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Novel 3-D Array Configuration based on CRLB Formulation for High-Resolution DOA Estimation

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1. Introduction

Direction-Of-Arrival (DOA) estimation is one of significant techniques for high-speed mobile communication to know wave propagation environment [1]. High resolution DOA estimation algorithms like MUSIC [2], and ESPRIT [3] have already been widely used in many wireless communication applications. Generally planar array structures are used for 2-D DOA estimation. Planar arrays often well estimate azimuth angles but cannot well estimate elevation angles because of short antenna aperture in elevation direction. Therefore we try to develop a 3-D array configuration which well estimate both azimuth and elevation angles.

This paper presents a simple 3-D array configuration by simply modifying the height of some array elements in planar array. We start from the definition of CRLB [4] of DOAs, and show that CRLB of elevation angles could be improved by modifying the height of some array elements in planar array through some examples of actual 3-D arrays. Based on the analysis of CRLB formulation and its dependency on the height of array elements, we develop a 3-D array structure which improves elevation angle estimation accuracy while preserving azimuth angle estimation accuracy. Performance of the proposed array structure is evaluated through some DOA estimation simulations.

2. Signal Model

Assume that uncorrelated K waves are received by P-elements array antenna under an AWGN (Additive White Gaussian Noise) environment. With the positioning vector $\boldsymbol{r}_p (p = 1, ..., P)$ of the p-th element, the array steering vector components $\boldsymbol{a}_p$ $(p = 1, ..., P)$ is represented by

$$\boldsymbol{a}_p(\theta, \phi) = \exp\left[j\frac{2\pi}{\lambda}\boldsymbol{r}_p^T \boldsymbol{\ell}(\theta, \phi)\right] \quad (1)$$

$$\boldsymbol{\ell}(\theta, \phi) = [\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta]^T \quad (2)$$

where θ, ϕ, λ respectively denote the elevation, the azimuth and the wavelength of the source wave. Signal waveform and the DOA of k-th incident waves is given by $s_k(t)$ and (θ_k, ϕ_k) for $k = 1, 2, 3, ..., K$. With the array steering vector $\boldsymbol{a}(\theta_k, \phi_k) = [a_1(\theta_k, \phi_k), ..., a_P(\theta_k, \phi_k)]^T$, the array input vector $\boldsymbol{x}(t)$ is represented by

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \quad (3)$$

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1, \phi_1), \boldsymbol{a}(\theta_2, \phi_2), ..., \boldsymbol{a}(\theta_K, \phi_K)] \quad (4)$$

$$\boldsymbol{s}(t) = [s_1(t), s_2(t), ..., s_K(t)]^T \quad (5)$$

where $\boldsymbol{n}(t)$ is the thermal noise vector of 0-mean, with the variance σ^2.

3. 3-D array configuration

This section first analyses CRLB formulation and its dependency on the height of array elements, and then develops novel 3-D array configuration. The purpose of this paper is to improve the estimation accuracy of elevation angle θ as introduced in Section 1. In this paper, a novel 3-D array structure is proposed by shifting some elements of planar array, not by adding some array elements. We discuss a configuration of novel 3-D structure array configuration via the formulation of CRLB.

3.1 Formulation of CRLB

Assume that the k-th incident signal whose complex amplitude is $s_k(t)$ arrives from elevation θ_k and azimuth ϕ_k. Then the CRLB of the estimated value elevation θ can be expressed by the combination of

$$\boldsymbol{V} = \begin{bmatrix} \mathrm{var}(\theta_1) & & 0 \\ & \ddots & \\ 0 & & \mathrm{var}(\theta_K) \end{bmatrix} = \frac{\sigma^2}{2N_s}\mathrm{Re}\left[(\boldsymbol{B}^H\boldsymbol{P}_N\boldsymbol{B})\odot\boldsymbol{S}^T\right]^{-1} \tag{6}$$

$$\boldsymbol{S} = E\left[\boldsymbol{s}(t)\boldsymbol{s}(t)^H\right] \tag{7}$$

$$\boldsymbol{P}_N = \boldsymbol{I} - \boldsymbol{A}(\boldsymbol{A}^H\boldsymbol{A})^{-1}\boldsymbol{A}^H \tag{8}$$

$$\boldsymbol{B} = [\boldsymbol{b}(\theta_1), \boldsymbol{b}(\theta_2), \ldots, \boldsymbol{b}(\theta_K)] \tag{9}$$

$$\boldsymbol{b}(\theta) = \frac{\partial\boldsymbol{a}}{\partial\theta} \tag{10}$$

where $\odot$, Re[·], N_s , $E[\cdot]$ and $\boldsymbol{I}$ denote Hadamard product, real part, the number of snapshots, ensamble average and the identity matrix, respectively [4]. Using (6), CRLB for the elevation angle θ in the case of one wave source is given by

$$\mathrm{var}(\theta) = \frac{\sigma^2}{2N_sP_s}\cdot\frac{1}{AR}\cdot\left(\frac{\lambda}{2\pi}\right)^2 \tag{11}$$

where P_s is the signal power, and the term AR depends on the array configuration.

