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Monopulse Angle Estimation For Unresolved Targets With A Fourth Order Cumulant

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1. Introduction

Monopulse angle estimation with the sum and difference antenna beam has been widely used in current radar and navigation systems. Unlike conical lobing, monopulse method can estimate the direction-of-arrival (DOA) of a target simultaneously with two receiver channels. DOA is obtained from the monopulse ratio, the ratio of the difference beam to the sum beam output, by referring to the monopulse error curve. Monopulse angle accuracy is generally very close to Cramer-Rao lower bound. However when multiple target signals are arriving at the mainbeam, the monopulse ratios for every single target signal cannot be calculated. As a result, the performance of angle estimation is dramatically degraded.

Applying the superresolution angle estimation method such as MUSIC [1] is one of the solutions for this problem only if more than three receiver channels are available. Moreover, the array manifold, array responses for every DOAs for required angle coverage, must be measured and stored. Generally the array responses are required to be measured at finer angle step than required angle accuracy, resulting in high expense and exhausting measurement.

Our goal in this paper is to develop an affordable method that can estimate DOAs of the unresolved multiple targets in mainbeam with monopulse-based method so that no additional receiver channel and no effort for obtaining array manifold are required. To this end, the monopulse method with the virtual ESPRIT algorithm (VESPA) [2] is proposed in this paper. Original VESPA by Dogan and Mendel is a novel cumulant-based DOA estimation algorithm and require a pair of 'guiding sensor' with known identical phase characteristics and relative distance vector. In the proposed method, a derivative VESPA with the sum and difference channel as guiding sensors is used for calculating multiple monopulse ratios of received targets. By introducing the derivative VESPA, the array's degree-of-freedom of monopulse beamformer is increased to four from two. Consequently DOAs for up-to three targets can be estimated via monopulse error curve with the monopulse ratios obtained by the derivative VESPA.

2. Proposed VESPA Monopulse

2.1 Conventional VESPA Algorithm

Consider a linear array with M channel and K target signals from angles $\{\theta_k\}_{k=1}^{K}$. Received vector $\mathbf{x}(n)$ can given as

$$\mathbf{x}(n) = \mathbf{A}\mathbf{s}(n) + \mathbf{n}(n) \tag{1}$$

where n is a snapshot number, $\mathbf{A}$ is a matrix with k-th steering vectors $\mathbf{a}(\theta_k)$ set as $\mathbf{A} = [\mathbf{a}(\theta_1),\cdots,\mathbf{a}(\theta_k),\cdots,\mathbf{a}(\theta_K)]$, $\mathbf{s}(n)$ is a signal vector as $\mathbf{s}(n) = [s_1(n),\cdots,s_k(n),\cdots,s_K(n)]^T$ and $\mathbf{n}(n)$ is a Gaussian receiver noise vector. Notice that it is assumed that $\{s_k(n)\}_{k=1}^{K}$ are non-Gaussian distributed and uncorrelated signals.

Let call the u-th and v-th channels of M channel array as the guiding channel and describe vector elements of $\mathbf{x}(n)$ as x_u, x_v, x_m, x_n. Snapshot number n is dropped for notional convenience. A fourth order cumulant for x_u, x_v, x_m, x_n is given as

$$cum[x_u, x_v^*, x_m, x_n^*] \equiv E[x_u x_v^* x_m x_n^*] - E[x_u x_v^*]E[x_m x_n^*] - E[x_u x_m]E[x_v^* x_n^*] - E[x_v^* x_m]E[x_u x_n^*] \tag{2}$$

By substituting (1) into (2), it can be rewritten as

$$cum[x_u, x_v^*, x_m, x_n^*] = \sum_{k=1}^{K} a_u(\theta_k) a_v^*(\theta_k) a_m(\theta_k) a_n^*(\theta_k) \gamma_{s_k} \tag{3}$$

where $a_i(\theta_k)$ is the i-th element of a steering vector $\mathbf{a}(\theta_k)$ and γ_{s_k} is a fourth order cumulant for the k-th signal given as

$$\gamma_{s_k} \equiv cum[s_k, s_k^*, s_k, s_k^*] \tag{4}$$

Note that noise term is cancelled due to $cum[n_u, n_v^*, n_m, n_n^*] = 0$ for Gaussian noise.

