Proceedings of ISAP2012, Nagoya, Japan

POS2-36

ANGULAR POWER SPECTRUM OF SCATTERED ELECTROMAGNETIC WAVES IN RANDOMLY INHOMOGENEOUS PLASMA WITH ELECTRON DENSITY FLUCTUATIONS

[#] **G.V. Jandieri** [1] **, Akira Ishimaru** [2] **, N.F. Mchedlishvili** [1]

[1] Physics department, Georgian Technical University,
77 Kostava str. 0175, Tbilisi, GEORGIA, jandieri@access.sanet.ge, galoba47@mail.ru

[2] Department of Electrical Engineering, University of Washington,
FT-10 Seattle, Washington 98 195, USA, e-mail: ishimaru@u.washington.edu

1. Introduction

Peculiarities of the electromagnetic waves propagation in randomly inhomogeneous media have been intensively studied [1,2]. Anisotropic irregularities characterize: lyotropic liquid crystals having hexagonal structure [3], the Earth's ionospheric plasma with filed-aligned irregularities [4]. Angular power spectrum (APS) of ray intensity at light propagation in random medium with prolate irregularities has been investigated in [5-9].

Peculiarities of the evolution of the APS of multiple scattered radiation in the ionospheric plasma taking into account diffraction effects caused by electron density fluctuations are investigated in this paper. It has been shown that the APS has a double-peaked shape at oblique illumination of medium by mono-directed incident radiation using the smooth perturbation method. Correlation functions of the phase fluctuations are calculated for arbitrary correlation functions of electron density fluctuations. Analytical and numerical calculations are carried out for anisotropic Gaussian correlation function.

2. Formulation of the problem

Let us consider the features of scattered waves in the ionospheric plasma with anisotropic electron density and external magnetic field. Initial is the following vector wave equation:

$$\left(\frac{\partial^2}{\partial x_i\,\partial x_j}-\Delta\delta_{ij}-k_0^2\,\varepsilon_{ij}(\mathbf{r})\right)\mathbf{E_j}(\mathbf{r})=0\,. \qquad (1)$$

Wave field we introduce as $E_j(\mathbf{r})=E_{0j}\exp(\varphi_1+\varphi_2+i\,k_\perp\,y+i\,k_0\,z)$ ($k_\perp << k_0$). If electromagnetic wave is propagating along z axis and the vector $\boldsymbol{\tau}$ lies in the yoz coordinate plane ($\mathbf{k}\,||\,z$, $<\mathbf{H_0}>\in yz$), the components of the second-rank tensor ε_{ij} of collisionless magnetized plasma have the following form [10]:

$$\varepsilon_{xx}=1-\frac{\mathrm{v}}{1-u}\,,\quad \varepsilon_{yy}=1-\frac{\mathrm{v}(1-u\sin^2\alpha)}{1-u}\,,\quad \varepsilon_{zz}=1-\frac{\mathrm{v}(1-u\cos^2\alpha)}{1-u}\,,$$

$$\varepsilon_{xy}=-\varepsilon_{yx}=i\,\frac{\mathrm{v}\sqrt{u}\cos\alpha}{1-u}\,,\quad \varepsilon_{yz}=\varepsilon_{zy}=\frac{u\,\mathrm{v}\sin\alpha\cos\alpha}{1-u},\quad \varepsilon_{xz}=-\varepsilon_{zx}=-\,i\,\frac{\mathrm{v}\sqrt{u}\sin\alpha}{1-u}\,,$$

where α is the angle between the vectors $\mathbf{k}$ and $\mathbf{H_0}$; $\varepsilon_{xy}=i\,\tilde{\varepsilon}_{xy}$, $\varepsilon_{xz}=-\,i\,\tilde{\varepsilon}_{xz}$, $u=(e\,H_0\,/\,m\,c\,\omega)^2$, $\mathrm{v}=\omega_p^2\,/\,\omega^2$ are the magneto-ionic parameters, $\omega_p=(4\pi N\,e^2\,/\,m)^{1/2}$ is the plasma frequency,

$\Omega_H = eH_0 / mc$ is the electron gyrofrequency. Dielectric permittivity of turbulent magnetized plasma is second rank tensor, with randomly varying spatial coordinates $\varepsilon_{ij}(\mathbf{r}) = \varepsilon_{ij}^{(0)} + \varepsilon_{ij}^{(1)}(\mathbf{r})$, $|\varepsilon_{ij}^{(1)}(\mathbf{r})| << 1$. First component represents zero-order approximation, second one takes into account fluctuations of both electron density; fluctuations of complex phase are of the order $\varphi_1 \sim \varepsilon_{ij}^{(1)}$, $\varphi_2 \sim \varepsilon_{ij}^{(1)\,2}$.

