POS2-9

# Asymmetric dipole antenna current integral equation

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**Abstract:** The approximate analytical formulas of Sommerfeld type integrals are derived from studies on the vector potential generated by vertical electric dipole and Sommerfeld type integrals. According to the correlation between vector potential generated by wire antenna and that generated by vertical electric dipole, the underground asymmetric dipole antenna current equation is derived which avoids calculation on the Sommerfeld type integrals and greatly simplifies the theoretical analysis and computational complexity.

**Keywords:** Asymmetric dipole antenna, Sommerfeld type integrals, current integral equation

## 1.Introduction:

To study properties of widely used wire antennas, the current distribution on the antenna must be clear. Theoretically speaking, Maxwell equations and corresponding boundary conditions can be used to deduce the analytical formula of current distribution on antennas in inhomogeneous medium. However, difficulty is in Sommerfeld type integrals (SI) calculation. Even if the underground space is assumed to be homogeneous medium, generally speaking, it is difficult to directly obtain analytical formula of current distribution from solutions of SI. Numerical methods also face difficulties because of the complexity of the integrand and various choices of the possible integration paths on the complex plane. In addition, such repeat calculation is time-consuming. There are studies trying to truncate the infinite SI [1][2], but SI calculation is unavoidable. Also, series of the complex function is used to approximate the kernel function in spectral domain, which avoids calculating the SI but decreases the speed because of convergence of the series.

In this paper, numerical method and analytical approximation are combined to deduce the current integral equation of underground asymmetric dipole antenna which excludes SI. With the help of method of moment, the results of the new method in this paper are in accord with the results from traditional numerical method.

## 2. Vector potential generated by underground vertical electric dipole

The simplest model in near-earth antenna is homogeneous semi-space model, shown in Figure1.

Assume that the vertical electric dipole $I(z')dz'$ is at z' on z-axis, time dependence is set to $e^{iwt}$. According to classical electromagnetic theories, the field component generated by electric dipoles can be expressed by Hertz electric vector. As for that in Figure 1, the Hertz electric vector only has a z component, which is set to$\boldsymbol{\Pi}_z$ ..$\boldsymbol{\Pi}_{ez}$ and $\boldsymbol{\Pi}_{oz}$ stand for $\boldsymbol{\Pi}_z$ in earth and in air respectively, so they satisfy the following equations:

$$(\nabla^2+k_e^2)\ \boldsymbol{\Pi}_{ez} = -\frac{i_z}{(\sigma_e + i\omega\varepsilon_e)}\ I(z')\ dz'\ \delta(\rho)\delta(z-z') \qquad (1)$$

$$\left(\nabla^2 + k_0^2\right) \boldsymbol{\Pi}_{oz}=0 \qquad (2)$$

where $k_e = \omega[\mu_e(\varepsilon_e - i\sigma_e/\omega)]^{1/2}$ , $k_0 = \omega(\mu_o\varepsilon_o)^{1/2}$ .

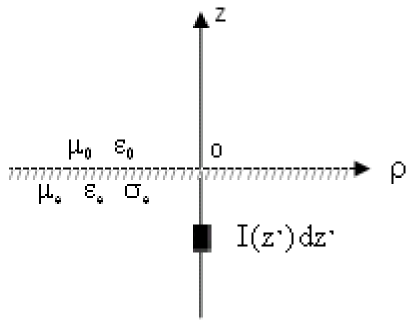

Figure 1 The current element in media

Taking into account the boundary conditions, the tangential component of the electric and magnetic fields generated by electric dipole is continues at the position where z=0. Therefore, $\boldsymbol{\Pi}_{ez}$ can be obtained. Owing to the relationship between vector potential $\boldsymbol{P}_{ez}$ generated by electric dipole in the soil and Hertz vector,, there is

$$\boldsymbol{P}_{ez} = i\omega\mu_e(\varepsilon_e - i\sigma_e/\omega)\ \boldsymbol{\Pi}_{ez} \qquad (3)$$

so the value of vector potential $\boldsymbol{P}_{ez}$ is the potential function $p_{ez}$ that:

$$p_{ez} = \frac{\mu_e}{4\pi} I(z')dz' \int_0^\infty [\frac{1}{r_e} e^{-r_e|z-z'|} + \frac{k_0{}^2 r_e - k_e{}^2 r_0}{r_e(k_0{}^2 r_e + k_e{}^2 r_0)} e^{-r_e(z+z')}]\ J_0(\lambda\rho)\lambda d\lambda \qquad (4)$$

where: $r_e = \sqrt{\lambda^2 - k_e{}^2}$ $\quad r_0 = \sqrt{\lambda^2 - k_0{}^2}$