A fourth order cumulant matrix $\mathbf{R}_{u,u}$ is given as

$$\mathbf{R}_{u,u} \equiv cum[x_u, x_u^*, \mathbf{x}, \mathbf{x}^H] = \mathbf{A}\,\Gamma_u\,\mathbf{A}^H \tag{5}$$

where Γ_u is a diagonal matrix as

$$\Gamma_u = diag\{|a_u(\theta_1)|^2 \gamma_{s_1}, \cdots, |a_u(\theta_k)|^2 \gamma_{s_k}, \cdots, |a_u(\theta_K)|^2 \gamma_{s_K}\} \tag{6}$$

Another fourth order cumulant matrix $\mathbf{R}_{u,v}$ is given as

$$\mathbf{R}_{u,v} \equiv cum[x_u, x_v^*, \mathbf{x}, \mathbf{x}^H] = \mathbf{A}\,\mathbf{\Phi}_{u,v}\,\Gamma_u\,\mathbf{A}^H \tag{7}$$

where $\mathbf{\Phi}_{u,v}$ is a diagonal matrix with response ratios of the guiding channels for angle θ_k given as

$$\mathbf{\Phi}_{u,v} = diag\left\{a_v^*(\theta_1)/a_u^*(\theta_1), \cdots, a_v^*(\theta_k)/a_u^*(\theta_k), \cdots, a_v^*(\theta_K)/a_u^*(\theta_K)\right\} \tag{8}$$

In a similar way, a fourth order cumulant matrix $\mathbf{R}_{v,v}$ is given as

$$\mathbf{R}_{v,v} \equiv cum[x_v, x_v^*, \mathbf{x}, \mathbf{x}^H] = \mathbf{A}\,\mathbf{\Phi}_{u,v}\,\mathbf{\Gamma}_u\,\mathbf{\Phi}_{u,v}^H\,\mathbf{A}^H \tag{9}$$

Now following a fourth order cumulant matrix $\bar{\mathbf{R}}_{u,v}$ can be given as

$$\bar{\mathbf{R}}_{u,v} \equiv \begin{bmatrix} \mathbf{R}_{u,u} & \mathbf{R}_{u,v}^H \\ \mathbf{R}_{u,v} & \mathbf{R}_{v,v} \end{bmatrix} = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}\mathbf{\Phi}_{u,v} \end{bmatrix} \mathbf{\Gamma}_u \begin{bmatrix} \mathbf{A} \\ \mathbf{A}\mathbf{\Phi}_{u,v} \end{bmatrix}^H \tag{10}$$

From the structure of $\bar{\mathbf{R}}_{u,v}$ as seen in right hand side of (10), one can notice that $\mathbf{\Phi}_{u,v}$ can be estimated by ESPRIT algorithm [3]. In conventional VESPA, the identical phase characteristics and relative distance vector of the guiding channel are known. Consequently DOA θ_k for the k-th target is estimated from following equation.

$$\arg\left\{\frac{a_v^*(\theta_k)}{a_u^*(\theta_k)}\right\} = -\frac{2\pi \sin\theta_k}{\lambda}(u-v)d \tag{11}$$

2.2 Proposed VESPA Monopulse Method

Let us focus (8) in the previous section. The k-th diagonal element of (8) is a response ratios of the guiding channels for the k-th target signal at angle θ_k. If the sum and difference channels are selected as guiding channels, the k-th diagonal element of (8) can be rewritten with a conjugate of the monopulse ratio for angle θ_k. From this standpoint, a derivative VESPA using the monopulse channel as guiding channel is used for calculating the multiple monopulse ratios for received targets in the proposed method.

Let define received vector $\mathbf{x}_{\Sigma,\Delta}(n) = \left[x_\Sigma(n) \quad x_\Delta(n)\right]^T$, where $x_\Sigma(n), x_\Delta(n)$ are received signals for the sum and difference channel respectively. $\mathbf{x}_{\Sigma,\Delta}(n)$ can be given like (1) as

$$\mathbf{x}_{\Sigma,\Delta}(n) = \mathbf{A}_{\Sigma,\Delta}\mathbf{s}(n) + \mathbf{n}_{\Sigma,\Delta}(n) \tag{12}$$

where $\mathbf{A}_{\Sigma,\Delta}$ is a matrix with the k-th monopulse steering vectors $\mathbf{a}_{\Sigma,\Delta}(\theta_k) = \left[a_\Sigma(\theta_k) \quad a_\Delta(\theta_k)\right]^T$ set as $\mathbf{A}_{\Sigma,\Delta} = [\mathbf{a}_{\Sigma,\Delta}(\theta_1), \cdots, \mathbf{a}_{\Sigma,\Delta}(\theta_k), \cdots, \mathbf{a}_{\Sigma,\Delta}(\theta_K)]$ and $\mathbf{n}_{\Sigma,\Delta}(n)$ is a Gaussian receiver noise vector of monopulse channel.

