MOD-P SIERPINSKI FRACTAL MULTIBAND ANTENNA

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1 ABSTRACT

Multiband antennas have an important role in the incoming telecommunication systems. A new set of multiband antennas based on fractal geometry, referred as mod-p Sierpinski gaskets is presented. These novel designs constitute a generalisation of the Sierpinski antenna already analysed, and present a log-periodic behaviour due to their fractal self-similarity properties.

2 INTRODUCTION

The behaviour of an antenna depends on the relation between its size and its operating frequency. This relation makes antenna designs highly frequency dependent, denoting the difficulty to use the same antenna at different bands. A good alternative to overcome this problem is the application of fractal structures to design multiband antennas, as is described in [1]. In the Sierpinski gasket monopole [2,3], both the input parameters and the radiation patterns present a log-periodic behaviour. The log-period matches the scale factor that characterises the geometrical self-similarity properties of the fractal object. The classical Sierpinski gasket is a particular case of a broader set of fractal structures, which are called Pascal-Sierpinski Gaskets [4]. In this paper a special case of this new class of geometric fractals is presented and is called mod-p Sierpinski gaskets. Mod-p Sierpinski gaskets, whose structure is concerned with the divisibility of the coefficients in the Pascal triangle by a factor p, were generated using an iterative algorithm, that is Iterated Function Systems (IFS) [5]. In this paper, the generation process and the main results of mod-2, mod-3 and mod-5 Sierpinski fractal monopole antennas will be presented. The results will permit the authors to introduce a new kind of fractal multiband antennas, mod-p Sierpinski gaskets, which present a self-similar behaviour at several bands.

3 MOD-P SIERPINSKI GASKETS GENERATION PROCESS

The antennas are generated using the concept of Iterated Function Systems (IFS), based on a set of geometrical transformations called linear affine transformations [1]. An affine transformation, also named contraction, consist of a linear mapping together with a translation. If we apply N contractions w_1, w_2, ..., w_N to a subset (A) and we assemble the results, the collage obtained can be expressed by the collage mapping

$$W(A) = w_1(A) \cup w_2(A) \cup ... w_N(A) \quad (1)$$

where W is the Hutchison operator. By applying repeatedly W, a sequence of sets is obtained, tending to a final set A_{∞}, called the attractor of the IFS. If we apply W to the attractor, it is left invariant.

Affine transformations can modify angles, contrary to similarity transformations, which maintain angles unchanged. Therefore, the second ones are a particular case of the first ones. Since fractals are self-similar, they can be generated by an IFS scheme involving similarity transformations. Mod-p Sierpinski gaskets are generated by the IFS scheme related to the divisibility of the coefficients in the Pascal triangle. This triangle is a triangular array of numbers composed of the coefficients of the expansion of the polynomial $(1+x)^n$. The mod-p structures are constructed eliminating from the array those elements that are divisible by p, a factor that generates self-similar fractal objects when it is a prime number [4]. We consider a unit square Q and subdivide it into p^2 congruent squares $Q_{a,b}$ with $a,b \in \{0,...,p-1\}$. With these assumptions, the IFS used to generate the mod-p Sierpinski gaskets is based on the following contractions $w_{a,b}$:

$$w_{a,b}(x,y) = \left(\frac{x+a}{p}, \frac{y+b}{p} \right) \quad (2)$$

Then we define a set of similarity contractions by imposing the mod-p condition

$$a + b \leq p - 1 \quad (3)$$

which yields a total number of N contractions given by

$$N = \frac{p(p+1)}{2} \quad (4)$$

each with a contraction factor of 1/p, and hence the scaling between mod-p fractal iterations is 1/p. For p=2 the classical Sierpinski gasket is obtained. In Fig.1 the first eight rows of Pascal's triangle with the coefficients which are divisible by p=2 coloured in white are shown. It is observed that the structure corresponds to a two-iteration Sierpinski gasket generated by an IFS. The three-iteration Sierpinski gasket is also presented in Fig. 1.

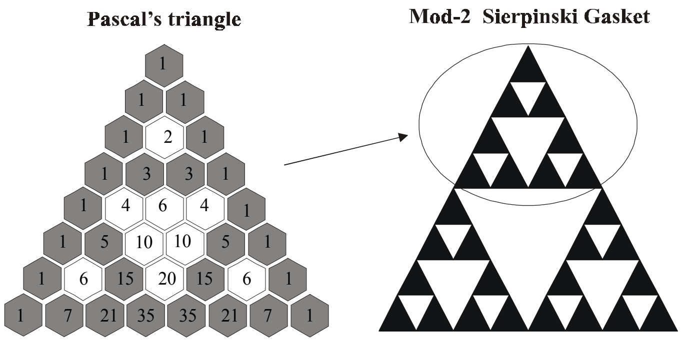

Fig.1. Generation process of the mod-2 Sierpinski gasket through the divisibility of the coefficients in the Pascal's triangle

4 ANTENNAS DESCRIPTION

Two mod-p Sierpinski monopoles, mod-3 and mod-5, were generated using the mentioned IFS scheme, and were printed over a thin dielectric substrate (ε_r=3.38, h=0.8mm), mounted over a 800x800 mm^2 square conductor ground plane and fed using a coaxial probe, Fig.2. It can be observed that the scale factors p are 3 and 5 for mod-3 and mod-5 antennas, respectively. As it occurs with the classical Sierpinski antenna, the triangular-like shape appears at different scales and consequently it is expected that mod-p antennas behave similarly to a triangular antenna but at different bands.

