

An Iterative CFO Estimator for QO-STBC Uplink MC-CDMA MIMO Systems

Jian-Shiuan Lin, # Tsui-Tsai Lin

Department of Electronics Engineering, National United University
 No. 1 Lien-Da, Kung-Ching Li, Miaoli 360, Taiwan, ttlin@nuu.edu.tw

1. Introduction

The combination of multiple-input multiple-output (MIMO) and multicarrier code-division multiple-access (MC-CDMA) is a more powerful solution of the physical layer for high-speed wireless communications [1]. However, such a hybrid scheme suffers from the unfavorable effects inherited from MC-CDMA systems. For instance, carrier frequency offset (CFO) [2] is commonly caused either by the mismatch of local oscillator frequency between the transmitter and receiver or the Doppler shift resulting from a user's mobility. Inevitably, CFO destroys the orthogonality of subcarriers and leads to system performance degradation if no countermeasure is adopted.

In the present paper, we propose an iterative blind CFO estimator for the hybrid quasi-orthogonal space-time block coding (QO-STBC) [3] MC-CDMA system with four transmit antennas. The design of the proposed scheme started with the development of a multiply constrained minimum output energy (MCMOE)-based blind detector, in which a set of correlators is constructed to collect each multipath signal from the user-of-interest and suppress multiple access interference (MAI). The CFO estimate can be determined by minimizing the output power of the MCMOE detector, which approximates to a quadratic function in terms of CFO by using the first-order Taylor series expansion. To reduce the computational complexity, we find the solution to the quadratic function as the CFO estimate, instead of searching the peak location of the output power spectrum of the MCMOE detector. Finally, further performance enhancement is achieved by using an iterative scheme in which the CFO-compensated data is sent to the MCMOE detector as the input of the next iteration. Numerical results have been conducted to verify that the proposed blind estimator can achieve the precise CFO estimation with a few iterations.

2. Proposed Blind Iterative CFO Estimator

In this section, we propose a blind CFO estimator, for QO-STBC MIMO systems to alleviate performance degradation due to CFO.

2.1 Signal Model

Consider an N -subcarrier MC-CDMA system employing QO-STBC [4], where four transmit antennas (TXs) and N_r receive antennas (RXs) are deployed for the transmission over a multipath fading channel with a length of L . Let the modulated signal of user k be spread with a unique spreading code $\mathbf{c}_k^{(p)}$ of length N for the p th TX. The cyclic prefix (CP) of length N_g is inserted to reduce the inter-symbol interference (ISI), where N_g is not less than the channel order L . Suppose that CFO is present at the receiver end. Following the CP removal and FFT operation, the $NN_r \times 1$ received array data vector at the i th time slot is given by

$$\begin{aligned} \mathbf{y}(i) &= \sum_{k=1}^K \sigma_k \phi^i(\varepsilon_k) \underbrace{\mathbf{I}_{N_r} \otimes \mathbf{S}(\varepsilon_k)}_{\Phi(\varepsilon_k)} \underbrace{[\mathbf{I}_{N_r} \otimes \mathbf{C}_k^{(1)} \mathbf{h}_k^{(1)}, \dots, \mathbf{I}_{N_r} \otimes \mathbf{C}_k^{(4)} \mathbf{h}_k^{(4)}]}_{\mathbf{\Gamma}_k^{(1)}} \mathbf{b}_k(i) + \mathbf{v}(i) \\ &= \sum_{k=1}^K \sigma_k \phi^i(\varepsilon_k) \underbrace{\Phi(\varepsilon_k)}_{\mathbf{\Gamma}_k} \underbrace{[\mathbf{\Gamma}_k^{(1)}, \mathbf{\Gamma}_k^{(2)}, \mathbf{\Gamma}_k^{(3)}, \mathbf{\Gamma}_k^{(4)}]}_{\mathbf{H}_k} \underbrace{\text{diag}\{\mathbf{h}_k^{(1)}, \mathbf{h}_k^{(2)}, \mathbf{h}_k^{(3)}, \mathbf{h}_k^{(4)}\}}_{\mathbf{H}_k} \mathbf{b}_k(i) + \mathbf{v}(i) \end{aligned} \quad (1)$$