Fourth order cumulant matrices are given as

$$\mathbf{R}_{\Sigma,\Sigma} \equiv cum[x_\Sigma, x_\Sigma^*, \mathbf{x}_{\Sigma,\Delta}, \mathbf{x}_{\Sigma,\Delta}{}^H] = \mathbf{A}_{\Sigma,\Delta}\mathbf{\Gamma}_\Sigma\mathbf{A}_{\Sigma,\Delta}{}^H \tag{13}$$

$$\mathbf{R}_{\Sigma,\Delta} \equiv \mathbf{A}_{\Sigma,\Delta}\mathbf{\Phi}_{\Sigma,\Delta}\mathbf{\Gamma}_\Sigma\mathbf{A}_{\Sigma,\Delta}{}^H \tag{14}$$

$$\mathbf{R}_{\Delta,\Delta} \equiv \mathbf{A}_{\Sigma,\Delta}\mathbf{\Phi}_{\Sigma,\Delta}\mathbf{\Gamma}_\Sigma\mathbf{\Phi}_{\Sigma,\Delta}^H\mathbf{A}_{\Sigma,\Delta}{}^H \tag{15}$$

where $\mathbf{\Gamma}_\Sigma, \mathbf{\Phi}_{\Sigma,\Delta}$ are given as

$$\mathbf{\Gamma}_\Sigma = diag\{|a_\Sigma(\theta_1)|^2 \gamma_{s_1}, \cdots, |a_\Sigma(\theta_k)|^2 \gamma_{s_k}, \cdots, |a_\Sigma(\theta_K)|^2 \gamma_{s_K}\} \tag{16}$$

$$\mathbf{\Phi}_{\Sigma,\Delta} = diag\left\{\frac{a_\Delta^*(\theta_1)}{a_\Sigma^*(\theta_1)}, \cdots, \frac{a_\Delta^*(\theta_k)}{a_\Sigma^*(\theta_k)}, \cdots, \frac{a_\Delta^*(\theta_K)}{a_\Sigma^*(\theta_K)}\right\} \tag{17}$$

Then the final fourth order cumulant matrix $\bar{\mathbf{R}}_{\Sigma,\Delta}$ is given as

$$\bar{\mathbf{R}}_{\Sigma,\Delta} \equiv \begin{bmatrix} \mathbf{R}_{\Sigma,\Sigma} & \mathbf{R}_{\Sigma,\Delta}{}^H \\ \mathbf{R}_{\Sigma,\Delta} & \mathbf{R}_{\Delta,\Delta} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{\Sigma,\Delta} \\ \mathbf{A}_{\Sigma,\Delta}\mathbf{\Phi}_{\Sigma,\Delta} \end{bmatrix} \mathbf{\Gamma}_\Sigma \begin{bmatrix} \mathbf{A}_{\Sigma,\Delta} \\ \mathbf{A}_{\Sigma,\Delta}\mathbf{\Phi}_{\Sigma,\Delta} \end{bmatrix}^H \tag{18}$$

$\mathbf{\Phi}_{\Sigma,\Delta}$ can be estimated by ESPRIT in a similar way as described in previous section.

Now let see the structure of $\bar{\mathbf{R}}_{\Sigma,\Delta}$ for discussion. Firstly because of 4×4 dimension of $\bar{\mathbf{R}}_{\Sigma,\Delta}$, the maximum signal number K for ESPRIT is apparently $K=3$. Secondly DOAs for K signals can be estimated by using monopulse error curve because the k-th diagonal element of $\mathbf{\Phi}_{\Sigma,\Delta}$ is the conjugate of the monopulse ratio $a_\Delta(\theta_k)/a_\Sigma(\theta_k)$.

