POS2-30

DOA Estimation Using Subspacing Tracking Method for Coherent Waves

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1. Introduction

Recently, by development of mobile communications, radio wave environments are made more complicated. In this situation, high-resolution DOA (direction of arrival) estimation algorithms such as MUSIC and ESPRIT [1] were proposed and currently they are attracting much attention. These algorithms make use of eigenvalue-decomposition (EVD) of array covariance matrix, which brings about high computational load. Therefore, the development of the sequential or iterative computation of the eigenvalues and eigenvectors has been expected. As one of the sequential computation methods, PAST (Projection Approximation Subspace Tracking) method [2] is proposed in recent years and the low computational load of it is noteworthy. In this paper, we improve the PAST method by using the sub-array scheme (spacial smoothing processing) and the unitary transformation so that the PAST method can deal with the coherent waves and the case of less snapshots. Computer simulation is carried out to demonstrate the effectiveness of the improved PAST method in DOA estimation with MUSIC algorithm.

2. Signal Model and DOA Estimation

2.1 Signal Model

Consider that the uniform linear array (ULA) is used for DOA estimation, which is composed of K isotropic elements with element spacing d as shown in Fig.1. Also, it is supposed that the ULA receive L ($L < K$) narrow-band waves with DOAs being $\theta_1,\ \theta_2,\ \cdots,\ \theta_L$, respectively, and with complex amplitudes being $s_1(t),\ s_2(t),\ \cdots,\ s_L(t)$, respectively. When the array response vector (mode vector) of the lth incoming wave is given by $\boldsymbol{a}(\theta_l)$ $(l = 1,\ 2,\ \cdots,\ L)$, the array input vector $\boldsymbol{x}(t)$ can be expressed as

$$\boldsymbol{x}(t) = \sum_{l=1}^{L} \boldsymbol{a}(\theta_l)s_l(t) + \boldsymbol{n}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{1}$$

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1),\ \boldsymbol{a}(\theta_2),\ \cdots, \boldsymbol{a}(\theta_L)],\ \boldsymbol{s}(t) = [s_1(t),\ s_2(t),\ \cdots,\ s_L(t)]^T \tag{2}$$

where $\boldsymbol{A}$ and $\boldsymbol{s}(t)$ are called the array response matrix (mode matrix) and the signal vector, respectively, and $\boldsymbol{n}(t)$ is the internal additive noise vector.

In using the spatial smoothing processing, the K-element ULA is divided into N ULAs (sub-arrays), each of which has the same number of array elements $M(= K - N + 1)$. Then, the nth sub-array input vector $\boldsymbol{x}_n(t)$ can be expressed as

$$\boldsymbol{x}_n(t) = [x_n(t), x_{n+1}(t), \cdots, x_{n+M-1}(t)]^T \ \ (n = 1, 2, \cdots, N) \tag{3}$$

2.2 PAST Method Using Sub-arrays

The PAST method using sub-arrays is base on the idea that $\boldsymbol{Q}_A$ minimizing the following cost function is the signal subspace matrix

$$J_s\left(\boldsymbol{Q}_A(t)\right) \triangleq \sum_{i=1}^{t}\sum_{n=1}^{N} \| \boldsymbol{x}_n(i) - \boldsymbol{Q}_A(t)\boldsymbol{h}_n(i) \|^2 \tag{4}$$

$$\boldsymbol{h}_n(t) \triangleq \boldsymbol{Q}_A^H(t-1)\boldsymbol{x}_n(t) \tag{5}$$

where $\boldsymbol{h}_n(t)$ is the compressed vector of the nth sub-array input vector. By minimizing eq.(4) with respect to $\boldsymbol{Q}_A(t)$, the optimal solution of $\boldsymbol{Q}_A$ is obtained as follows:

$$\boldsymbol{Q}_A(t) = \boldsymbol{C}_{xh}(t)\boldsymbol{C}_{hh}^{-1}(t) \tag{6}$$

$$\boldsymbol{C}_{xh}(t) = \sum_{i=1}^{t}\sum_{n=1}^{N} \boldsymbol{x}_n(i)\boldsymbol{h}_n^H(i),\ \boldsymbol{C}_{hh}(t) = \sum_{i=1}^{t}\sum_{n=1}^{N} \boldsymbol{h}_n(i)\boldsymbol{h}_n^H(i) \tag{7}$$

Introducing the forgetting factor $\alpha(0 < \alpha < 1)$, eq.(7) is expressed in the recursive form as

$$\boldsymbol{C}_{xh}(t) = \alpha\boldsymbol{C}_{xh}(t-1) + (1-\alpha)\sum_{n=1}^{N} \boldsymbol{x}_n(i)\boldsymbol{h}_n^H(i) \tag{8}$$

$$\boldsymbol{C}_{hh}(t) = \alpha\boldsymbol{C}_{hh}(t-1) + (1-\alpha)\sum_{n=1}^{N} \boldsymbol{h}_n(i)\boldsymbol{h}_n^H(i) \tag{9}$$