where $\mathbf{C}_k^{(p)} = \text{diag}\{\mathbf{c}_k^{(p)}\mathbf{F}(:,1:L)\}$ and K is the number of users. σ_k^2 is the signal power with the parameter k being the user index. $\mathbf{b}_k(i) = [b_k^{(1)}(i), b_k^{(2)}(i), b_k^{(3)}(i), b_k^{(4)}(i)]^T$ and $b_k^{(p)}(i)$ is the transmitted symbol through the p th TX at the i th time slot which is assumed to be independent and identically distributed (i.i.d.) with zero mean and unit variance. The transmitted data and zero-mean complex white Gaussian noise $\mathbf{v}(i)$ with power $\sigma_n^2 \mathbf{I}_{NN_r}$ are assumed to be mutually independent. The $N \times N$ matrix \mathbf{F} represents the discrete Fourier transform (DFT) matrix. The $LN_r \times 1$ vector $\mathbf{h}_k^{(p)}$, denotes the channel response for the path from the p th TX to RXs. We assume that the channel parameters keep constant within the time span used to calculate the time-averaged correlation matrix, but vary from one time span to another. $\phi(\varepsilon_k) = e^{j2\pi\varepsilon_k(N+N_g)/N}$ is defined as the phase difference between two consecutive symbols where ε_k is the corresponding frequency offset normalized with respect to the subcarrier spacing. The impact of frequency offset in (1) among the subcarriers can be expressed as a Toeplitz unitary matrix $\mathbf{S}(\varepsilon_k) = \mathbf{F}\mathbf{E}(\varepsilon_k)\mathbf{F}^H$, where $\mathbf{E}(\varepsilon_k) = \text{diag}\{1, e^{j2\pi\varepsilon_k/N}, \dots, e^{j2(n-1)\pi\varepsilon_k/N}\}$.

2.2 Proposed Iterative Blind CFO Estimation

In [4], the min/max approach has been employed to estimate CFO over an AWGN channel according to the principle of MAI suppression for an MOE receiver. An alternative for multipath fading channels is to incorporate multiple constraints with the to-be-determined CFO by following the MOE optimization approach, which decouples each multipath signal introduced by the use-of-interest from the others so as to avoid a mutual cancellation. Specifically, each multipath signal is retained with a unit gain from the received data, and the other signals are rejected. Using the intrinsic structure of the QO-STBC system and applying the linear constraints to the minimization of the MOE receiver, the constrained problem leads to

$$\min_{\mathbf{w}_{1,l}} \sum_{m=0}^3 E \left\{ |\mathbf{w}_{1,l}^H \mathbf{y}(4i+m)|^2 \right\} \equiv \mathbf{w}_{1,l}^H \mathbf{R} \mathbf{w}_{1,l}, \quad \text{subject to: } \mathbf{w}_{1,l}^H \mathbf{\Phi}(\varepsilon) \mathbf{\Gamma}_1 = \mathbf{e}_l^T, l=1,2,\dots,4LN_r, \quad (2)$$

where $\mathbf{R} = \sum_{m=0}^3 E \left\{ \mathbf{y}(4i+m) \mathbf{y}^H(4i+m) \right\}$, and \mathbf{e}_l is the l th column of \mathbf{I}_{4LN_r} . The solution to (2) is given by $\mathbf{w}_{1,l} = \mathbf{R}^{-1} \mathbf{\Phi}(\varepsilon) \mathbf{\Gamma}_1 [\mathbf{\Gamma}_1^H \mathbf{\Phi}^H(\varepsilon) \mathbf{R}^{-1} \mathbf{\Phi}(\varepsilon) \mathbf{\Gamma}_1]^{-1} \mathbf{e}_l$ for $l=1,2,\dots,4LN_r$. The total output power of the MCMOE detector is derived by

$$P(\varepsilon) = \sum_{l=1}^{4LN_r} \sum_{m=0}^3 E \left\{ |\mathbf{w}_{1,l}^H \mathbf{y}(4i+m)|^2 \right\} = \text{tr} \left\{ [\mathbf{\Gamma}_1^H \mathbf{\Phi}^H(\varepsilon) \mathbf{R}^{-1} \mathbf{\Phi}(\varepsilon) \mathbf{\Gamma}_1]^{-1} \right\}. \quad (3)$$