In zero-order approximation we have wave equation

$$\left[\frac{\partial^2 \varphi_0}{\partial x_i \,\partial x_j} + \frac{\partial \varphi_0}{\partial x_i}\frac{\partial \varphi_0}{\partial x_j} + (k_\perp^2 + k_0^2)\,\delta_{ij} - k_0^2\,\varepsilon_{ij}^{(0)} \right] E_{0j} = 0 \,, \tag{2}$$

containing the set of three algebraic equations for E_{0j} regular field components:

$$\left(\varepsilon_{xx}^{(0)} - 1 - \frac{k_\perp^2}{k_0^2} \right) E_{0x} + i\,\tilde{\varepsilon}_{xy}^{(0)} E_{0y} - i\,\tilde{\varepsilon}_{xz}^{(0)} E_{0z} = 0 \,, \qquad i\,\tilde{\varepsilon}_{xy}^{(0)} E_{0x} + (1 - \varepsilon_{yy}^{(0)}) E_{0y} - \left(\frac{k_\perp}{k_0} + \varepsilon_{yz}^{(0)} \right) E_{0z} = 0 \,,$$

$$i\,\tilde{\varepsilon}_{xz}^{(0)} E_{0x} + \left(\varepsilon_{yz}^{(0)} + \frac{k_\perp}{k_0} \right) E_{0y} - \left(\frac{k_\perp^2}{k_0^2} - \varepsilon_{zz}^{(0)} \right) E_{0z} = 0 \,.$$

Solution of determinant imposes the restriction on the non-dimensional parameter $\mu = k_\perp / k_0$:

$$(2 - \varepsilon_{yy})\,\mu^4 + 2\varepsilon_{yz}\,\mu^3 + (2 - 2\varepsilon_{xx} - \varepsilon_{yy} - \varepsilon_{zz} + \varepsilon_{xx}\,\varepsilon_{yy} + \varepsilon_{yy}\,\varepsilon_{zz} - \varepsilon_{yz}^2 - \tilde{\varepsilon}_{xy}^2)\,\mu^2 + 2\left[\tilde{\varepsilon}_{xy}\,\tilde{\varepsilon}_{xz} + \varepsilon_{yz}(1 - \varepsilon_{xx}) \right] \cdot$$

$$\cdot\,\mu + \left[\varepsilon_{zz}(\varepsilon_{xx} - \varepsilon_{xx}\varepsilon_{yy} - 1 + \varepsilon_{yy}) + 2\tilde{\varepsilon}_{xy}\,\tilde{\varepsilon}_{xz}\,\varepsilon_{yz} + \tilde{\varepsilon}_{xz}^2(\varepsilon_{yy} - 1) + \varepsilon_{yz}^2(\varepsilon_{xx} - 1) + \tilde{\varepsilon}_{xy}^2\,\varepsilon_{zz} \right] = 0 \,.$$

Fluctuations of dielectric permittivity are caused by electron density fluctuations, which are random functions of the spatial coordinates: $\mathrm{v}(\mathbf{r}) = \mathrm{v}_0[1 + n_1(\mathbf{r})]$. Transverse correlation function of a scattered field has the following form [6] $W_{EE^*}(\boldsymbol{\rho}) = \langle E(\mathbf{r})E^*(\mathbf{r}+\boldsymbol{\rho}) \rangle$ taking into account that the observation points are spaced apart at a small distance $\boldsymbol{\rho} = \{\rho_x, \rho_y\}$:

$$W_{EE^*}(\boldsymbol{\rho}, k_\perp) = E_0^2 \exp\left\{ \mathrm{Re}\left[\frac{1}{2}\left(\langle \varphi_1^2(\mathbf{r}) \rangle + \langle \varphi_1^{2*}(\mathbf{r}+\boldsymbol{\rho}) \rangle \right) + \langle \varphi_1(\mathbf{r})\,\varphi_1^*(\mathbf{r}+\boldsymbol{\rho}) \rangle + 2\langle \varphi_2 \rangle \right] \right\} \cdot$$

