Analysis of Magnetic Photonic Crystals Using Complex Envelope ADI-FDTD Method

Sang-Gyu Ha, Jeahoon Cho, Minseok Park, Jaewoo Baek and Kyung-Young Jung
Dept. of Electronic Computer and Engineering, Hanyang University, Seoul Korea
kyjung3@hanyang.ac.kr

***Abstract* – We develop the complex envelop alternating-direction-implicit finite-difference time-domain (CE-ADI-FDTD) suitable for analysis of magnetic photonic crystals (MPhCs) which require highly refined grids. MPhCs are periodic structure with unit cell composed of two misaligned anisotropic dielectric layers and one ferromagnetic layer. Numerical results show that the proposed CE-ADI-FDTD results agree very well with conventional FDTD results at various Courant number values.**

***Index Terms* — Complex-envelop-alternating-direction-implicit finite-difference time-domain (CE-ADI-FDTD), anisotropic dielectrics, magnetic photonic crystals (MPhCs).**

1. Introduction

The alternating-direction-implicit finite-difference time-domain (ADI-FDTD) method is unconditionally stable algorithm regardless of Courant-limit time step size Δt [1] and is suitable for analysis of electromagnetic fields in highly refined grids compared to the wavelength of the frequency of interest. However, numerical dispersion error of ADI-FDTD increase rapidly as Δt increase. In general, Δt is limited by accuracy consideration. In this work, we have proposed the complex envelope (CE) ADI-FDTD method [2], [3], which is useful for analysis of magnetic photonic crystals (MPhCs) with froze mode [4], to improve the numerical dispersion error of the ADI-FDTD. The CE-ADI-FDTD incorporates an analytical expression of the carrier frequency by means of modified field update equations and uses a finite-difference approximation of complex field envelops instead of the actual fields. The proposed method can be useful for efficient transient analysis of MPhCs.

2. Formulation

We assume an $e^{j\omega t}$ time dependence in what follows. We consider $\mathbf{F} = \mathrm{Re}[\tilde{\mathbf{F}} \cdot e^{j\omega_c t}]$, where ω_c is the carrier frequency, $\tilde{\mathbf{F}}$ and $\mathbf{F}$ indicate the complex envelope and the actual field, respectively. Therefore, complex envelope (CE) Maxwell's curl equations can be expressed as [3]

$$\nabla \times \tilde{\mathbf{H}} = \frac{\partial \tilde{\mathbf{D}}}{\partial t} + j\omega_c \tilde{\mathbf{D}}$$
$$\nabla \times \tilde{\mathbf{E}} = \frac{\partial \tilde{\mathbf{B}}}{\partial t} + j\omega_c \tilde{\mathbf{B}} \qquad (1)$$

In order to obtain the corresponding time-domain equations, we apply an inverse Fourier transform and then express the resulting equations in terms of complex envelopes.

For MPhCs, $\tilde{\mathbf{D}}$ is $\bar{\varepsilon}_A(\omega)\tilde{\mathbf{E}}(\omega)$ and $\tilde{\mathbf{B}}$ is $\bar{\mu}_F(\omega)\tilde{\mathbf{H}}(\omega)$. The constitutive tensor can be expressed as

$$\bar{\varepsilon}_A(\omega) = \varepsilon_o \begin{bmatrix} \varepsilon_A + \delta_A \cos 2\phi_A & \delta_A \sin 2\phi_A & 0 \\ \delta_A \sin 2\phi_A & \varepsilon_A - \delta_A \cos 2\phi_A & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix} \qquad (2)$$

$$\bar{\mu}_F(\omega) = \mu_o \begin{bmatrix} 1+\chi_{xx}(\omega) & \chi_{xy}(\omega) & 0 \\ \chi_{yx}(\omega) & 1+\chi_{yy}(\omega) & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (3)$$

with $\bar{\mu}_A(\omega) = \mu_o \mu_r \bar{I}$ and $\bar{\varepsilon}_F(\omega) = \varepsilon_o \varepsilon_r \bar{I}$. More details on the parameters can be found in [4].

3. Numerical examples

Periodic structure with anisotropic materials can produce unique electromagnetic wave propagation such as frozen modes, pulse compression, and unidirectionality. The best way to yield a MPhCs is a periodic structure whose unit cell is composed of two misaligned anisotropic dielectric layers (*A*-layers) and one ferromagnetic layer (*F*-layer) as shown in Fig. 1. the dc biasing magnetic field is -z direction in the *F* layers.

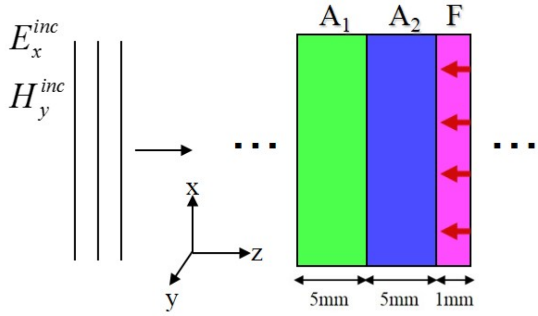

Fig. 1. Geometry of spectrally asymmetric MPhCs .

To validate the CE-ADI-FDTD algorithm, CE-ADI-FDTD results are compared with FDTD results. For *A* layers, the constitutive tensor parameters are $\varepsilon_{A1} = \varepsilon_{A2} = 7$; $\delta_{A1} = \delta_{A2} = 6$; $\phi_{A1} = 0°$, $\phi_{A2} = 36.0963°$; $\varepsilon_{zz,A_1} = \varepsilon_{zz,A_2} = 0$; $\mu_{r,A_1} = \mu_{r,A_2} = 1$. For *F* layers, the constitutive tensor parameters are given by $\varepsilon_r = 5$; $\omega_0 = 36.503 \times 10^9$ rad/s; $\omega_m = 73.006 \times 10^9$ rad/s. The excitation is a sine wave modulated Gaussian pulse with 0.01% half-power fractional bandwidth. Fig. 2 shows the time response of E_x which represents both the actual field for FDTD and the field envelope for CE-ADI-FDTD. Good agreement is shown between FDTD method and CE-ADI-FDTD method at various Courant number (CN) values in Fig. 2. At CN = 50, CPU time consumed by the CE-ADI-FDTD method is 75% of CPU time consumed by the FDTD method and the root mean square error is 0.86%.

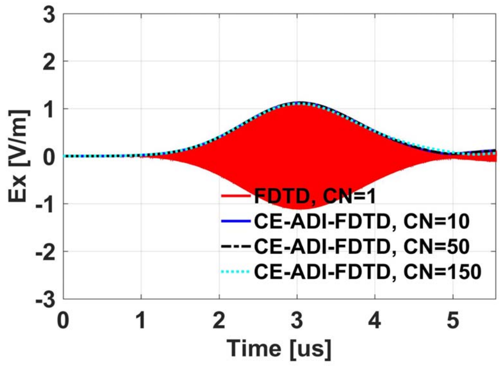

Fig. 2. Time response of E_x.

4. Conclusion

We have developed the CE-ADI-FDTD algorithm suitable for analysis of MPhCs. The CE-ADI-FDTD method is unconditionally stable algorithm regardless of the time step size and/or the maximum frequency. Numerical examples are used to illustrate the computational accuracy of CE-ADI-FDTD method compared to conventional FDTD method. It is shown that the CE-ADI-FDTD is significantly suited for narrowband problems requiring highly refined grids.

Acknowledgment

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