The output power in (3) decreases as the CFO values increase. Thus, the CFO estimate can be estimated by finding the frequency at which the maximum output power is reached:

$$\hat{\varepsilon}_1 = \arg \min_{\varepsilon} P(\varepsilon) = \arg \min_{\varepsilon} \text{tr} \left\{ [\mathbf{\Gamma}_1^H \mathbf{\Phi}^H(\varepsilon) \mathbf{R}^{-1} \mathbf{\Phi}(\varepsilon) \mathbf{\Gamma}_1]^{-1} \right\}. \quad (4)$$

The computational complexity of this approach is inversely proportional to the value of searching granularity. Consequently, there is a trade-off between accuracy and computational complexity for the above-mentioned CFO estimation. To guarantee acceptable performance in the CFO estimation, the granularity has to be chosen small enough. Unfortunately, this brings extreme computational complexity and may impede practical application.

To reduce computational complexity in the proposed estimation approach, we introduce an efficient method, which performs the Taylor series expansion on the CFO-driven matrix $\mathbf{S}(\varepsilon)$. For small ε , $\mathbf{S}(\varepsilon)$ approximates to the first-order Taylor series expansion:

$$\mathbf{S}(\varepsilon) \approx \mathbf{I}_N + \varepsilon \frac{d}{d\varepsilon} \mathbf{S}(\varepsilon) = \mathbf{I}_N + \varepsilon \dot{\mathbf{S}}_0, \quad (5)$$

Such that we have $\Phi(\varepsilon) \approx \mathbf{I}_{NN_r} + \varepsilon(\mathbf{I}_{N_r} \otimes \dot{\mathbf{S}}_0)$, where $\dot{\mathbf{S}}_0 = \left. \frac{d}{d\varepsilon} \mathbf{S}(\varepsilon) \right|_{\varepsilon=0}$. The output power in (3) can be rewritten by

$$\begin{aligned} P(\varepsilon) &\approx \text{tr} \left\{ \left\{ \mathbf{\Gamma}_1^H [\mathbf{I}_{NN_r} + \varepsilon(\mathbf{I}_{N_r} \otimes \dot{\mathbf{S}}_0)]^H \mathbf{R}^{-1} [\mathbf{I}_{NN_r} + \varepsilon(\mathbf{I}_{N_r} \otimes \dot{\mathbf{S}}_0)] \mathbf{\Gamma}_1 \right\}^{-1} \right\} \\ &= \text{tr} \{ \mathbf{T} \} - \text{tr} \left\{ \mathbf{T} \mathbf{\Gamma}_1^H (\mathbf{R}^{-1} \dot{\mathbf{S}}_0 + \dot{\mathbf{S}}_0^H \mathbf{R}^{-1}) \mathbf{\Gamma}_1 \mathbf{T} \right\} \varepsilon - \text{tr} \left\{ \mathbf{T} \mathbf{\Gamma}_1^H \dot{\mathbf{S}}_0^H \mathbf{R}^{-1} \dot{\mathbf{S}}_0 \mathbf{\Gamma}_1 \mathbf{T} \right\} \varepsilon^2, \end{aligned} \quad (6)$$

where $\mathbf{T} = (\mathbf{\Gamma}_1^H \mathbf{R}^{-1} \mathbf{\Gamma}_1)^{-1}$. We use the first-order Taylor expansion series to approximate the inversion of matrix. The output power in (6) reveals a simple 1-D optimization problem in searching the maximum output power. The optimal selection of $\hat{\varepsilon}_1$ is derived by

$$\hat{\varepsilon}_1 = - \frac{\text{tr} \left\{ \mathbf{T} \mathbf{\Gamma}_1^H (\mathbf{R}^{-1} \dot{\mathbf{S}}_0 + \dot{\mathbf{S}}_0^H \mathbf{R}^{-1}) \mathbf{\Gamma}_1 \mathbf{T} \right\}}{2 \times \text{tr} \left\{ \mathbf{T} \mathbf{\Gamma}_1^H \dot{\mathbf{S}}_0^H \mathbf{R}^{-1} \dot{\mathbf{S}}_0 \mathbf{\Gamma}_1 \mathbf{T} \right\}} = - \frac{\text{Re} \left\{ \text{tr} \left\{ \mathbf{T} \mathbf{\Gamma}_1^H \mathbf{R}^{-1} \dot{\mathbf{S}}_0 \mathbf{\Gamma}_1 \mathbf{T} \right\} \right\}}{\text{tr} \left\{ \mathbf{T} \mathbf{\Gamma}_1^H \dot{\mathbf{S}}_0^H \mathbf{R}^{-1} \dot{\mathbf{S}}_0 \mathbf{\Gamma}_1 \mathbf{T} \right\}}. \quad (7)$$

