Multiresolution Algorithm for the analysis of Printed Antennas

P. Pirinoli, G. Vecchi, L. Matekovits, M. Orefice
Dipartimento di Elettronica, Politecnico di Torino, I-10129 Torino, Italy
Tel: +39 011 564 4090, Fax: +39 011 564 4015
e-mail: pirinoli@polito.it, vecchi@polito.it, matekovots@polito.it, orefice@polito.it

Introduction

In the analysis of complex printed structures, as real life antennas and arrays with radiating elements of arbitrary shape and beam forming networks, the reduction of the computational efforts mantaining high accuracy in representing the solution is mandatory. The Integral Equation (IE) - Method of Moments (MoM) approach seems to be the only able to guarantee accurate results; nevertheless, this task is reached generating very large, densely populated matrices, with a high condition number, i.e. with a considerable computational cost. In view of solving these problems, in the last years different alternatives have been proposed. Among these, several wavelet-based or multiresolution (MR) approaches have appeared in the electromagnetic literature, mostly motivated by the appealing properties of these functions. However, the intrinsic difficulties of generating and employing vector MR functions in three-dimensional problems, have up to now limited their applications to scalar or one-dimensional structures [1]-[3]; a very small number of wavelet-based analysis of printed circuits exists, in which wavelets are applied separately to the two cartesian components of the current [4].

The approach recently proposed by these authors [5, 6, 7] leads instead to the introduction of vector miltiresolution functions that can be defined on arbitrary shape geometries; they are constructed using the concepts of the wavelet representation, yet keeping as much as possible of the "physical" information contained in the IE-MoM format: the resulting vector basis functions have a good degree of multiresolution, and generate MoM matrices with a large dynamics. This allows the use of a diagonal preconditioning to reduce the condition number of the MoM matrix: as a consequence, the scheme appears to be stable against perturbations, and notably against sparsification: it means that all the matrix entries below a fixed threshold can be zoroed, keeping the error on the solution under control. Furthermore, the stability of the linear system makes the use of iterative solvers, as the Conjugate Gradient (CG), convenient.

Generation of the MR scheme

The starting point in defining the generation procedure for the MR basis functions is a consideration on the conditioning of the MoM matrix: since it depends on the singular, quasi-static behavior of the EFIE [8, 9] and since in vector problems there are two of such terms, it is convenient to split the current into two parts, a solenoidal (TE) term and a non-solenoidal (quasi-irrotational, qTM) remainder, that exhibit different properties [8]. These two parts can be brought into correspondence with scalar functions that (unlike the components of the vector basis functions usually employed to represent the current) are "isotropically" scalar, i.e. they show the same degree of regularity in both spatial directions. The TE current is in relation with a solenoidal potential, piecewise linear, expressed by scalar "pyramidal" functions [8]; when a rectangular mesh is adopted, as here, they are defined on four adjacent cells and the functions naturally generated from them for the current are the "loop" functions [8, 9]. The non-solenoidal part of the current is instead related with a piecewise constant quantity, the charge. The MR vector functions proposed here are generated defining first multiresolution, two-dimensional scalar functions for these scalar quantities, and then mapping these ones back onto the current. For this reason the basis formed by the resulting vector MR functions is called "dual-isoscalar".

The second issue that has to be taken into account in the generation of the MR functions is a geometric one, i.e. we are interested in the analysis of structures of arbitrary shapes, conforming to a grid with rectangular cells, here called "pixels", on which the usual interpolating vector linear functions are defined; the MR vector basis functions are defined as grouping of these "elemental" functions. The arbitrariety of the geometry implies that the MR functions generation process is divided into two steps, both separately for the TE and the qTM parts of the current: the first step is the so called "domain decomposition", in which the structure is broken down into a reasonable number of rectangles; they form the coarsest level mesh, on which the functions connecting the different domains are built. The second step consists in the generation of the MR functions inside the rectangles: they are separable domains, in which the two-dimensional scalar functions (for the solenoidal potential or the charge) can be obtained as tensor product of two one-dimensional scalar sets of functions, defined on the two sides of the rectangle. In the present scheme, on each side a scalar and unsymmetric Haar basis is defined, but other choices can be adopted.

