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Performance Analysis of SURE Method for DOA Estimation of Coherent Sources by Uniform Linear Array

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1. Introduction

DOA (Direction-Of-Arrival) estimation is one of mandatory techniques for accurately detecting radio wave propagation environment around base station and mobile terminals. Many DOA estimation algorithms have already been studied like MUSIC (eigendecomposition & angular search), Root-MUSIC (eigendecomposition), TLS-ESPRIT (some eigendecompositions) and MODE (maximum likelihood). These algorithms generally achieve high accuracy but also need large computational cost. Moreover, most of them require space averaging technique in estimating DOAs of coherent sources. Such technique is worth to reduce signal correlation but often leads worse DOA estimation accuracy due to decreasing the effective number of array elements.

The authors have proposed SURE (SimUltaneous Root Extraction) method for high-resolution and low-cost DOA estimation of multiple coherent sources by ULA (Uniform Linear Array) [1], [2]. This paper introduces SURE method and analyses its estimation performance in comparison with conventional algorithms through computer simulation.

2. SURE Method [2]

Assume that L incident waves $s_i(i = 1, 2, \cdots, L)$ arrive at K-element ULA of half-wave length interval each. The complex received signal x_k at the k-th antenna is written as

$$x_k(n) \quad = \quad \sum_{i=1}^{L} s_i(n) e^{-j2\pi \frac{(k-1)d}{\lambda} \sin\theta_i} + u_k(nn) = \sum_{i=1}^{L} s_i(n) e^{-j\pi(k-1)\sin\theta_i} + u_k(n),$$

for $k = 1, 2, \ldots, K$, where λ and $d = \lambda/2$ denote the wavelength and the element interval, respectively. The angle θ_i is the DOA of i-th incident wave which changes within $(-90°, 90°)$. Besides, u_k denotes the additive (white Gaussian) noise of k-th element. The array input vector $\boldsymbol{x}(n)$ and its correlation matrix $\boldsymbol{R}_{xx}$ (approximated by sampled correlation matrix) are respectively given as follows.

$$\boldsymbol{R}_{xx} \quad = \quad E\left[\boldsymbol{x}(n)\boldsymbol{x}^H(n)\right] \simeq \frac{1}{N}\sum_{n=1}^{N} \boldsymbol{x}(n)\boldsymbol{x}^H(n),$$

where $\boldsymbol{x}(n) = [x_1(n), x_2(n), \ldots, x_K(n)]^T$, N is the number of snapshots, and $E[\cdot]$ denotes the ensemble average. Here we make the following proposition. Here, the k-th row ℓ-th column component $R_{k\ell}$ of the correlation matrix $\boldsymbol{R}_{xx}$ can be represented as a function of DOAs as follows [1].

$$R_{k\ell} = E[x_k(n)x_\ell^*(n)] \simeq \sum_{i=1}^{L}\sum_{j=1}^{L}\left[S_{ij}\Theta_i^{-(k-1)}\Theta_j^{\ell-1}\right] + \sigma^2\delta_{k\ell} \tag{1}$$

where $S_{ij} = E\left[s_i(n)s_j^*(n)\right] \simeq \frac{1}{N}\sum_{n=1}^{N} s_i(n)s_j^*(n)$ and $\Theta_i = e^{-j\pi\sin\theta_i}$. Besides, σ^2 and $\delta_{k\ell}$ denote noise power and Kronecker's delta, respectively.

The correlation values between the received signals which are the components of the correlation matrix can be rewrote as the functions of the DOAs of the signals as the formula (1). As in [1], we resolve the DOA estimation problem by deriving the solutions of nonlinear simultaneous equations which contain the DOA parameters and the components of the correlation matrix. We finally found that the following $(5L-12)$ equations.