It is shown that for small CFO, CFO estimate $\hat{\varepsilon}_1$ for the desired user can be done accurately by (7). However, large CFO makes significant disparity between $\mathbf{S}(\varepsilon_1)$ and the approximate result in (5) and leads to substantial estimation error. To remedy this, an iterative scheme is introduced in which the CFO-compensated data is sent to the MCMOE detector as the input of next iteration. By using the $(j-1)$ th iteration result $\hat{\varepsilon}_1^{(j)}$, along some linear algebraic manipulations, we have the CFO estimate of the j th iteration

$$\hat{\varepsilon}_1^{(j)} = \hat{\varepsilon}_1^{(j-1)} - \frac{\text{Re} \left\{ \text{tr} \left\{ \mathbf{T}^{(j-1)} \mathbf{\Gamma}_1^H \Phi^H(\hat{\varepsilon}_1^{(j-1)}) \mathbf{R}^{-1} \Phi(\hat{\varepsilon}_1^{(j-1)}) \dot{\mathbf{S}}_0 \mathbf{\Gamma}_1 \mathbf{T}^{(j-1)} \right\} \right\}}{\text{tr} \left\{ \mathbf{T}^{(j-1)} \mathbf{\Gamma}_1^H \dot{\mathbf{S}}_0^H \Phi^H(\hat{\varepsilon}_1^{(j-1)}) \mathbf{R}^{-1} \Phi(\hat{\varepsilon}_1^{(j-1)}) \dot{\mathbf{S}}_0 \mathbf{\Gamma}_1 \mathbf{T}^{(j-1)} \right\}}, \quad (8)$$

where $\mathbf{T}^{(j)} = [\mathbf{\Gamma}_1^H \Phi^H(\hat{\varepsilon}_1^{(j)}) \mathbf{R}^{-1} \Phi(\hat{\varepsilon}_1^{(j)}) \dot{\mathbf{S}}_0 \mathbf{\Gamma}_1]^{-1}$ and $\hat{\varepsilon}_1^{(0)} = 0$. The above described procedure can be iterated $J > 2$ times, if necessary, to gain further improvement.

3. Computer Simulations

This section provides computer simulations to verify the efficiency of the proposed CFO estimation. Consider a 10-user MIMO MC-CDMA uplink that employs two RXs over a four-path Rayleigh fading channel. The transmission uses QPSK modulation and the random spreading codes $\mathbf{c}_k^{(p)}$ of length 32 are normalized to one. For ISI suppression, a CP of length eight is used. Suppose all users have an identical power level for simplicity. For each simulation trial, N_s symbols are used to compute \mathbf{R} . To realize the characteristic of the proposed CFO estimator, the mean square errors (MSEs), $E\{|\varepsilon_1 - \hat{\varepsilon}_1|^2\}$, are investigated. In the first set of simulations, the SNR effect is examined for CFO=0.2 and $N_s=10^3$. Fig. 1(a) plots the MSE versus the number of iterations, J . It shows that, for all SNR, the MSE decreases while the J value increases. In addition, a few iterations are required to reach stable performance for high SNR. The resulting MSE versus SNR plot depicted in Fig. 1(b) shows that the proposed CFO estimation achieves better performance as iterations proceed. However, CFO leads to large error when performing the first-order Taylor series expansion on $\mathbf{S}(\varepsilon)$ and makes the proposed scheme with two iterations fail to acquire an accurate CFO estimate. In the next set of simulations, we examine the effects of CFO and received symbol size N_s for SNR=10 dB. The MSE versus CFO plot for $N_s=10^3$, shown in Fig. 2(a), indicates that, for small CFO less than 0.05, the proposed scheme with two iterations can offer reliable performance. However, large CFO significantly degrades its accuracy in CFO estimation. This problem is greatly alleviated by using a few iterations. Fig. 2(b) plots the MSE versus N_s for CFO=0.2. As expected, the MSE values decrease as the number of received symbols increases for $J=5$ and 10. On the contrary, increment in sample size N_s does not seem to help much for the proposed scheme with two iterations. This is mainly due to the significant disparity caused by CFO when using the approximation in (5).