Set the integral part of (4) as follows:

$$T(z,z') = \int_0^\infty \frac{k_0{}^2 r_e - k_e{}^2 r_0}{r_e(k_0{}^2 r_e + k_e{}^2 r_0)} e^{-r_e(z+z')} J_0(\lambda\rho)\lambda d\lambda \qquad (5)$$

This is a SI which cannot be solved simply by ordinary numerical integration, so it needs valuations. With the properties of cylindrical functions

$$J_0(\lambda\rho) = \frac{1}{2}(H_0^{(1)}(\lambda\rho) + H_0^{(2)}(\lambda\rho)) \qquad H_0^{(1)}(\lambda\rho) = -H_0^{(2)}(-\lambda\rho)$$

(5) can be rewritten as:

$$T(z,z') = \frac{1}{2}\int_{-\infty}^\infty \frac{k_0{}^2 r_e - k_e{}^2 r_0}{r_e(k_0{}^2 r_e + k_e{}^2 r_0)} e^{-r_e(z+z')} H_0^{(2)}(\lambda\rho)\lambda d\lambda \qquad (6)$$

As many practical antennas are close to the ground, working at the frequency band of a few Hz to tens of Hz, we can assume that the ground is still the near-field region in such low frequency band, even if the radiation source is set several kilometers beneath the ground source, where $k_e\rho << 1$,and $k_e >> k_0$ . Hence, the Hankel function in (6) has a pair of fulcrum where $\lambda$=0, and another pair of fulcrum where $\lambda = \pm k_o$,, $\lambda = \pm k_e$ .According to the complex function theory, if a number of slots are made in the complex plane, the integrand function will have no polar point and be a single-valued analytic function.[5] Quasi-static method approximates (6) by [6]: $k_0 = 0$.

$$T(z,z') = \int_0^\infty \frac{1}{r_e} e^{-r_e(z+z')} J_0(\lambda\rho)\lambda d\lambda \qquad (7)$$

Put (7) into (4), using the identity mentioned above,

$$\int_0^{\infty} \frac{1}{r_e} e^{-ir_e|z-z'|} J_0(\lambda\rho)\lambda d\lambda = \frac{e^{-ikeR}}{R} \tag{8}$$

Then the approximate analytical expression of the vector potential values of underground vertical electric dipole is obtained as

$$p_{ez} = \frac{\mu_e}{4\pi}\left(\frac{e^{-ikeR}}{R} - \frac{e^{-ikeR'}}{R'}\right) I(z')dz' \tag{9}$$

where $R=\sqrt{\rho^2+(z-z')^2}$ $R'=\sqrt{\rho^2+(z+z')^2}$

## 3. Current integral equation of underground asymmetric dipole antenna

Suppose that the asymmetric antenna is vertically buried, radius a is much smaller than its length, there is an insulating gap H meters underground. The source $V_0$, in the gap, powers the two side feed. The antenna consists of the two parts located on the top and bottom of the feed point shown in Figure 2. The current electric vector on the antenna is $\boldsymbol{P_{ez}(a,z)}$ ,and z component is the potential function $p_{ez}(a,z)$.

At the point where ρ= a, electric field Eez = - Voδ(z).where, δ(z)is Dirac δ function. Hence there is:

$$\left(\frac{\partial^2}{\partial z^2} + k_z^2\right) p_{ez}(a,z) = 0 \quad (-h << z << H, z \neq 0) \tag{10}$$