$$R_{L-1-k,L} - R_{L-1,L+k})^* = \sum_{i=1}^{k-1} c_i (R_{L-1,L-i+k} - R_{L+i-1-k,L})^* - \sum_{j=k+1}^{L} c_j (R_{L,L+j-1-k} - R_{L-j+k,L-1}), \quad k = 1, 2, ..., L-2, \tag{2}$$

$$(R_{L-k-2,L-1} - R_{L-1,L+k})^* = \sum_{i=1}^{k} c_i (R_{L-1,L-i+k} - R_{L+i-2-k,L-1})^* - \sum_{j=k+2}^{L} c_j (R_{L-1,L+j-2-k} - R_{L-j+k,L-1}), \quad k = 1, 2, ..., L-3, \tag{3}$$

$$(R_{L-3+k,L-2+k} - R_{L-1+k,L+k})^* = c_1 (R_{L-1+k,L-1+k} - R_{L-2+k,L-2+k}) - \sum_{i=3}^{L} c_i (R_{L-2+k,L+i-3+k} - R_{L-i+k,L-1+k}), \quad k = 1, 2, ..., K-2L+3, \tag{4}$$

$$(R_{L-3+k,L-3+k} - R_{L+k,L+k}) = \sum_{i=1}^{2} c_i (R_{L-i+k,L+k} - R_{L-3+k,L+i-3+k}) - \sum_{j=4}^{L} c_j (R_{L-3+k,L+j-3+k} - R_{L-j+k,L+k}) \quad k = 1, 2, ..., K-2L+3, \tag{5}$$

$$(R_{L+k,L+k} - R_{L+1+k,L+1+k}) = \sum_{i=1}^{L} c_i (R_{L-i+1+k,L+1+k} - R_{L-i+k,L+k}), \quad k = 1, 2, ..., K-L-1, \tag{6}$$

$$(R_{2+k,L+k} - R_{3+k,L+1+k})^* = \sum_{i=1}^{L-3} c_i (R_{3+k,L-i+1+k} - R_{2+k,L-i+k})^* + \sum_{j=L-2}^{L} c_j (R_{L-j+1+k,3+k} - R_{L-j+k,2+k}), \quad k = 1, 2, ..., K-L-1, \tag{7}$$

$$(R_{1+k,L+k} - R_{2+k,L+1+k})^* = \sum_{i=1}^{L-2} c_i (R_{2+k,L-i+1+k} - R_{1+k,L-i+k})^* + \sum_{j=L-1}^{L} c_j (R_{L-j+1+k,2+k} - R_{L-j+k,1+k}), \quad k = 1, 2, ..., K-L-1, \tag{8}$$

where c_k is related to Vieta's formula

$$c_k = \sum_{i_1=1}^{L-k+1} \sum_{i_2=i_1+1}^{L-k+2} \cdots \sum_{i_k=i_{k-1}+1}^{L} \left(\prod_{j=1}^{k} \Theta_{i_j}^{-1} \right), \quad k = 1, 2, \ldots, L,$$

says that the complex DOAs $\{\Theta_i\}_{i=1}^{L}$ are derived as roots of a polynomial equation

$$\sum_{i=0}^{L} (-1)^{L-i} c_i \Theta^i = 0. \tag{9}$$

Equation (9) means that the DOAs can be estimated by finding roots of a polynomial like Root-MUSIC method. Therefore the DOA estimation problem can be reduced into finding $\{c_i\}_{i=1}^{L}$ in (2)–(8). Rearranging the equations (2)–(8) in a vector/matrix form, we have

$$\boldsymbol{F}\boldsymbol{c} = \boldsymbol{g} \tag{10}$$

where $\boldsymbol{c} = [c_1, c_2, \ldots, c_L]^T$ is an $L \times 1$ vector. Note that the $(5L-12) \times L$ matrix $\boldsymbol{F}$ and the $(5L-12) \times 1$ vector $\boldsymbol{g}$ do not contain the term of c_k.