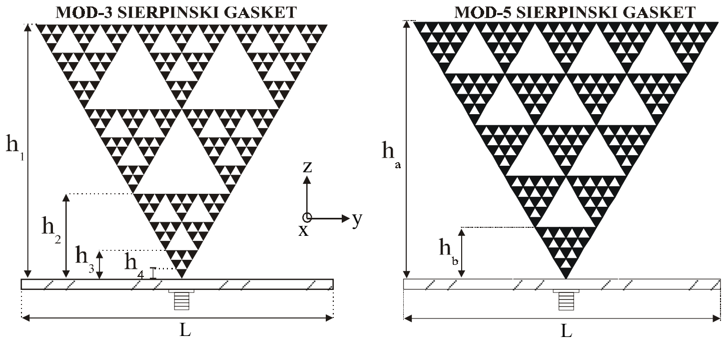

Fig.2. Three-iteration mod-3 and two-iteration mod-5 Sierpinski monopoles

5 EXPERIMENTAL RESULTS

The input parameters of the mod-3 and mod-5 antennas were measured using an HP8510B from 0.2 to 13 GHz. The input reflection coefficient relative to 50 Ω ($\Gamma_{in}^{50\Omega}$) together with the real and imaginary parts (R_{in},X_{in}) of the input impedance is plotted in Fig. 3. From the different coloured zones it can be observed that the resonant frequencies are log-periodically spaced by a factor which matches the scale factor (p). The mod-3 antenna has been built after three iterations, and hence it exhibits three bands. This log-periodic behaviour is also repeated for the mod-5 antenna, where two matched bands are observed. In this case, only two iterations were considered due to technological reasons. The presence of multiple resonant frequencies inside each band is caused by the multiple triangles, which appear between fractal iterations.

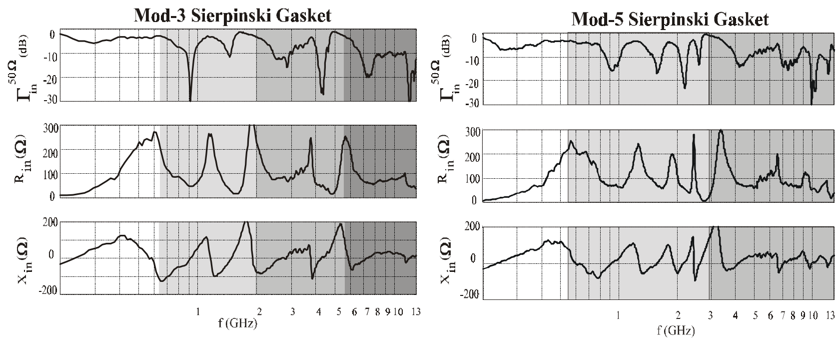

Fig.3. Input reflection coefficient, input resistance and reactance of the two fractal antennas

From the radiation patterns, the main cuts (ϕ=0º, ϕ=90º) of the mod-3 and mod-5 fractal monopoles were measured in an anechoic chamber. For the mod-3 monopole the patterns were measured at the first resonance of the three bands; that are, 0.92, 2.74 and 7.46 GHz. For the mod-5 antenna, the diagrams were measured at the first and last resonances for the two bands: 0.91 and 2.6 GHz for the first and 4.24 and 12.08 for the second. The patterns keep a notable degree of similarity among bands, especially in ϕ=90º plane, Fig.4. It should be noticed the effect of the finite size of the ground plane at the upper bands, where a slight increase of ripples is observed.

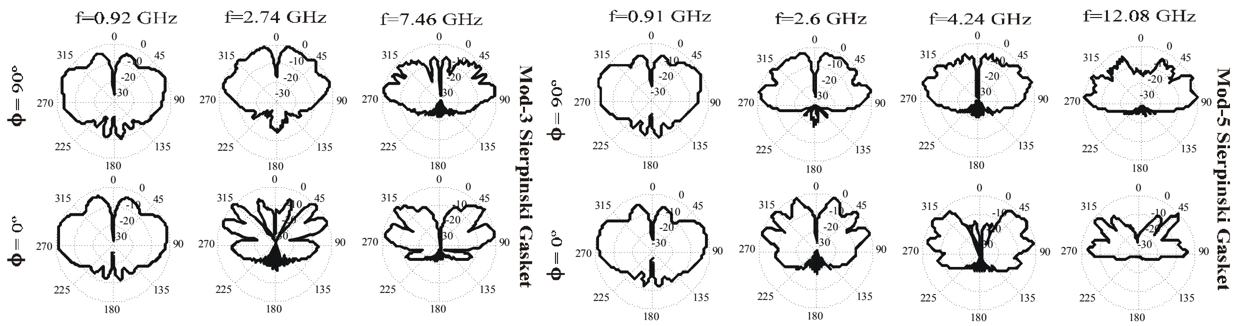

Fig.4. Main cuts (ϕ=0º, ϕ=90º) of radiation patterns for the mod-3 and mod-5 Sierpinski monopoles

6 CONCLUSIONS

A new set of fractal multiband antennas called mod-p Sierpinski gaskets has been presented. The antennas were designed through and IFS scheme. Experimental results show that the spacing between bands is related to the characteristic scale factor of the fractal structure. Besides, the number of iterations of the structure is concerned with the number of bands at which the antenna keeps its log-periodic behaviour. Consequently, mod-p Sierpinski gaskets constitute a new set of fractal multiband antennas where the number of bands and the spacing between them can be adjusted.

7 ACKNOWLEDGEMENTS

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