$$\cdot\,\exp(-i\,\rho_y\,k_\perp) \,, \tag{3}$$

where E_0^2 is the intensity of an incident radiation. APS of scattered field in case of incident plane wave $W(k', k_\perp)$ is easily calculated by Fourier transform of the transversal correlation function of a scattered field:

$$W(k', k_\perp) = \int_{-\infty}^{\infty} d\rho_y\, W_{EE^*}(\rho_y, k_\perp) \exp(i\,k'\rho_y) \,. \tag{4}$$

On the other hand, if the APS of an incident wave has a finite width and its maximum is directed along the z axis, intensity of scattered radiation is given by [6]:

$$I(k') = \int_{-\infty}^{\infty} dk_\perp\, W(k', k_\perp) \exp(-k_\perp^2\,\beta^2) \,, \tag{5}$$

where β characterizes the dispersal of an incident radiation (disorder of an incident radiation).

In the absence of an external magnetic field we obtain:

$$\langle \varphi_1^2(\mathbf{r}) \rangle + \langle \varphi_1^{*\,2}(\mathbf{r}+\boldsymbol{\rho}) \rangle = -\pi k_0^3 \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y\, \frac{1}{k_y^2}\, \sin\left(\frac{k_y^2}{k_0} L \right) V\left(k_x, k_y, -\frac{k_\perp}{k_0}\,k_y \right).$$

$$\langle \varphi_1(\mathbf{r})\,\varphi_1^*(\mathbf{r}+\boldsymbol{\rho}) \rangle = \frac{\pi k_0^2 L}{2} \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y\; V\left[k_x, k_y, -\frac{k_y(k_y + 2k_\perp)}{2k_0} \right] \exp(-i\,k_x\,\rho_x - i\,k_y\,\rho_y) \,,$$

$$\mathrm{Re} < \varphi_2(\mathbf{r}) > = -\frac{\pi k_0^2 L}{4} \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \left[1 - \frac{k_0}{L k_y^2} \sin\left(\frac{k_y^2}{k_0} L \right) \right] V\left(k_x, k_y, -\frac{k_\perp}{k_0} k_y \right). \quad (6)$$

3. Numerical calculations

For investigation of the influence of electron density fluctuations on the evaluation of the APS of scattered radiation in analytical and numerical calculations we will use the anisotropic Gaussian correlation function having in the principle yoz plane the following form:

$$V_n(k_x, k_y, k_z) = \sigma_n^2 \frac{l_\perp^2 l_\parallel}{8\pi^{3/2}} \exp\left(-\frac{k_x^2 l_\perp^2}{4} - p_1 \frac{k_y^2 l_\parallel^2}{4} - p_2 \frac{k_z^2 l_\parallel^2}{4} - p_3 k_y k_z l_\parallel^2 \right). \quad (7)$$

This function is characterized by anisotropy factor of irregularities $\chi = l_\parallel / l_\perp$ (ratio of longitudinal and transverse linear scales of plasma irregularities with respect to the external magnetic field) and the inclination angle of prolate irregularities with respect to the external magnetic field γ_0. $p_1 = \left[1 + (1-\chi^2)^2 \sin^2\gamma_0 \cos^2\gamma_0 \right] (\sin^2\gamma_0 + \chi^2 \cos^2\gamma_0)^{-1}$, $p_2 = (\sin^2\gamma_0 + \chi^2\cos^2\gamma_0)/\chi^2$, $p_3 = (1-\chi^2) \cdot$ $\cdot \sin\gamma_0 \cos\gamma_0 / 2\chi^2$. Substituting (7) into equations (6) and (3) for normalized anisotropic Gaussian correlation function of scattered electromagnetic waves we obtain:

$$\frac{W_{EE^*}(\xi, \eta, \mu)}{E_0^2} = \exp(-i\eta\mu) \cdot \exp\left[-2\pi\sqrt{2}\, B_0\, (p_1 + p_2\mu^2 + 4p_3\mu)^{-1/2} \right] \cdot \exp\left\{ \exp\left(-\frac{\chi^2}{T^2}\xi^2 \right) \frac{T B_0}{2\sqrt{\pi}} \cdot \right.$$