Since the simultaneous equation (10) becomes overdetermined for the case of $L > 2$, the closed-form DOA estimation formula [1] is used instead for the case of $L = 1, 2$. We have the coefficient vector $\boldsymbol{c}$ by $\boldsymbol{c} = \boldsymbol{F}^{-1}\boldsymbol{g}$, in the case $L = 1, 2$, and by $\boldsymbol{c} = \boldsymbol{F}^{+}\boldsymbol{g} = \left(\boldsymbol{F}^H\boldsymbol{F}\right)^{-1} \boldsymbol{F}^H\boldsymbol{g}$ in the case $L > 2$, where $\boldsymbol{F}^{+}$

denotes Moore-Penrose pseudo-inverse of $\boldsymbol{F}$. Then we have Θ_i by finding the roots of (9), and finally the estimated DOAs are given by

$$\theta_i = -\sin^{-1}\left\{\frac{1}{\pi}\tan^{-1}\left(\frac{\mathrm{Im}[\Theta_i]}{\mathrm{Re}[\Theta_i]}\right)\right\}, \quad i = 1, 2, \ldots, L, \tag{11}$$

where $\tan^{-1}(\cdot)$ denotes four-quadrant arctangent [1].

3. Simulation

In this section, the estimation accuracy and the computational cost of the SURE method are evaluated through computer simulation in comparison with those by Root-MUSIC and MODE methods. The estimation accuracy is evaluated by RMSE (Root Mean Square Error) of DOA estimation, which is calculated as the average of 100 Monte-Carlo simulation results. Besides, the spatial averaging technique is applied to Root-MUSIC method for estimating DOAs of coherent sources.

3.1 SNR dependency

We first study the SNR dependency of the SURE method in comparison with those by Root-MUSIC and MODE methods. Specifications of simulation are shown in Table. 1.

Figures 1(a) and 2(a) show the comparison of the DOA estimation accuracy for various values of SNRs in the case of 5 elements ULA / 3 waves in Fig. 1(a), and 8 elements ULA / 5 waves in Fig. 3(b), respectively. From Figs. 1(a) and 2(a), we see that the DOA estimation accuracy of the SURE method is better than Root-MUSIC method and as accurate as MODE method.

Also note that MODE method has an inherent problem that it does not estimate DOAs accurately when the two of the DOAs have same absolute values but different signs, i.e., $\pm\theta$ [2].

3.2 Snapshot dependency

Then we focus on the effect of the number of snapshots to the DOA estimation accuracy. Specifications of simulation are again shown in Table. 1.

Figures 1(b) and 2(b) show the comparison of the DOA estimation accuracy for various values of the number of snapshots in the case of 5 elements ULA / 3 waves in Fig. 1(b), and 8 elements ULA / 5 waves in Fig. 2(b), respectively. From Figs. 1(b) and 2(b), we see that the DOA estimation accuracy of the SURE method is much better than Root-MUSIC method and as accurate as MODE method.

We also found from Figs. 1(b) and 2(b) that the accuracy of Root-MUSIC method does not come close to that of the other two methods, even in large number of snapshots. The reason why the performance of the Root-MUSIC method becomes worse would be due to the spatial averaging for estimating DOAs of coherent sources. SURE method accurately estimtes DOAs in the case of large number of snapshots, as accurate as MODE method.

3.3 Computational cost

The computation time required to estimate DOAs of 1,000 times is indicated in Table 2. Note that the unit of computation time is [sec], and $(\cdot)$ represents the ratio when the time required to the SURE method is set to one, and all the simulations have been done on MATLAB. We see from Table 2 that SURE method requires the smallest computational cost in three methods, and realize as high accuracy as MODE method. Indeed the number of operations (additions and multiplications) can be written as $O(K^3)$ for Root-MUSIC and MODE methods, and $O(KL^2)$ for the SURE method, in the case of K elements / L sources. As from the fact $K > L$, we can confirm that $KL^2 < K^3$.

4. Concluding Remarks

We introduced SURE method; a novel high-resolution DOA estimation algorithm of multiple coherent waves for Uniform Linear array. The accuracy of SURE method is close to MODE method and much better than Root-MUSIC method, while preserving small computational load.

References

[1] K. Ichige, N. Takabe, H. Arai, "An Explicit High-Resolution DOA Estimation Formula For Two Wave Sources," Proc. Int'l Conf. Acoustics, Speech and Signal Processing, vol. IV, pp. 893-896, May 2006.

[2] K. Ichige, H. Li, H. Arai, "SURE: SimUltaneous Root Extraction Method for DOA Estimation of Coherent Sources by ULA," to be presented at IEEE Sensor Array & Multichannel Workshop, June 2012.