4. Conclusion

In this paper, an iterative blind CFO estimator is proposed for QO-STBC uplink MC-CDMA systems. Numerical results demonstrate that the proposed scheme without requirement of training sequence can achieve reliable performance in CFO estimation as iterations proceed.

Acknowledgments

This work was sponsored by the National Science Council, R.O.C, under the Contract NSC 100-2221-7-239-027.

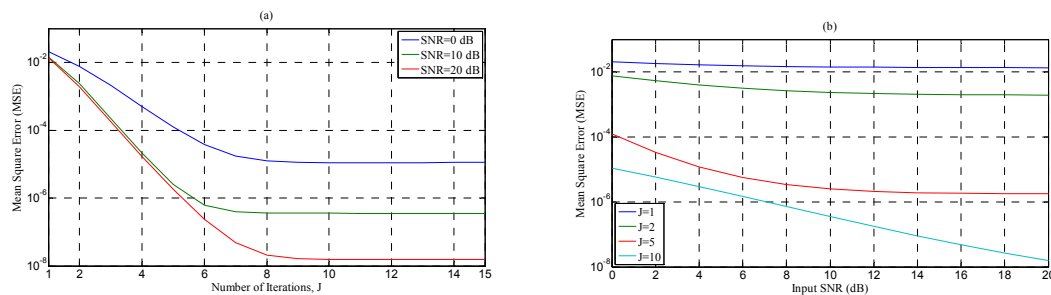


Figure 1: MSE performance of the proposed iterative CFO estimation with CFO=0.2 and $N_s=10^3$. (a) MSE versus iteration number and (b) MSE versus SNR.

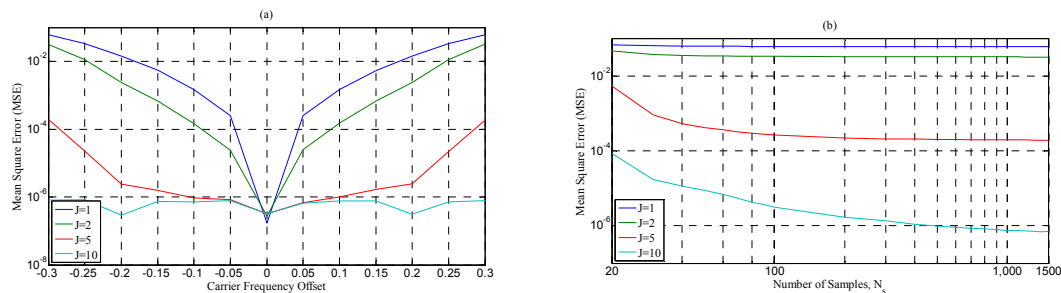


Figure 2: MSE performance of the proposed iterative CFO estimation with SNR=10 dB. (a) MSE versus carrier frequency offset (CFO) and (b) MSE versus received symbol size, N_s .

References

- [1] H. Hsinsuke, P. Ramjee, *Multicarrier Techniques for 4G Mobile Communications*, Artech House, Boston London, 2003.
- [2] Y. Mam, R. Rafazolli, "Estimation of carrier frequency offset for multicarrier CDMA uplink," *IEEE Trans. Signal Process.*, Vol. 55, No. 6, pp. 2671-2627, Jun. 2007.
- [3] H. Jafarkhani, "A quasi-orthogonal space-time block code," *IEEE Trans. Commun.*, Vol. 49, No. 1, pp. 1-4, Jan. 2001.
- [4] B. Seo, H.M. Kim, "Frequency offset estimation and multiuser detection for MC-CDMA systems," *Proc. IEEE MILCOM '02*, Vol. 2, pp. 804-807, Oct. 2002.