In the following, the generating procedure of the MR functions for the TE and the qTM currents are summerized: for a more detailed treatment, see [5, 6, 7].

The MR vector TE functions are obtained as a linear combination of the loops. The coefficient of this linear combination are the same derived from the 2D scalar MR functions for the solenoidal potential: in a rectangular domain, they are given by the tensor product of the one-dimensional scalar functions defined along the two sides. The mesh for the solenoidal current and potential is node-based, i.e. in a rectangular mesh there is a number of functions equal to the number of *inner* nodes of the mesh. In a one dimension, the pyramids correspond to triangular interpolating functions: therefore, the one-dimensional scalar functions on each side of the rectangular domain are given as the sampling of the Haar basis defined on a 1D domain divided in a number of interval equal to the number of inner nodes.

When two neighbouring domains have to be connected, the number of 2D indipendent functions is equal to the number N_b of nodes belonging to both domains: therefore the connecting functions are given by the tensor product of N_b functions in the directions along the boundary and of one function in the direction across the boundary, defined over the 1D domain that extends across both domains.

In the case of the qTM functions, the mesh is cell-based, and over a N_C cell mesh it is possible to define $N_C - 1$ functions for the charge. Inside each rectangle a hierarchy of meshes of different levels, i.e. meshes with increasing level of resolution j, the finest with cells coincident with the pixels, is first generated. The cells of the mesh at level j are composed of an integer number of cells of the mesh of finer level $j+1$. At each level, the domain is divided in four subdomains, that represent the cells of the level. On this four-cell mesh one can define three functions giving total null charge, that can be directly represented as a current, therefore generating the three MR vector functions for the current (see [5]) belonging to this level, while the constant charge state is discarded, since it has no correspondence with the current. The procedure is then hierarchically repeated inside each subdomain and it stops when a domain with dimension 2×2 or $N \times 1$ is encountered.

As concerns the connecting functions, two different schemes have been investigated: in the first option, the connections are not realized using MR functions, but instead the "star" basis functions generated through the automatic procedure detailed in [9]. Their presence interrupt the multilevel spectral localization sequence of the MR scheme, and therefore the introduction of a "pre-regularization" procedure, consisting in the elimination the N_S star functions from the linear system is necessary before the diagonal preconditioning. The second scheme is complete MR, and consists in the definition of MR scalar functions over a one-dimensional, non-uniform mesh, which cells are in number and have dimension equal to the rectangular domains. Mapping back these functions on the two-dimensional mesh, we obtained a two-dimensional representation for the charge and then for the current. The performances of the two schemes are comparable, both in terms of the error introduced by the sparsification, and for what concerns the convergence of the iterative solvers. The advantage of using a complete MR scheme is that no pre-regularization is needed, and this can reduce the computational time in case of complex structures, for which a high number of domain has to be defined.

Numerical example

The MR scheme described above has been applied to the analysis of different configurations, in order to test the capability of MR representations on realistic cases of non-canonical shapes, and in large dimension problems. Here, we report the results relative to an array of four series fed stacked-patches (see inset of Fig.1). The number of total unknown is 1302 and the domain decomposition for the qTM current is called for, resulting in $N_S = 15$ matching functions.

As concerns the conditioning of the MoM matrix, we find that the condition number of the matrix in the rooftop basis is $\kappa_{rft} = 650$, while in MR basis it is $\tilde{\kappa}_{MR} = 57$, i.e. it has been reduced of one order of magnitude. The effects of the MR functions are shown also in Fig. 1, where the error on the MoM matrix Z (on the left) and on its inverse Z^{-1} (on the right) versus the sparsification (here obtained setting to zero the entries below a fixed threshold) is reported; the curves show that while the error on Z in the rooftop basis and in MR basis has almost the same trend, the introduction of the MR scheme makes also the error on Z^{-1} controllable.