3.2 URA-Based 3-D Array

We develop 3-D array configuration modified from 4(2× 2) and 9(3× 3) elements URA whose interelements spacing is d_x (x direction), d_y (y direction). It is often necessary for DOA estimator to estimate arbitrary (whole 360°) azimuth angles. Therefore 3-D array configuration should be symmetric structure to improve estimation accuracy for the waves from arbitrary angles. The above symmetric concept also holds for the array configurations with larger number of array elements. We propose URA-based 3-D arrays described in Fig.1(a),(b). In the case of simple 4(2× 2) and 9(3× 3) elements URA, AR of the elevation angle θ, AR_{θ_4} and AR_{θ_9}, are given by

$$AR_{\theta\_4} = (d_x\cos\theta\cos\phi)^2 + (d_y\cos\theta\sin\phi)^2 \tag{12}$$

$$AR_{\theta\_9} = 6(d_x\cos\theta\cos\phi)^2 + 6(d_y\cos\theta\sin\phi)^2 \tag{13}$$

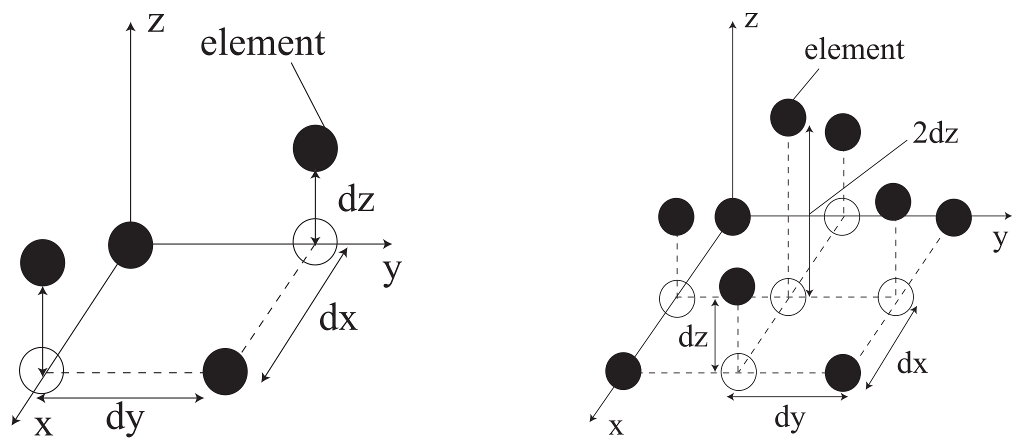
Next the term AR for the construction in Fig.1(a),(b) are given by

$$AR_{\theta\#1} = (d_x\cos\theta\cos\phi)^2 + (d_y\cos\theta\sin\phi)^2 + (d_z\sin\theta)^2. \tag{14}$$

$$AR_{\theta\#2} = 6(d_x\cos\theta\cos\phi)^2 + 6(d_y\cos\theta\sin\phi)^2 + 4(d_z\sin\theta)^2. \tag{15}$$

In this case, the first and second terms in (14), (15) are same with those in (12), (13). Then we see that the term $AR_{\theta\#1}$ and $AR_{\theta\#2}$ are certainly larger than AR_{θ_4} and AR_{θ_9} because the third term always becomes nonnegative. Indeed the CRLB of Fig.1(a),(b) becomes smaller than that of URA. If these array configurations are not symmetric structure, after third term of AR are not always positive and could be negative depending on DOA. Therefore the CRLB of asymmetric array configuration does not become always smaller than that of URA.

We can expand these ideas to $(2N+1)\times(2N+1)$ elements URA (page limit does not permit to describe details).



(a) 3-D array #1 (4-elements)

(b) 3-D array #2 (9-elements)

Figure 1: Configuration of 3-D arrays

Table 1: Specifications of simulation

array configuration	Fig.1(a)	Fig.1(b)
interelement space, d_x, d_y	0.45λ	
# of array elements, P	4	9
snapshot	200	
DOA estimator	MUSIC	
# of trials	1,000	
DOAs	uniform distribution	

4. Simulation

We investigate the effect of 3-D structure array antenna on DOA estimation in this section. Three 3-D array antennas in Figs.1(a),(b) are employed for simulation. Specifications of simulation is listed in Table 1.

