Noise Robust Time of Arrival Estimation Method Using Hierarchical Bayesian Based Compressed Sensing Algorithm

Akira Moro, Fang Shang, Shouhei Kidera, and Tetsuo Kirimoto
Graduate School of Informatics and Engineering, The University of Electro-Communications, Japan

Abstract **- A microwave radar system is a useful tool for all-weather type remote sensing, such as terrain surface measurement. It is well known fact that a resolution of time-of-arrival (TOA) is strictly determined by the frequency bandwidth of transmitted signal. As super-resolution technique beyond such limitation, a compressed sensing (CS) algorithm has come under spotlight because a sparse assumption is well established in typical radar situations. However, the original CS method suffers from lower TOA resolution in insufficient SNR level. To address with such problem, this paper introduces a hierarchical Bayesian based CS algorithm. This method introduces the stochastic model derived from cross-correlation response as *a priori* information for CS reconstruction as hyper-prior distribution. The results of numerical simulation show that the proposed method enhances an accuracy for signal reconstruction, even in lower SNR situations.**

Index Terms **— Radar signal processing, Compressed sensing, TOA estimation, hierarchical Bayesian model.**

1. Introduction

Since a microwave radar system is one of the most useful tools for terrain surface observations being applicable to all-weather situations. It is promising for monitoring landslides after cataracts of rain or volcanic activity, where the optical sensor is hardly applicable [1]. Assuming a propagation media as an air, ranging problem is converted to time of arrival (TOA) estimation issue. On the other hand, the frequency bandwidth of transmitted signal is limited due to the legal regulations for radio wave emission, which causes ambiguity response in applying the cross-correlation based TOA estimation. To achieve more accurate and higher range resolution, promising algorithms have been proposed, known as Capon or MUSIC (MUltiple SIgnal Classification) methods.

As a promising approach, a number of sparse regularization approaches have been developed, often called as compressed sensing [2]. This type of method requires a simple assumption for signal reconstruction, that spatial or temporal distribution of targets should be sparse, compared with total sampled area. Such assumption is realized in general TOA estimation issues, where the number of targets is considerably lower than that of range bin. The literature [3] has reported that l_1-norm minimization based CS estimation was converted Bayesian estimation, that is, it is reformulated as probabilistic model with relevance vector machine solution. The recent literature [4] indicated the other advantages for determining significant parameters, such as an estimation for noise and target precision values in calculating hyper-priori distribution by using hierarchical Bayesian estimation. These estimated parameters are useful to evaluate the performance of estimation method.

According to this background, this paper introduces the hierarchical Bayesian model into CS method to enhance TOA accuracy in lower SNR situation. This paper incorporates a cross-correlation output into sparse regularization through hyper-prior distribution in estimating maximum posterior probability. The results from numerical simulations show that the range resolution and reconstruction accuracy of our proposed method are considerably enhanced compared to those obtained by the conventional CS method even in significantly lower SNR situation.

2. System Model

It assumes that a monostatic radar and single observation. Received N sampled data as $\boldsymbol{y} \in C^{N\times 1}$ is expressed as

$$\boldsymbol{y} = \boldsymbol{S}\boldsymbol{r} + \boldsymbol{n}, \quad (1)$$

where $\boldsymbol{r} \in C^{M\times 1}$ denotes the temporal target distribution. $\boldsymbol{S}$ denotes the observation matrix, which is often expressed by the time shifted sequences of transmitted signal.

3. Proposed Method

It is well established fact in radar observation model that the number of total range bins is much greater than that of actual target distribution. Namely, sparseness assumption is suitable for signal reconstruction in the ranging problem. This CS based optimization problem is formulated as the l_1-norm minimization. To enhance noise-robustness of such kind of method, this paper introduces a hierarchical Bayesian based CS algorithm [4], where the response from cross-correlation is introduced as hyper-prior information. Based on Bayes' theorem the joint posterior probability is expressed as;

$$P(\boldsymbol{r}, \boldsymbol{\alpha}|\boldsymbol{y}) = (P(\boldsymbol{y}|\boldsymbol{r})P(\boldsymbol{r}|\boldsymbol{\alpha})P(\boldsymbol{\alpha})) / P(\boldsymbol{y}). \quad (2)$$

Here $P(\boldsymbol{y}|\boldsymbol{r})$ is expressed;

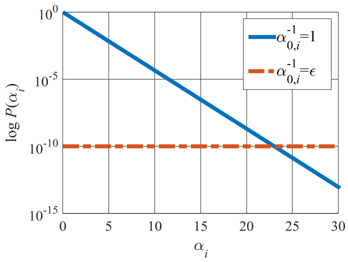

Fig. 1. Gamma distribution (in the case of changing hyper parameter $\alpha_{0,i}^{-1}$ and $\gamma_{0,i} = 1$ holds).

