

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