Then we define the matrix $\boldsymbol{D}_n$ as follows;

$$\boldsymbol{D}_0(t) = \alpha\boldsymbol{C}_{xh}(t-1) \tag{10}$$

$$\boldsymbol{D}_n(t) = \boldsymbol{D}_{n-1}(t) + (1-\alpha)\boldsymbol{h}_n(i)\boldsymbol{h}_n^H(i) \quad (1 \leqq n \leqq N) \tag{11}$$

where $\boldsymbol{D}_N(t) = \boldsymbol{C}_{hh}(t)$. Using a matrix formula [3], the inverse matrix of $\boldsymbol{P}_n(t)$ can be expressed in the recursive form as follows:.

$$\boldsymbol{P}_0(t) = \frac{1}{\alpha}\boldsymbol{P}_N(t-1) \tag{12}$$

$$\boldsymbol{P}_n(t) = \boldsymbol{P}_{n-1}(t) - \frac{(1-\alpha)\boldsymbol{q}_n(t)\boldsymbol{q}_n^H(t)}{1 + (1-\alpha)\boldsymbol{h}_n^H(t)\boldsymbol{q}_n(t)} \quad (1 \leqq n \leqq N) \tag{13}$$

where

$$\boldsymbol{P}_n(t) = \boldsymbol{D}_n^{-1}(t),\ \boldsymbol{q}_n(t) = \boldsymbol{P}_{n-1}(t)\boldsymbol{h}_n(t) \tag{14}$$

Since $\boldsymbol{P}_N(t)$ is equal to the inverse of $\boldsymbol{C}_{hh}(t)$, substituting eqs.(9) and (13) into eq.(6) and manipulating the resultant gives the recursive form as follows.

$$\boldsymbol{Q}_A(t) = \boldsymbol{Q}_A(t-1) + (1-\alpha)\sum_{n=1}^{N} \boldsymbol{x}_{n\perp}(t)\boldsymbol{g}_n^H(t) \tag{15}$$

where

$$\boldsymbol{g}_n(t) = \frac{\boldsymbol{q}_n(t)}{1 + (1-\alpha)\boldsymbol{h}_n^H(t)\boldsymbol{q}_n(t)},\quad \boldsymbol{x}_{n\perp}(t) = \boldsymbol{x}_n(t) - \boldsymbol{Q}_A(t-1)\boldsymbol{h}_n(t) \tag{16}$$

In this paper, the initial value of $\boldsymbol{Q}_A(t)$ and $\boldsymbol{P}(t)$ are given by

$$\boldsymbol{Q}_A(0) = \begin{bmatrix} \boldsymbol{I}_L \\ \boldsymbol{0} \end{bmatrix},\quad \boldsymbol{P}(0) = \boldsymbol{I}_L \times 10^{-2} \tag{17}$$

where $\boldsymbol{I}_L$ is an L-dimensional identity matrix.

2.3 PAST Method Using the Unitary Transformation

The unitary transformation is performed to the input vector as follows:

$$\boldsymbol{Q}_M^H\boldsymbol{x}(t) = \boldsymbol{y}(t) \tag{18}$$

where $\boldsymbol{Q}_M$ is a unitary matrix which is given by [1].

$$\boldsymbol{Q}_M = \frac{1}{\sqrt{2}}\begin{bmatrix} \boldsymbol{I}_m & j\boldsymbol{I}_m \\ \boldsymbol{\Pi}_m & -\boldsymbol{\Pi}_m \end{bmatrix} (M = 2m),\ \boldsymbol{Q}_M = \frac{1}{\sqrt{2}}\begin{bmatrix} \boldsymbol{I}_m & \boldsymbol{0} & j\boldsymbol{I}_m \\ \boldsymbol{0}^T & \sqrt{2} & \boldsymbol{0} \\ \boldsymbol{\Pi}_m & \boldsymbol{0} & -\boldsymbol{\Pi}_m \end{bmatrix} (M = 2m+1) \tag{19}$$

where $\boldsymbol{\Pi}_m$ is a square matrix of m-dimension as shown below.

$$\boldsymbol{\Pi}_m = \begin{bmatrix} 0 & \cdots & 0 & 1 \\ 0 & \cdots & 1 & 0 \\ \vdots & \iddots & \vdots & \vdots \\ 1 & \cdots & 0 & 0 \end{bmatrix} \tag{20}$$

The unitary transformation can be introduced to the PAST method by regarding the real part and imaginary part of transformed vector $\boldsymbol{y}(t)$ as an independent input vector. In this case, it is noted that $\boldsymbol{Q}_A(t)$, Re$\boldsymbol{y}$(t), Im$\boldsymbol{y}$(t) are all real-valued. By minimizing this cost function, the optimul solution of PAST method using the unitary transformation is obtained, and the recursive algorithm has the same form as the one using sub-arrays, in which the real part and imaginary part of transformed input vector are utilized as two input vectors.