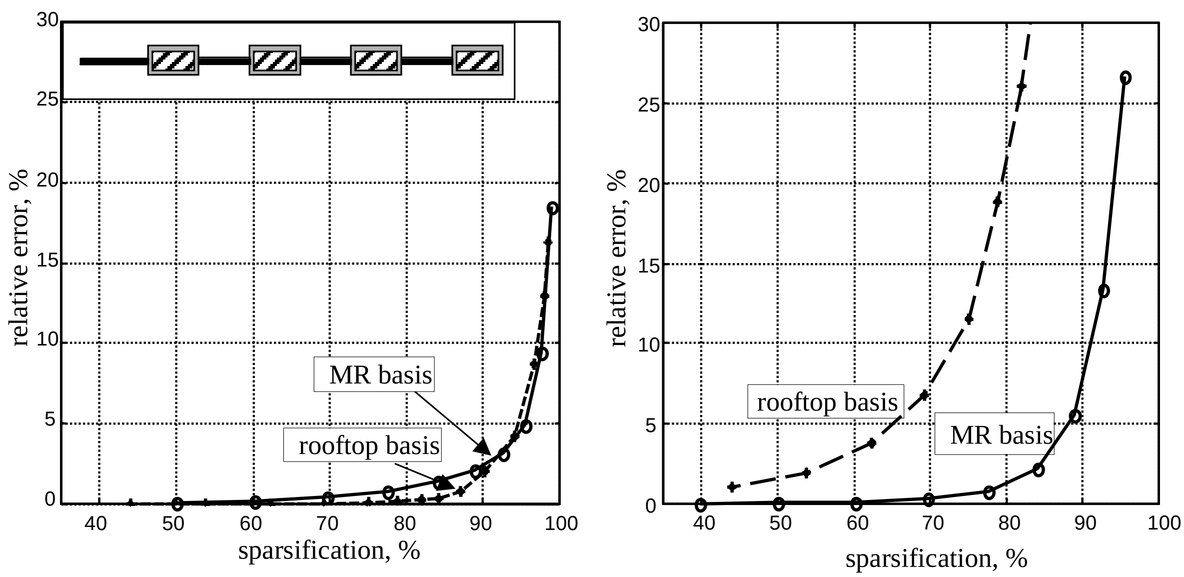

Figure 1 – Array of four stacked patches (inset). Left: per cent error on the MoM matrix vs. sparsification. Right: per cent error on the inverse MoM matrix vs. sparsification.

The effects of the introduction of the MR functions on the convergence history of the conjugate gradient are shown in Fig. 2: from this plot it appears that the MR scheme drastically reduces the convergence of the CG with a consequent reduction of the computational effort. This effort has been evaluated in terms of the floating point operations (flops) necessary to obtain the solution. In rooftop basis, the linear system is solved using LU, while when MR basis is adopted, the system is solved with the CG and the number of flops is the total one, i.e. it takes into account the number of operations for the generation of the MR basis and for the change of basis. From the plot in Fig. 2 it appears that in the case of sparsification of 90%, that guarantees an error on Z^{-1}, i.e. on the current, of the order of 5%, the reduction of flops with respect to the use of rooftops is almost of one order of magnitude.

