MOMENT METHOD ANALYSIS OF E-H PLANE TEE JUNCTION USING PULSE FUNCTION AND POINT MATCHING

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1. Introduction :

Variational analysis of E-H plane T-junction [1] coupled through inclined slot in the narrow wall of a rectangular waveguide assuming sinusoidal field distribution [2] has been carried out. A moment method solution using pulse basis function and point matching [3] which leads to more accurate solution for the field distribution in the aperture plane of the slot is used in the present paper. Correction for finite wall thickness in the moment method formulation is applied following the procedure suggested by Josefsson [4]. This junction permits realisation of resonance of coupling inclined slot in the frequency range of interest and makes array excitation without cross polarisation feasible.

2. General Analysis :

Consider the E-H plane Tee junction of Fig. 1 together with the co-ordinate system. An expanded view of the coupling slot in a thick waveguide wall in the presence of incident and reflected waves in the coupling slot considered as a stub waveguide is shown in Fig. 2.

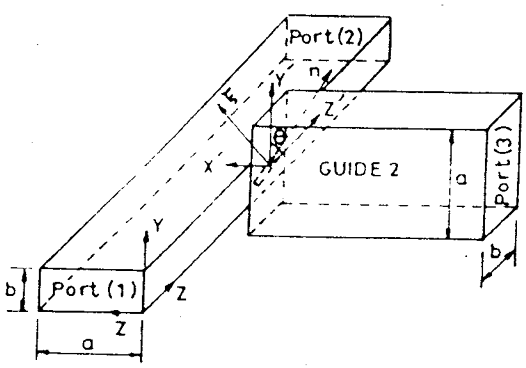

Fig. 1 : An E-H plane Tee Junction between two rectangular waveguides.

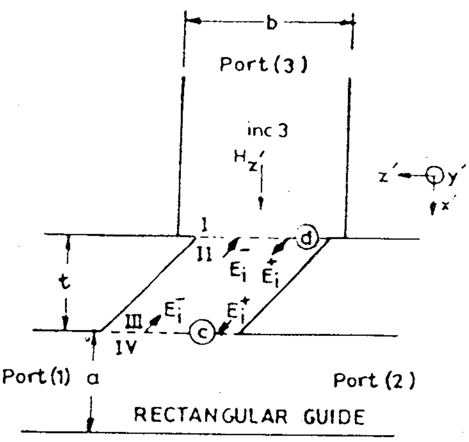

Fig. 2 : An expanded view of the coupling slot in two thick wall of the waveguide showing incident and reflected waves in the stub waveguide.

The ξ-directed electric field in the aperture field of Figs 1 & 2 are expressed as a superposition of appropriate basis function e_η where η is the variable along the axis of the slot. Considering the incident and reflected fields of the ith mode in the region of the stub waveguide as well as superposition of the components related to the basis functions, the boundary conditions for the η-component of the tangential magnetic field in the aperture planes of the slot at the two interfaces are of the form [4]

$$\sum_{p} \left[H_{\eta}^{(2)}\left(e_{\eta}^{p}\right)+E_{p}^{+} Y_{p} e_{\eta}^{p}\right]-\sum_{i} \left[Y_{i} E_{i}^{-} e_{\eta}^{i}-H_{\eta}^{(2)}\left(e_{\eta}^{i}\right)\right]=0 \qquad \ldots(1)$$

at the interface A, and

$$-\sum_{p} \left[E_{p}^{-} Y_{p} e_{\eta}^{p}-H_{\eta}^{(1)}\left(e_{\eta}^{p}\right)\right]+\sum_{i} \left[E_{i}^{+} Y_{i} e_{\eta}^{i}+H_{\eta}^{(1)}\left(e_{\eta}^{i}\right)\right]=H_{\eta}^{inc} \qquad \ldots(2)$$

at the interface B

where E^{+} and E^{-} indicate the amplitude co-efficients of the modal fields propagating in the – and + directions of the slot waveguide as shown in Fig. 2. e_{η}^{i} and e_{η}^{p} are the basis functions of the aperture field, Y_i, Y_p are the modal admittances of the fields propagating along the stub waveguide.

$H_{\eta}^{(2)}(e_{\eta})$ and $H_{\eta}^{(1)}(e_{\eta})$ are respectively the magnetic fields in the aperture plane of the slot in the T-arm and primary side of the guide respectively.

The magnetic fields $H_{\eta}^{(2)}(e_{\eta})$ and $H_{\eta}^{(1)}(e_{\eta})$ are expressed in terms of respective Green's function as

$$H_{\eta}^{(2)}\left(e_{\eta}\right)=\iint_{slot} \overline{\overline{U}}_{n}\cdot e_{n}\, ds' \qquad \ldots(3)$$

where Green's function $\overline{\overline{U}}_{n}$ in the Tee-arm is of the form [5]

$$\overline{\overline{U}}_{n}=\sum_{i} h_{i}(\eta,\xi) h_{i}(\eta',\xi') Y_{i} \qquad \ldots(4)$$

and

$$H_{\eta}^{(1)}\left(e_{n}\right)=\iint_{slot} \overline{\overline{B}}\cdot e_{\eta}\, ds' \qquad \ldots(5)$$

where $\overline{\overline{B}}$ is obtained from eqn. (6) of ref. [6].