4.1 Dependency on d_z

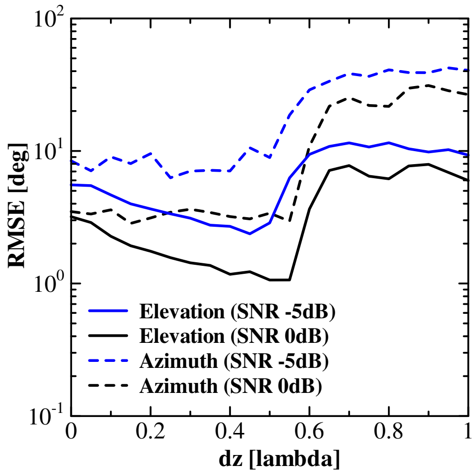
First we show the dependency of DOA estimation accuracy in the sense of Root Mean Square Error (RMSE) on d_z in Fig.2(a), for the configuration in Fig.1(a). In the case of 4 elements, as the height d_z becomes larger, the better the elevation angle estimation accuracy is achieved but it is observed only when $d_z < 0.5\lambda$, and the accuracy becomes remarkably worse for $d_z > 0.5\lambda$. This is because of pseudo peaks (grating lobes) when interelement spacing is larger than half-wavelength. Furthermore, azimuth estimation accuracy does not become worse and estimation accuracy is close to that of planar URA.

Next we consider 9-elements 3-D array antenna as in Figs.2(b). Figure 2(b) shows the behavior of RMSE in estimating DOAs. As to the case of 4-elements, we found that the better elevation angle estimation accuracy is achieved as the height d_z becomes larger when $d_z < 0.5\lambda$. Therefore the discussion in Section 3 is applicable even if the number of elements or array configuration is changed.

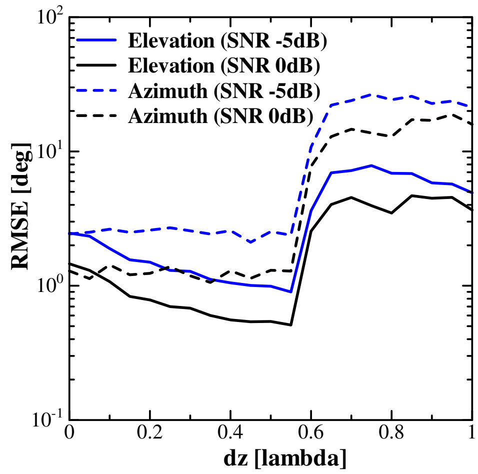
4.2 SNR Characteristics

We also show the SNR characteristics of RMSE in Figs.3(a),(b) for the case of $d_z = 0.45\lambda$. From Fig. 3, we see that the elevation angle estimation accuracy of 3-D arrays becomes much better than that of planar URA in the cases of both 4 and 9 elements. These results also confirm the results of the discussion in Section 3.

Indeed the azimuth estimation accuracy does not become worse when the height of each element is only shifted.

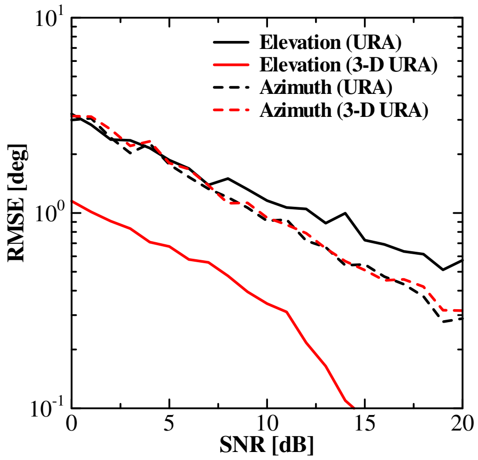


(a) 4-elements configuration

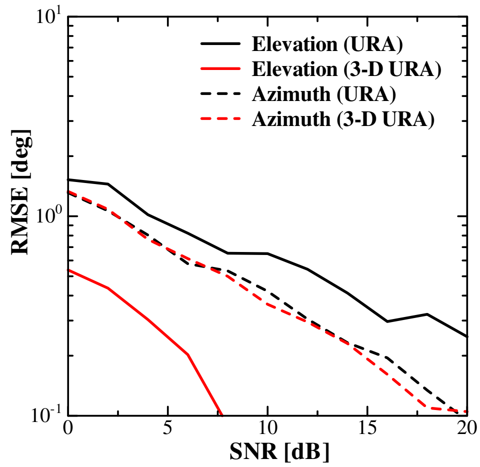


(b) 9-elements configuration

Figure 2: Behavior of RMSE for various values of the height d_z



(a) 4-elements configuration



(b) 9-elements configuration

Figure 3: Behavior of RMSE for various values of SNR

5. Concluding Remarks

In this paper, we proposed a way of developing 3-D array configuration which could improve elevation angle estimation accuracy of planar array while preserving azimuth angle estimation accuracy, without increasing the number of array elements. According to the formulation of CRLB, we found that the elevation angle estimation accuracy could be improved only by shifting the height of some array elements. One of future studies would be how to determine the optimum shift value d_z for more complicated array structures.

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