3. Simulations

Under conditions shown in Tables1-3, computer simulation is carried out to clarify the performance of the improved method. For DOA estimation, Root-MUSIC[1] using the PAST method is employed. In each simulation, the conventional method, the one using sub-arrays, and the one using the unitary transformation are compared. Directions of arrival are changed by 0.01 degrees in each sample. As the evaluation measure of estimated results, RMSE (root mean square error) of DOA estimates is used, which is calculated through 1000 independent trials. The number of incoming waves are two and is assumed to be estimated exactly in any simulation of DOA estimation.

First, the convergence characteristics of the PAST methods are examined when the two incoming waves are coherent or incoherent with each other. The radio environment is described in Tables 2. RMSE of DOA estimates vs. discrete time in samples are shown in Figs.2 and 3 along with Cramer-Rao bound (CRB) [5]. From Fig.2, it is found that the convergence rate and estimation accuracy in the coherent case are improved by the proposed methods, especially by using sub-arrays. From Fig.3, it is observed that the convergence rate becomes rapid in the incoherent case both by using sub-arrays and by using the unitary transformation.

Next, computation times of the PAST methods are examined. The specification of computer used is described in Table3. The computation times vs. the number of elements are shown in Fig.4. Although the differences of computation times among three methods are recognized as shown in Fig.4, they are very small, so we can say the three methods have almost the same computation load.

Table 1: Simulation condition.

Array configuration	Uniform inear array of isotropic elements
Number of elements	6
Number of elements of sub-array	5
Element spacing	0.5λ
SNR	20dB
forgetting factor	0.9

Table 2: Radio enviroment.

	Power[dB]	DOA[degree]
1st wave	0	-30
2nd wave	0	50

Table 3: Specification of a computer.

CPU	Intel Core(TM) i5 2.80GHz
OS	Windows 7 professional
Software	MATLAB 2007b

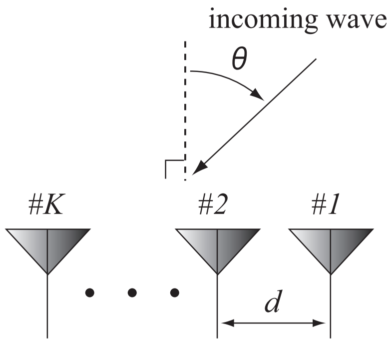

Figure 1: *K*-element uniform linear array. (element spacing: *d*)

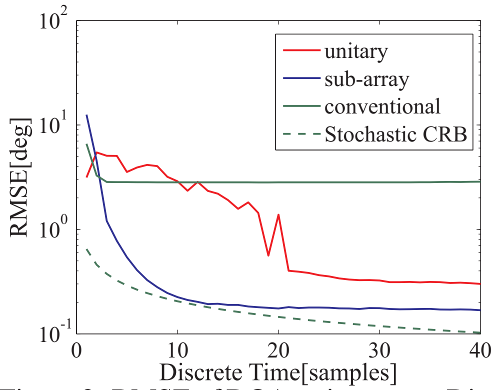

Figure 2: RMSE of DOA estimates vs. Discrete Time. (coherent case)

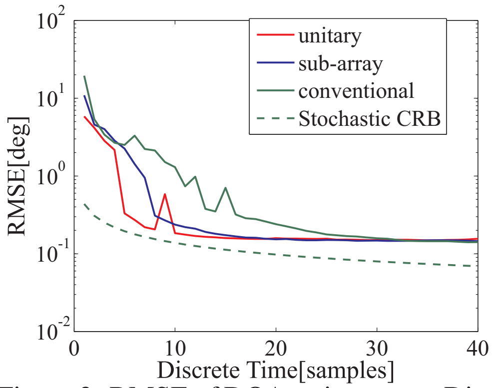

Figure 3: RMSE of DOA estimates vs. Discrete Time. (incoherent case)

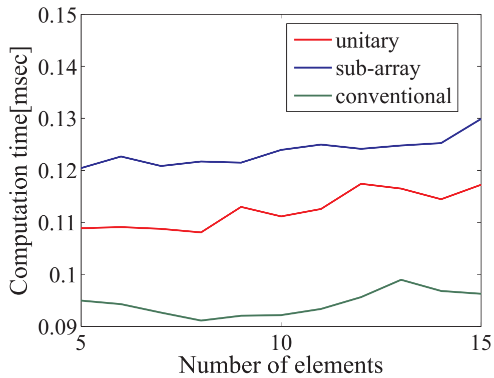

Figure 4: Computation time vs. Number of elements.

4. Conclusion

In this paper, we have improved the PAST method by incorporating the spatial smoothing technique using sub-arrays and the unitary transformation into the PAST method. Via computer simulation of DOA estimation, higher estimation accuracy and more rapid convergence characteristics of the improved PAST method has been confirmed. As the future work, we will merge the sub-array method and the unitary transformation method for more than two coherent waves, and try to estimate the number of waves using the subspace tracking method for iterative DOA estimation.

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