In other words, by implementing VESPA software in conventional two-channel monopulse system, an ability to estimate multiple DOAs up-to three targets can be added in an affordable way.

3. Computer Simulation

Computer simulations are carried out to validate angle accuracy of the proposed VESPA monopulse comparing with MUSIC. MUSIC is chosen as a representative superresolution method. A scenario that two uncorrelated non-Gaussian closely-spaced targets impinging on the uniform linear array (ULA) consisting of 12 elements with half-wavelength element spacing is considered. SNR is ranged from -10 to 40 dB. Note that SNR is defined after coherent integration of a single target signal. Angular differences of the targets are set at $0.1BW$, $0.3BW$, $0.5BW$, $0.7BW$ and $0.9BW$, where BW stands for a beamwidth of the sum beam. Since the proposed method has four array's degree-of-freedom, namely 4×4 dimension of $\bar{\mathbf{R}}_{\Sigma,\Delta}$, the four dimensional beamspace MUSIC is used for this simulation. Beamspace matrix is formulated by dividing 12 elements ULA into 4 regular subarrays. Detail condition for the simulation is summarized in Table 1 and Figure 1.

Table 1. SIMULATION CONDITION

Antenna type	ULA with the sum and difference beamformer.
Number of antenna elements	12 elements positioned by half-wavelength spacing
Angular difference for two targets	$0.1BW$, $0.3BW$, $0.5BW$, $0.7BW$ and $0.9BW$
Number of snapshot	8
SNR	-10 dB to 40 dB by 2 dB increment
Reference method	Single target: Conventional monopulse Two targets: Four dimensional beamspace MUSIC
Number of independent trial	500 Trial

In Figure 2, all 500 trial result by the proposed VESPA monopulse for SNR=40 dB is presented. DOAs for target signal #1 and #2 are at -0.25BW and +0.25BW respectively and the angular difference is 0.5BW. It is illustrated that the proposed method with two receiver channel can estimate distinct DOAs from two targets.

RMSE of estimated DOA by the proposed method for various angular differences is presented in Figure 3. Note that RMSE is normalized by the sum beamwidth and calculated from 'resolved' trials of 500 trials. In this simulation, trials that the two estimated DOAs are within $2BW$ are defined as 'resolved'. From Figure 3, it is observed that RMSE is improved by increasing SNR from around 10 dB. In this SNR region, no distinct differences of RMSE among $0.5BW$, $0.7BW$ and $0.9BW$ are found.

RMSE of estimated DOA for $0.5BW$ case by the proposed method and the beamspace MUSIC is compared in Figure 4. It is observed that the proposed method show higher accuracy than the MUSIC. Note that the accuracy gap is reduced as the angular difference is increased (No Figures are presented in this paper due to space limitation.).

Finally we added a single target scenario whose target angle is array normal. RMSE of estimated DOA by the proposed method comparing with conventional monopulse is presented in Figure 5. It is observed that the RMSE by both methods are comparable and the proposed VESPA monopulse shows no superiority to the conventional method. The reason can be illustrated by noticing that monopulse ratio is $a_\Delta(\theta_1)/a_\Sigma(\theta_1)=0$ in this scenario. Substituting this into (17) results in $\mathbf{\Phi}_{\Sigma,\Delta}=0$. Consequently $\bar{\mathbf{R}}_{\Sigma,\Delta}$ reduces to $\bar{\mathbf{R}}_{\Sigma,\Delta}=\mathbf{A}_{\Sigma,\Delta}\mathbf{\Gamma}_{\Sigma}\mathbf{A}_{\Sigma,\Delta}{}^{H}$, namely $\bar{\mathbf{R}}_{\Sigma,\Delta}=\mathbf{R}_{\Sigma,\Sigma}$ as seen in (13). Moreover there is an additive residual noise term in (13) by finite snapshot effect. As a result, the structure of sampled cumulant matrix would have the similar structure of the covariance matrix for conventional monopulse method.

4. Concluding Remarks

Monopulse angle estimation for unresolved multiple targets in antenna mainbeam was proposed in this paper. A fourth order cumulant was introduced to increase the array's degree-of-freedom of monopulse beamformer from two to four. Subsequently, a derivative VESPA algorithm was used for calculating the multiple monopulse ratios to enable angle estimation by existing monopulse processing. Results of computer simulation were provided to illustrate the performance comparison with the four dimensional beamspace MUSIC. The results show higher accuracy than the beamspace MUSIC for the unresolved two targets scenario.