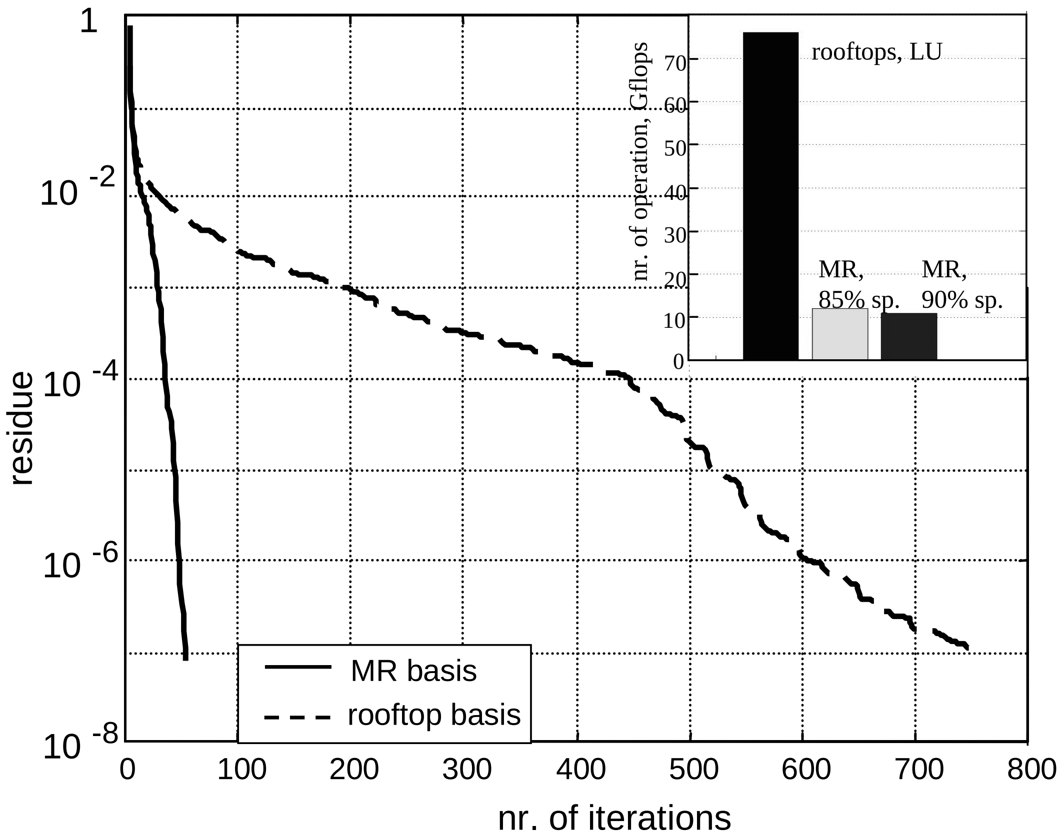

Figure 2 – Array of four stacked patches. Convergence history of the Conjugate Gradient. Inset: number of flops using rooftop basis or MR basis.

References

[1] Z. Baharav, Y. Leviatan, *IEEE Trans. Ant. Prop.*, vol. 46 (1998), pp. 226-233.

[2] G. Wang, G.W. Pan, *IEEE Trans. Microw. Theory Tech.*, vol .43 (1995), pp. 31-145.

[3] G. Oberschmidt, K. Bubke, A.F. Jacob, *IEEE Trans. Microw. Theory Tech.*, vol. 46 (1998), pp. 558-561.

[4] K. F. Sabet, L. P. B. Katehi, *Int. J. of Num. Model.*, vol. 11 (1998), pp. 3-19.

[5] P. Pirinoli, L. Matekovits, M. Sereno Garino, *Microw. and Optical Techn. Letters*, Vol. 23, n. 1 (1999), pp. 49-51.

[6] P. Pirinoli, G. Vecchi, L. Matekovits, M. Orefice, *Proc. of AP2000*, Davos (CH), Apr. 2000.

[7] P. Pirinoli, G. Vecchi, L. Matekovits, accepted for the publication on *IEEE Trans. Ant. Prop.*

[8] G. Vecchi, L. Matekovits, P. Pirinoli, M. Orefice, *Int. J. of Microw. and Mill. Wave Computer-aided Engineering*, vol. 7, pp. 410-431, Nov. 1997.

[9] G. Vecchi, “Loop-star decomposition of basis functions in the dscretization of EFIE”, *IEEE Trans. Ant. Prop.*, vol. 47 (1999), 339-346.