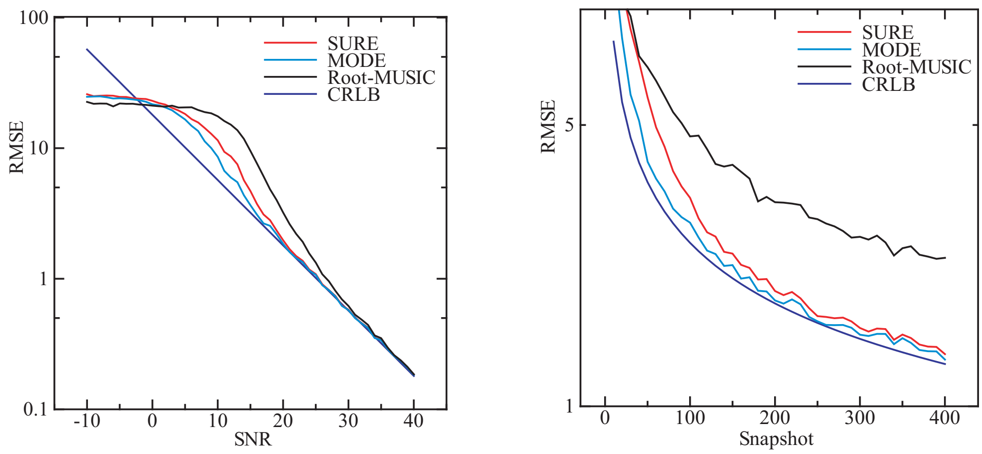
Table 1: specifications of simulation

Fig.	1(a)	1(b)	2(a)	2(b)
# of array elements	5		8	
# of incident waves	3		5	
DOAs [deg]	0,10,20		−30,0,10,20,40	
# of snapshots	200	change	200	change
SNR	change	20dB	change	10dB

Table 2: Comparison of computational cost

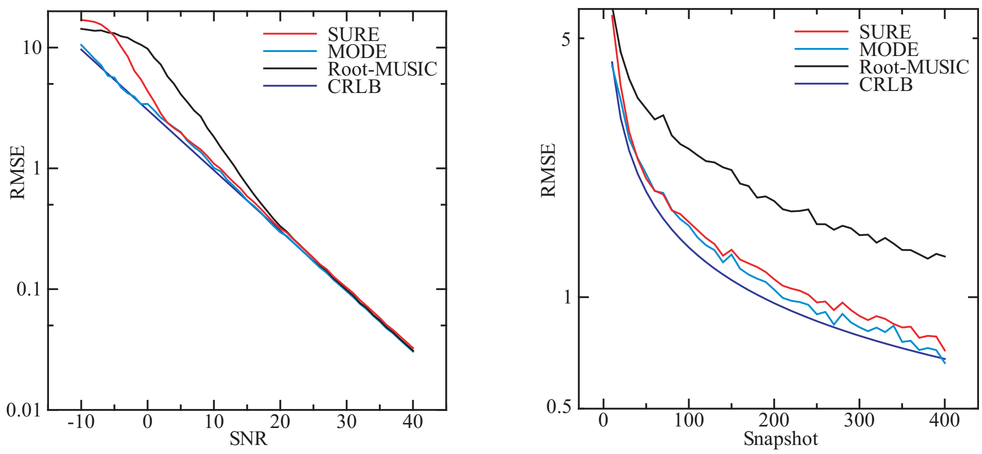
	Root-MUSIC	MODE	SURE
⋆1	0.005 (1.7)	0.015 (5.0)	0.003 (1.0)
⋆2	0.011 (2.8)	0.019 (4.8)	0.004 (1.0)

⋆1: case of 5 elements ULA / 3 coherent waves
⋆2: case of 10 elements ULA / 6 coherent waves



(a) SNR dependency (b) Snapshot dependency

Figure 1: Behavior of RMSE – in the case of 5 elements, 3 waves from 0, 10, 20 degrees



(a) SNR dependency (b) Snapshot dependency

Figure 2: Behavior of RMSE – in the case of 8 elements, 5 waves from −30, 0, 10, 20, 40 degrees