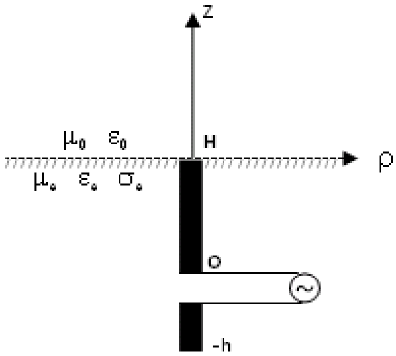

Figure 2: The model of underground asymmetric dipole antenna

The general solution of second order linear ordinary differential homogeneous equation (10) is

$$p_{ez}(a,z) = \begin{cases} i\frac{k_e}{\omega}(C_1 \cos k_e z + C_2 \sin k_e z) & 0 \langle z \leq H \\ i\frac{k_e}{\omega}(C_3 \cos k_e z + C_4 \sin k_e z) & -h \leq z \langle 0 \end{cases} \tag{11}$$

where Ci(i=1,2,3,4)is constant. With Lorentz condition where z=0: $C_2$-$C_4$=$V_0$ . Because of the continuity of the vector potential: $C_1$=$C_3$ . Set the origin at the feed point. From (9) we have:

$$p_{ez} == \frac{\mu_e}{4\pi} I(z')G(z,z')\, dz' \tag{12}$$

$$\text{where } G_e(z,z') = \frac{e^{-ike\sqrt{a^2+(z-z')^2}}}{\sqrt{a^2+(z-z')^2}} - \frac{e^{-ike\sqrt{a^2+(z+z'-2H)^2}}}{\sqrt{a^2+(z+z'-2H)^2}} \tag{13}$$

The vector potential generated by entire current distributed on the antenna is equivalent to the superimposed vector generated by the electric dipole. Therefore, the integration values of (12) equal the right side of (11). Moreover, when taking into account G(H, z')=0 we can deduce that:

$$\int_{-h}^{H} I(z')\left[G(z,z') - G(-h,z')\frac{\sin k_e(H-z)}{\sin k_e(H+h)}\right]dz' = \begin{cases} \frac{i4\pi k_e}{\varpi\mu_e} V_0 \sin k_e h \frac{\sin k_e(H-z)}{\sin k_e(H+h)} & 0 \langle z \leq H \\ \frac{i4\pi k_e}{\varpi\mu_e} V_0 \sin k_e H \frac{\sin k_e(h+z)}{\sin k_e(H+h)} & -h \leq z \langle 0 \end{cases} \tag{14}$$

This is the underground asymmetric dipole antenna's current integral equation.

## 4. Calculation and conclusion

The moment method is used to solve the current integral equation, and when we set driving voltage $V_0$=1 V the properties of the issues discussed in this case is still universal. Figure 3 shows the comparison of the results from this paper and the numerical results using the method mentioned in the paper [2]. The two curves are in good agreements, confirming the reliability of the theoretical derivation and the calculating method in this paper. Besides, Figure 4 gives the current distribution on the antenna with typical engineering parameters by the current integral equation derived in this paper. From the curves we can see the current distribution on the antenna decreases if the conductivity decreases, which is consistent with the actual situation.

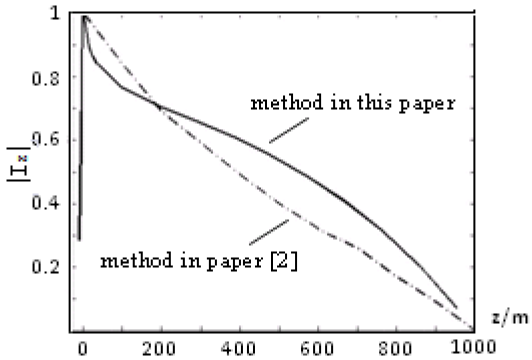

Figure 3:The comparsison of the current distribution
f=20Hz, a=0.1m, H=1000m, h=10m

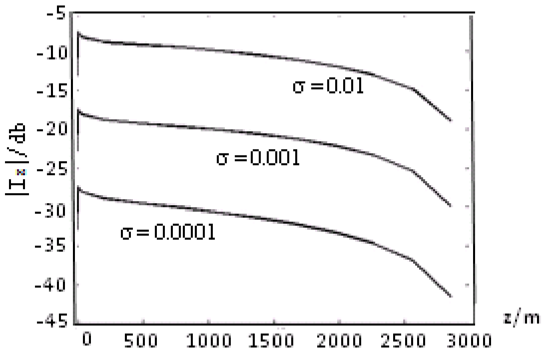

Figure 4: The current distribution with different σ
f=10Hz, a=0.1m, H=3000m, h=10m

In this paper, the underground asymmetric dipole antenna current integral equation is deduced to avoid the SI calculus, which greatly simplifies the theoretical analysis and computational complexity and better meet the needs of practical engineering. This is a method of great engineering value.

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