References

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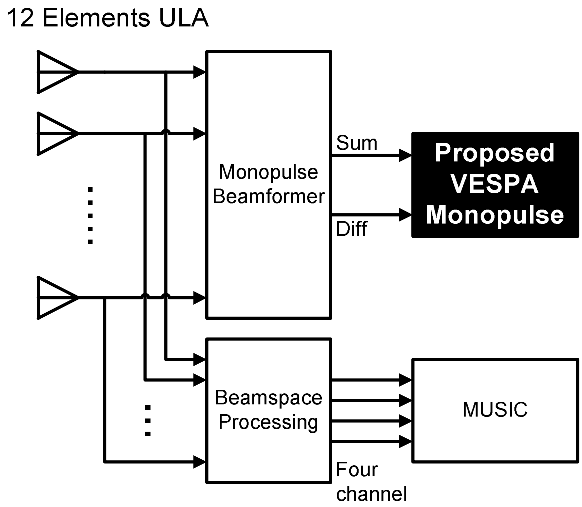

Figure 1. Processing block diagram of proposed VESPA monopulse. Beamspace MUSIC is used for comparison.

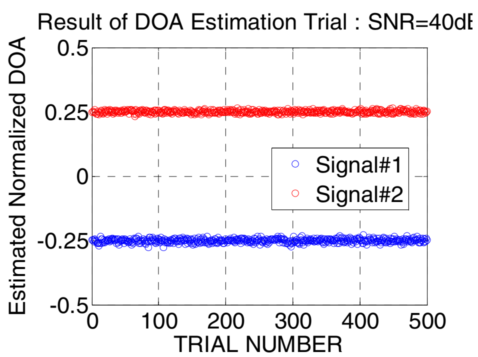

Figure 2. 500 trial result by the proposed VESPA monopulse for SNR=40 dB. DOA for signal #1 and #2 are -0.25BW and +0.25BW respectively and the angular difference is 0.5BW.

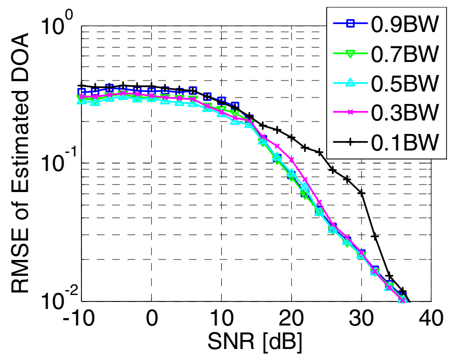

Figure 3. Normalized RMSE of angle estimation by the proposed VESPA monopulse for various SNR with different DOAs case.

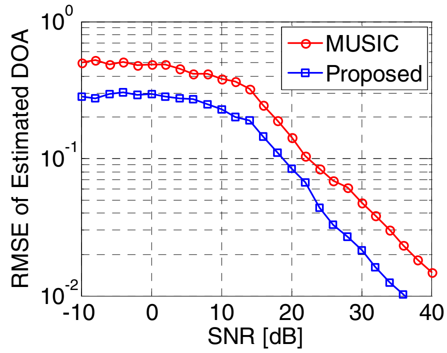

Figure 4. Normalized RMSE of angle estimation by the proposed VESPA monopulse and the four dimensional beamspace MUSIC for various SNR. Angular difference of DOA is a half of the beamwidth (0.5BW).

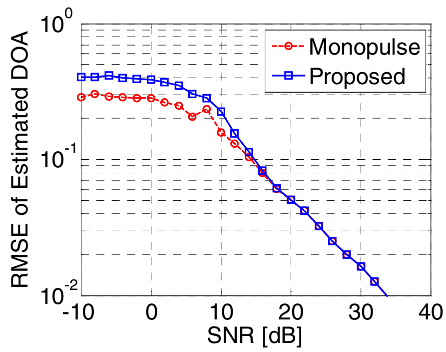

Figure 5. Normalized RMSE of angle estimation by the proposed VESPA monopulse and the conventional monopulse for various SNR. Single signal is impinging from array normal direction.