$$\left. \cdot \int_{-\infty}^{\infty} ds\, \exp\left(-\frac{T^2}{4} 2 \left[\frac{1}{4} p_2 s^4 + (p_2\mu + 2p_3) s^3 + (p_1 + p_2\mu^2 + 4p_3\mu) s^2 \right] \right) \exp(-i\eta s) \right\}, \quad (8)$$

where: $T = k_0 l_\parallel$, $B_0 = \sigma_n^2 \frac{\sqrt{\pi}}{4} \frac{T k_0 L}{\chi}$, $s = \frac{k_y}{k_0}$, $\eta = k_0 \rho_y$, $\xi = k_0 \rho_x$.

Figure 1 illustrates APS versus non-dimensional wavy parameter k for fixed parameters $T = 4000$, $\mu = 0.06$, $\gamma_0 = 0^0$, $\xi = 0$, $\chi = 167$ and different parameter B_0. Curves are normalized on theirs maximum value. From the numerical calculations it follows that a dip of the curves is getting much more pronounced, location of spectrum maximum slightly varies. The width substantially

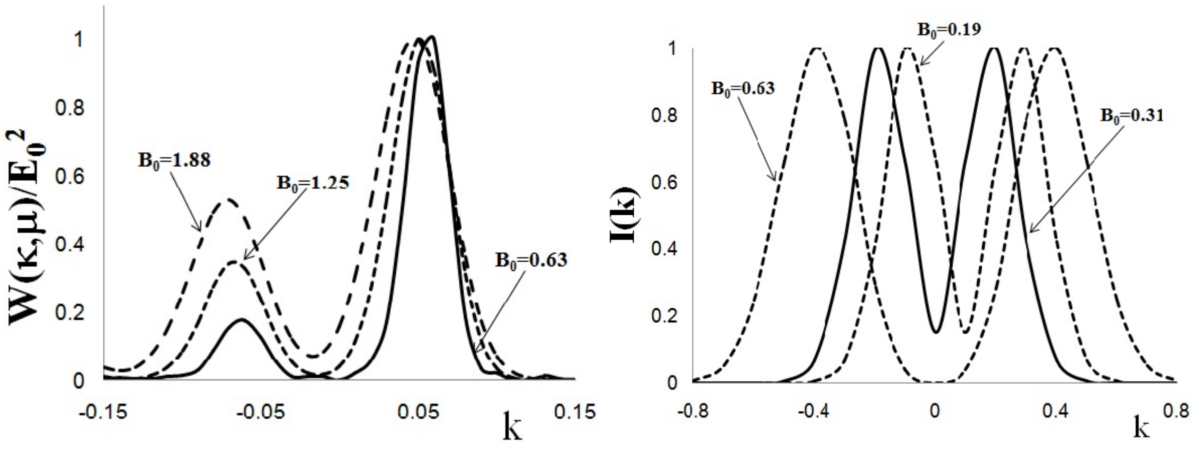

Figure 1: Dependence of normalized 2D APS and intensity of scattered electromagnetic field versus non-dimensional wave parameter for different parameter B_0 and fixed parameters: $T = 4000$, $\mu = 0.06$, $\gamma_0 = 0^0$, $B_0 = 1.5$, $\xi = 0$, $\chi = 167$.