The eqns. (1) – (5) are solved using pulse function and point matching [3], assuming a basis function of the form

$$e_{\eta}=\sum_{p=0}^{2M-1} E_{p} f_{p} \qquad \ldots(6)$$

where f_p is the pulse function which has non zero value only in the pth cell obtained on dividing the slot extending from –L to +L into M subsections, each of length L/M and width 2w.

Using η-directed incident magnetic field as

$$H_{\eta}^{inc}=\left[\cos\theta \frac{\pi / a}{j\omega\mu} \cos\frac{\pi x}{a}+\sin\theta \frac{\beta_{10}}{\omega\mu} \sin\frac{\pi x}{a}\right] e^{-j\beta_{10} z} \qquad \ldots(7)$$

and following the procedures of ref. [3,4] and matching the boundary condition for the magnetic field at the centre of sth subsection with co-ordinates $\eta=-L+p\frac{L}{M}+\frac{L}{2M}, \xi=0$, the amplitude co-efficients in eqn. [1,2,6] are obtained as a column matrix $E_p]$ expressed in terms of column matrix $h_{inc}]$ as

$$E_{p}]=\left\{[U]+[B]\left[R^{2}\right][B]\right\}\left\{\left[R^{1}\right][B]\left[R^{2}\right]-[U]\right\}^{-1}\left[Y^{1w}\right]^{-1} h_{inc}] \qquad \ldots(8)$$

where $Y^{1w} = \left\langle H^1_\eta, f_p \right\rangle + Y_p \left\langle f_p, \delta(s) \right\rangle$

$$\left[R^2\right] = \left[\left\langle H^{(2)}_\eta(e_p), f_p \right\rangle - Y_p\left\langle f_p, \delta(s)\right\rangle\right]^{-1}\left[\left\langle H^{(2)}_\eta(e_i), f_p \right\rangle - Y_p\left\langle e_i, \delta(s)\right\rangle\right]$$

$$\left[R^1\right] = \left[\left\langle H^{(1)}_\eta(e_p), f_p \right\rangle + Y_p\left\langle e_p, \delta(s)\right\rangle\right]^{-1}\left[\left\langle Y_i(e_i), \delta(s)\right\rangle - \left\langle H^{(1)}_\eta(e_i), f_p \right\rangle\right]$$

and [B] is a diagonal matrix, whose elements are $e^{-\gamma_{n0} t}$, t being the thickness of the waveguide wall and [U] is the unit matrix. γ_{n0} is the propagation constant of the TE_{n0} mode in the stub waveguide. The symbol < > indicates integration over a subsection of length L/M and width 2w. In the integral limits of η are from $-L + p\frac{L}{M}$ to $-L+(p+1)$ L/M (p=0,...,2M-1) and of ξ from $-w$ to $+w$. η and ξ are related x to z through co-ordinates transformation as shown in Fig. 1.

$$h_{inc} = \left\langle H^{inc}_\eta, \delta(s) \right\rangle \quad \text{for } \xi = 0 \qquad \text{...(9)}$$

3. Numerical Results and Discussion :

Using eqn. (1) – (9), the amplitude co-efficients in the centres of different subsections are evalued for M = 32; a=2.286 cm; b=1.016 cm, 2L=1.4cm and 2w= 0.08 cm. and waveguide wall thickness of 0.127 cm. The reflection co-efficient Γ is evaluated by following the method of ref. (15) in ref. [3]. The transmission co-efficient is found from the relation T = 1+Γ, assuming equivalent shunt representation. The return loss $20\log|\Gamma|$, and coupling $10\ \log\left[1-|\Gamma|^2-|T|^2\right]$ are evaluated as a function of frequency and the results are presented in Figs. 3 and 4 respectively. In the same figures, the experimental data and numerical data evaluated by the method of ref. [1] are also presented.

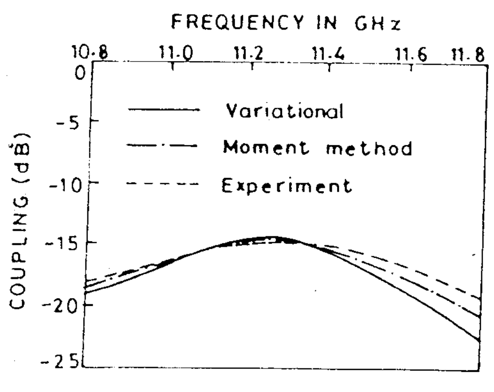

Fig. 3 : Variation return loss at port 1 with frequency

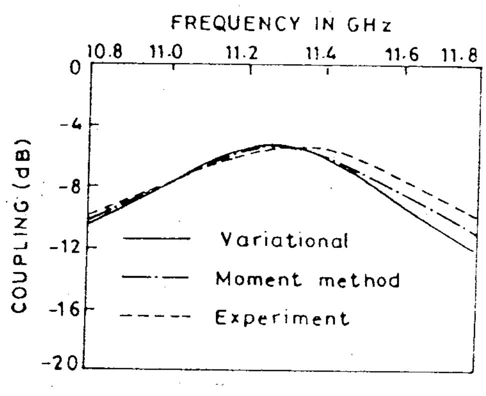

Fig. 4 : Variation of coupling between port 1 and port 3 with frequency

Fairly good agreement is observed in the frequency range of interest. Resonance occurs at 11.3 GHz, which is higher than the free space resonant frequency 10.2 GHz for a slot length 1.4 cm. Thus 2L/λ for resonance is higher than 0.5, which is the trend for slots coupling two waveguides. For a slot radiating into free space, this ratio is less than 0.5.

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