Plane Wave Scattering from Omega-medium Cylindrical Objects of Arbitrary Cross-section

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Abstract **– This paper presents the analysis of electromagnetic wave scattering from cylindrical objects of arbitrary cross-section made of Omega-medium, pseudochiral material located in a free space. Perpendicular illumination of a plane wave is assumed. The approach is based on the direct field matching technique with the usage of projection of the fields at the boundary on the fixed orthogonal basis functions. The structures can be utilized to shape the antenna radiated field. Several numerical examples are presented.**

Index Terms **— Cylindrical structure, Field matching, Omega medium, Scattering.**

1. Introduction

The properties of chiral and pseudochiral media have gained considerable attention over the last few decades [1]–[7]. The pseudochiral material contains Omega-shaped microstructures in which both the loop and stamps lie in the same plane. The induced electric and magnetic field polarizations in pseudochiral medium are perpendicular to each other. Inserting Omega particles into the medium results in a notable field displacement [2] and phase shift [3]. These phenomena can be utilized e.g. to shape the scattered electromagnetic field. This paper presents the analysis of plane wave scattering from the cylindrical objects located in a free space. The investigated scatterer has an arbitrary cross-section and is made of pseudochiral medium. The main idea of the analysis is based on the direct field matching technique in neighboring regions. The matching is based on the projection of the field at the boundary on the fixed orthogonal basis functions.

2. Formulation of the Problem

The schematic geometry of the investigated structure is presented in Fig. 1. The plane wave illuminates the object perpendicularly to its axis and it is TM^z polarized. The Omega particles in the pseudochiral medium are inserted axially into the object as illustrated in Fig. 1.

Assuming the homogeneity of the field along z axis, the constitutive equations for the investigated medium are defined as follows:

$$\mathbf{D} = \varepsilon_0 \boldsymbol{\varepsilon}_c \mathbf{E} + j\boldsymbol{\Omega}_{z\varphi}\mathbf{B}, \quad \mathbf{B} = \mu_0 \boldsymbol{\mu}_c \mathbf{H} - j\mu_0 \boldsymbol{\mu}_c \boldsymbol{\Omega}_{\varphi z}\mathbf{E}, \qquad (1)$$

where the relative permittivity and permeability have dyadic form: $\boldsymbol{\varepsilon}_c=\varepsilon(\mathbf{i}_\rho\mathbf{i}_\rho+\mathbf{i}_\varphi\mathbf{i}_\varphi)+\varepsilon\mathbf{i}_z\mathbf{i}_z$, $\boldsymbol{\mu}_c=\mu(\mathbf{i}_\rho\mathbf{i}_\rho+\mathbf{i}_z\mathbf{i}_z)+\mu_\varphi\mathbf{i}_\varphi\mathbf{i}_\varphi$, and the coupling dyadics are defined as $\boldsymbol{\Omega}_{z\varphi}=\Omega_c\mathbf{i}_z\mathbf{i}_\varphi$, $\boldsymbol{\Omega}_{\varphi z}=\Omega_c\mathbf{i}_\varphi\mathbf{i}_z$. The parameter Ω_c is a pseudochiral admittance and represents the coupling between electric and magnetic field along z and φ axes.

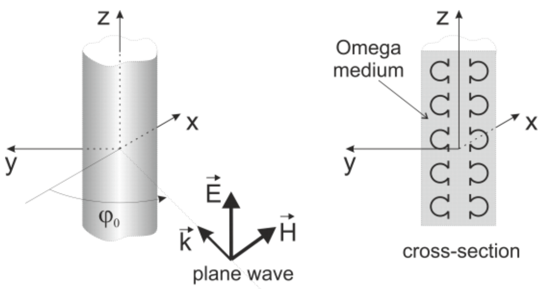

Fig. 1. General configuration of the investigated structure and the orientation of the Omega particles.

From the Maxwell's equation we can derive the differential equation presented in [7]. From the solution of this equation and the solution of the differential equation defined in free space we can formulate the axial components of the electric field inside the Omega-medium and outside the structure as follows:

$$E_z^{\Omega} = \sum_{m=-M}^{M} A_m P_{\upsilon,\alpha}^{J}(k_\rho^{\Omega}\rho)e^{jm\varphi}, \qquad (2)$$

$$E_z^{air} = \sum_{m=-M}^{M}\left(B_m J_m(k_0\rho) + C_m H_m^{(2)}(k_0\rho)\right)e^{jm\varphi}, \qquad (3)$$

where

$$P_{\upsilon,\alpha}^{J}(\xi) = \frac{2^{2\upsilon} j^{\upsilon}}{\Gamma(\upsilon+1)\sqrt{-2j\xi}} M_{\alpha,\upsilon}(-2j\xi), \qquad (4)$$

with $\upsilon=\mathrm{sqrt}(\mu_\varphi/\mu)m$, $k_\rho^{\Omega}=k_0\,\mathrm{sqrt}(\varepsilon_z\mu_\varphi)$, $\alpha=\Omega_c\,\mathrm{sqrt}(\mu_\varphi/\varepsilon_z)\eta_0$, k_0 is a wave number in free space and $\eta_0=\mathrm{sqrt}(\mu_0/\varepsilon_0)$. $M_{\alpha,\upsilon}(\cdot)$ is a Whittaker function of type M, $J_m(\cdot)$ and $H_m^{(2)}(\cdot)$ are Bessel and Hankel functions, respectively, A_m, and C_m are unknown field coefficients and B_m are known coefficient of incident plane wave. The φ and ρ components of the magnetic field in Omega-medium can be calculated from:

$$H_\varphi^{\Omega} = \left(\frac{-j}{k_0\eta_0\mu_\varphi}\frac{\partial}{\partial\rho} + j\Omega_c\right)E_z^{\Omega}, \quad H_\rho^{\Omega} = \frac{j}{k_0\eta_0\mu\rho}\frac{\partial}{\partial\varphi}E_z^{\Omega}. \qquad (5)$$

In order to calculate the scattered field, we need to satisfy the continuity conditions for the tangential field components on the surface of the investigated object: $E_z^{\Omega}=E_z^{air}$, $H_t^{\Omega}=H_t^{air}$ at interface, where subscript t denotes component tangential to the boundary, which can be expressed by the combination of φ and ρ components. Utilizing the projection of the fields at the boundary on the fixed orthogonal basis functions we can rewrite the boundary equations in matrix notation. Next, eliminating the coefficients of the field in Omega-medium we can derive the relation between unknown coefficients of scattered field and known coefficients of incident field:

$$[\mathbf{C}_m] = [\mathbf{T}]\,[\mathbf{B}_m], \qquad (1)$$

where [**T**] is a T-matrix describing the object. Using procedure described in [8] we can calculate scattered field from the object. It is worth noting that only for cylinder with circular cross section matrix [**T**] is diagonal. For any other shape of the cylinder the off-diagonal terms of the matrix appear and it is necessary to perform $(2M+1)\times(2M+1)$ integrations to calculate their values.

3. Numerical Results

Two configurations of the plane wave scattering from pseudochiral cylinder have been investigated. In the first example the plane wave scattering from cylinder with square cross-section is considered. The cylinder has dimensions a=0.3λ and is made of material with parameters ε_z=8, μ=4.6, μ=1 and different Ω_c. The scattering characteristic in the far zone and the electric field distribution inside and in the vicinity of the objects are illustrated in Figs. 2 and 3. It can be seen that the change of the sign of pseudochiral admittance Ω_c significantly alters the scattered field.

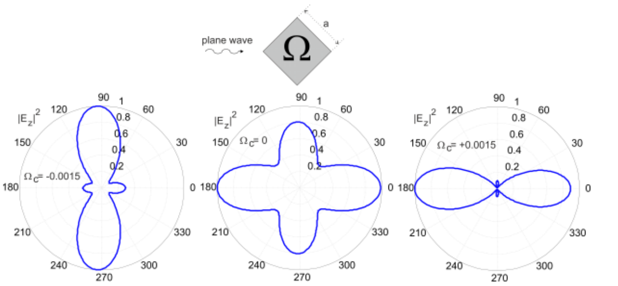

Fig. 2. Normalized amplitude of scattered electric field from the square cylinder of dimensions a=0.3λ for different values of pseudochiral admittance Ω_c.

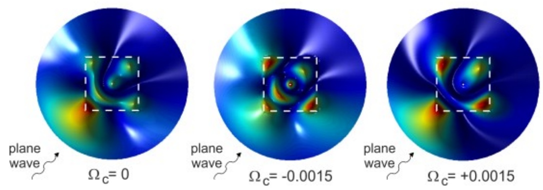

Fig. 3. Distribution of the electric field inside and in the vicinity of the object.

Similar effect of beam shaping is observed for the second example which considers the scattering from triangle cylinder of dimensions a=0.4λ and made of material with parameters ε_z=9, μ_φ=3, μ=1 and different Ω_c. The far field scattering characteristics are illustrated in Fig. 4.

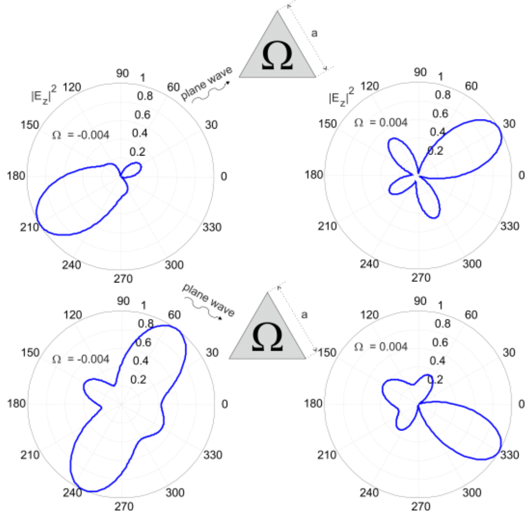

Fig. 4. Normalized amplitude of scattered electric field from the triangle cylinder of dimensions a=0.4λ for different values of pseudochiral admittance Ω_c.

4. Conclusion

The analysis of electromagnetic wave scattering from cylindrical, pseudochiral object of arbitrary cross-section has been presented. The approach is based on the direct field matching technique. The influence of the pseudochirality on the scattering patterns has been shown.

Acknowledgment

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