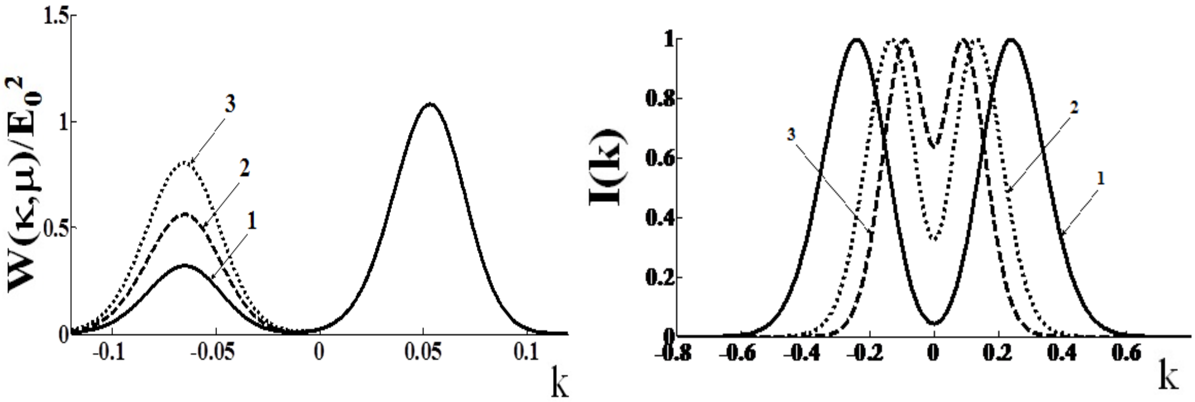

Figure 2: Dependence of normalized 2D APS and intensity of scattered radiation when the incident wave beam has a finite thickness ($\beta = 10$) for fixed parameters: $\xi = 0$, $T = 4000$, $\mu = 0.06$, $\gamma_0 = 0^0$, $B_0 = 1.5$ and different parameter of anisotropy 1 - $\chi = 167$, 2 - $\chi = 180$, 3 - $\chi = 190$ (a); 1 - $\chi = 250$, 2 - $\chi = 100$, 3 - $\chi = 80$ (b).

broadens with increasing a distance passing by the wave in anisotropic plasma and narrows with increasing parameter χ. Neglecting diffraction effects, i.e. neglecting the term $k_y^2 / 2k_0^2$ in the arguments of 2D spectrum of dielectric permittivity, "double-humping" effect in APS disappears. Numerical analyses show that when the incident wave beam has a finite thickness $\beta = 10$, the intensity of scattered radiation has a strongly pronounced dip along a direction of prolate irregularities; its maximum moves apart in proportion to a distance passing by the wave in plasma. Figure 2 shows the behaviour of 2D APS and intensity of scattered radiation for different parameter of anisotropy parameter χ. Depth of the APS increases in proportion χ.

References

[1] A. Ishimaru, A. *Wave Propagation and Scattering in Random Media*, *Vol. 2, Multiple Scattering, Turbulence, Rough Surfaces and Remote Sensing*, IEEE Press, Piscataway, New Jersey, USA, 1997.

[2] S.M. Rytov, Yu.A. Kravtsov, V.I. Tatarskii, *Principles of Statistical Radiophysics. vol.4. Waves Propagation Through Random Media*. Berlin, New York, Springer, 1989.

[3] G.H. Brown, J.J. Wolken, *Liquid Crystals and Biological Structures*, New York, 1979.

[4] B.N. Gershman, L.M. Eruxhimov, Yu.Ya. Iashin, *Wavy Phenomena in the Ionosphere and Cosmic Plasma*, Moscow, Nauka, 1984 (in Russian).

[5] V.G. Gavrilenko, A.V. Aistov, G.V. Jandieri, "Some peculiarities of wave multiple scattering in a statistically anisotropic medium," Waves Random Media, vol. 10, pp. 435-445, 2000.

[6] V.G. Gavrilenko, G.V. Jandieri, A. Ishimaru, V.G. Jandieri, "Peculiarities of spatial spectrum of scattered electromagnetic waves in anisotropic inhomogeneous medium," Progress In Electromagnetics Research B, PIER B, vol. 7, pp. 191–208, 2008.

[7] G.V. Jandieri, A. Ishimaru, V.G. Jandieri, A.G. Khantadze, Zh.M. Diasamidze, "Model computations of angular power spectra for anisotropic absorptive turbulent magnetized plasma," Progress In Electromagnetics Research, PIER, vol. 70, pp. 307–328, 2007.

[8] G.V. Jandieri, A. Ishimaru, N.N. Zhukova, T.N. Bzhalava, M.R. Diasamidze, "On the influence of fluctuations of the direction of an external magnetic field an phase and amplitude correlation functions of scattered radiation by magnetized plasma slab," Progress In Electromagnetics Research B, PIER B, vol. B22, pp. 121–143, 2010.

[9] G.V. Jandieri, A. Ishimaru, V.G. Jandieri, "Depolarization effects of incoherently scattered electromagnetic waves by inhomogeneous magnetized plasma slab," Journal of Electromagnetic Analysis and Application, vol. 3, pp. 471–478, 2011.

[10] V.L. Ginzburg, *Propagation of Electromagnetic Waves in Plasma*. New York: Gordon and Beach, 1961.