$$P(\boldsymbol{y}|\boldsymbol{r}) = (\beta/2\pi)^{N/2} \exp(-\beta/2\pi\|\boldsymbol{y} - \boldsymbol{S}\boldsymbol{r}\|^2), \quad (3)$$

where β denotes the noise precision parameter (inverse variance). $P(\boldsymbol{r}|\boldsymbol{\alpha})$ is defined as a zero-mean Gaussian prior on each element of $\boldsymbol{r}$

$$P(\boldsymbol{r}|\boldsymbol{\alpha}) = \prod^{N}_{i=1}\mathcal{N}(r_i\,;\,0,\,\alpha_i^{-1}), \quad (4)$$

where $\mathcal{N}(r_i;0,\alpha_i^{-1})$ denotes a zero-mean Gaussian distribution with $\boldsymbol{\alpha} = [\alpha_1,\ldots,\alpha_N]$ and i denotes the index number for time. Probability distribution in Eq. (4) is called as ARD (Automatic Relevance Determination) prior distribution, where most elements of $\boldsymbol{r}$ become zero. Also, $P(\boldsymbol{\alpha})$ is regarded as a hyper-prior distribution as;

$$P(\boldsymbol{\alpha}) = \prod^{N}_{i=1}\Gamma(\alpha_i\,;\,\alpha_{0,i},\,\gamma_{0,i}), \quad (5)$$

where $P(\boldsymbol{\alpha})$ denotes the multi-finite dimensional distribution for the discrete time series as $\boldsymbol{\alpha}$. $\Gamma(\alpha_i\,;\,\alpha_{0,i},\,\gamma_{0,i})$ denotes a Gamma distribution and $\gamma_{0,i}$ denotes a shape parameter which controls the spread of a Gamma distribution, and reflects the reliability of the prior knowledge derived from output of the cross-correlation. For determination of hyper parameter $\alpha_{0,i}$ in Eq. (5), the output of cross-correlation as s^{M}_i is employed as

$$\alpha_{0,i}^{-1} = \begin{cases} 1 & (|i - I_{\max}| \le I_{\mathrm{th}}), \\ \varepsilon & (\text{otherwise}) \end{cases} \quad (6)$$

where $I_{\max} = \arg\max_i |s^{\mathrm{M}}_i|$ and I_{th} is the threshold value adjusted by considering a theoretical time resolution and sampling interval. In addition, $\varepsilon \ll 1$ holds. If $\alpha_{0,i}^{-1}$ is close to 1, it denotes that a target response is around the index i. Figure 1 shows the hyper prior distribution in Eq. (5) in the case of hyper parameter $\alpha_{0,i}^{-1} = 1$ and $\alpha_{0,i}^{-1} = \varepsilon$. This figure indicates our proposed method can localize the target range by introducing the response from cross-correlation into $\alpha_{0,i}^{-1} = 1$. This is because the α_i tends to take small value in the case of $\alpha_{0,i}^{-1} = 1$ compared with $\alpha_{0,i}^{-1} = \varepsilon$. Then, noise-robustness for TOA estimation should be further improved. For maximizing posterior distribution in Eq. (2), we employs variational Bayesian algorithm to reconstruct $\boldsymbol{r}$ [5].

4. Performance Evaluation in Numerical Simulation

This section describes the results of performance evaluations for each method in numerical simulations. The minimum and maximum frequencies are fixed as $f_{\min} = 8.9$GHz and $f_{\max} = 9.3$GHz, respectively. Assuming actual radar signal band-limited due to legal regulation, it assumes the band divided transmitting signal. The effective bandwidth is 30 MHz, in this case. The sampling frequency f_s is 1GHz. The number of targets is 2 and the interval of two targets becomes $0.8\Delta t$, Δt denotes a theoretical time resolution. $I_{\mathrm{th}} = 2$ and $\varepsilon = 1\times10^{-10}$ are set in this case. Moreover, we set a shape parameter $\gamma_{0,i} = 1$. Figure 2 shows the TOA estimation results of the conventional and our proposed methods at the case of SNR = 30dB. This figure demonstrates that our proposed method enables us to decompose the two targets responses with high accuracy located within range bins.

As statistical viewpoint, where the noise pattern is changed 50 times for each SNR, Fig. 3 shows that the RMSE for each method, and it shows that our proposed method considerably reduces the RMSE by the original CS in any SNR cases.

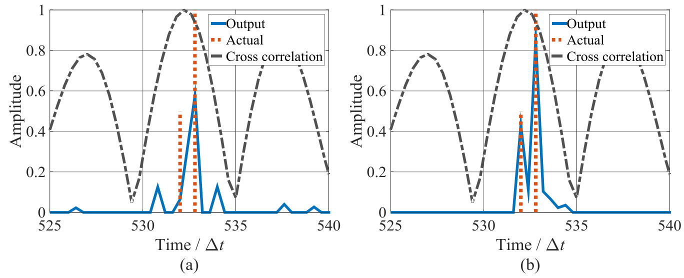

Fig. 2. Reconstruction examples ((a) Conventional (b) Proposed) at S/N = 20dB.

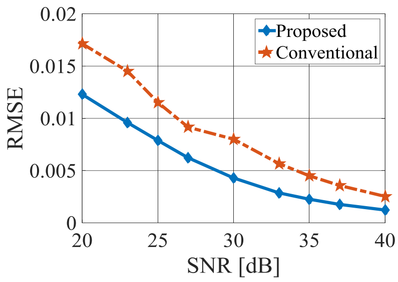

Fig. 3. RMSE for target reconstruction versus SNR.

5. Conclusion

This paper proposed hierarchical Bayesian based CS algorithm for achieving noise-robust TOA estimation in radar application. The results of the numerical simulations demonstrate the effectiveness of our method, and the experimental validation is our future work.

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