## Considerations on a Frequency Correlation for Distributed Antenna Systems

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## 1. Introduction

Radio technology has been developed extensively for mobile communications and wireless LANs. In a wireless communication channel, we have multipath propagation due to reflections and scatterings. When the delay difference between multipath waves is large, we have frequency selective fading. Thus far, a frequency correlation defined for a single antenna has been used for evaluating the frequency selectivity [1], [2]. Multiple antennas have been introduced to realize space diversity and/or spatial multiplexing. The conventional frequency correlation cannot be used for multiple-antenna systems.

Furthermore, in the case of MIMO-OFDM (Multiple Input Multiple Output-Orthogonal Frequency Division Multiplexing), it needs a weight for each subcarrier to realize space diversity and/or spatial filtering. However, since pilot symbols necessary for weight generation decrease the transmission efficiency, channel state information with a pilot symbol is used for neighbouring subcarriers. How many subcarriers can use the same weight depends on the behavior of the channel in the frequency domain. As a result, this issue is related to the frequency correlation.

Moreover, the frequency selective fading causes a degradation of a received signal level at some frequencies. A frequency diversity which uses the fact that received levels vary among different frequencies is a technique to solve this problem. When using the frequency diversity, independent or weak correlation branches are required to achieve the effect of the diversity. Therefore, it can be said that the effect of the frequency diversity in a multiple-antenna system depends on the proposed frequency correlation.

Thus far, we have proposed a frequency correlation for a centralized multiple-antenna system, and compared it with the conventional frequency correlation defined for a single antenna [3]. It has been clarified that when the antenna spacing is large, the proposed frequency correlation is different from the conventional one depending on the arrival angles. In this paper, we analyze the frequency correlation for distributed antenna systems [4] which have a large antenna spacing. Moreover, we introduce a degradation coefficient of signal detection to evaluate the effect of the frequency selectivity, and compare it with the frequency correlation.

# 2. Definitions of Frequency Correlation and Degradation Coefficient of Signal Detection

To facilitate understanding of the frequency correlation in a multiple antenna system, we state the definition proposed in the reference [3]. Let us consider a multiple-antenna system with N elements as shown in Fig. 1. We express channels between transmit and receive antennas at frequency f as  $h_1(f), h_2(f), \dots, h_N(f)$  and weights for the receive antennas as  $w_1, w_2, \dots, w_N$ . We define the vectors which consist of these as follows:

$$\boldsymbol{h}(f) = [h_1(f), h_2(f), \dots, h_N(f)]^T$$
(1)

$$\boldsymbol{w} = [w_1, w_2, \dots, w_N]^T,$$
 (2)



Figure 1: Multiple-antenna system configuration.

where  $[\cdot]^T$  denotes transpose. From (1) (2), the channel at the array output is given by

$$y(f) = \sum_{i=1}^{N} w_i h_i(f) = \boldsymbol{w}^T \boldsymbol{h}(f).$$
(3)

Using the conventional frequency correlation defined for a single antenna, we obtain

$$C(\Delta f) = \frac{\langle y^*(f)y(f + \Delta f) \rangle}{\langle y^*(f)y(f) \rangle}.$$
(4)

Substituting (3) into (4), we obtain

$$C(\Delta f) = \frac{\langle \boldsymbol{w}^H \boldsymbol{h}^*(f) \ \boldsymbol{w}^T \boldsymbol{h}(f + \Delta f) \rangle}{\langle \boldsymbol{w}^H \boldsymbol{h}^*(f) \ \boldsymbol{w}^T \boldsymbol{h}(f) \rangle}.$$
(5)

The above equation is the frequency correlation for a multiple-antenna system having weight vector w. This is a natural extension of the conventional frequency correlation.

In this paper, we use maximum ratio combining (MRC) weights for w. Thus, we have

$$\boldsymbol{w} = \boldsymbol{h}^*(f). \tag{6}$$

Then, substituting (6) into (5), we obtain

$$C(\Delta f) = \frac{\langle \boldsymbol{h}^{T}(f)\boldsymbol{h}^{*}(f)\boldsymbol{h}^{H}(f)\boldsymbol{h}(f+\Delta f)\rangle}{\langle \{\boldsymbol{h}^{H}(f)\boldsymbol{h}(f)\}^{2}\rangle}.$$
(7)

In the remainder of this paper, we consider  $C(\Delta f)$  given by (7) as the frequency correlation for a multipleantenna system.

In the above discussion, we defined the frequency correlation as a parameter to evaluate the frequency selectivity. One of the problems related to the frequency selectivity is signal detection in the case of OFDM. In this paper, in order to consider this problem, we propose a parameter to evaluate the effect of the frequency difference between a signal to be detected and a pilot symbol for channel estimation.

Here, let us consider that a symbol s is transmitted by a subcarrier at frequency  $f + \Delta f$ . We assume that this signal is received with a single antenna and detected by using channel state information obtained by a pilot subcarrier at frequency f. Then, we obtain the following equation as the input to a signal detector.

$$\frac{h(f+\Delta f)}{h(f)}s.$$
(8)



Figure 2: Distributed antenna system; 2 RAU case.

Figure 3: Distributed antenna system; 7 RAU case.

⋆ x

If the coefficient  $h(f + \Delta f)/h(f)$  in (8) is not 1, we have an error for signal detection. Here, we express  $h(f + \Delta f)/h(f)$  as  $E'(\Delta f)$  and consider it as a degradation coefficient of the signal detection. Next, we extend this equation to the multiple-antenna system shown in Fig. 1. We replace h(f) and  $h(f + \Delta f)$  in (8) with y(f) and  $y(f + \Delta f)$  given by (3), respectively. Then, substituting (6) into (8), we obtain

$$E'(\Delta f) = \frac{\boldsymbol{h}^{H}(f)\boldsymbol{h}(f+\Delta f)}{\boldsymbol{h}^{H}(f)\boldsymbol{h}(f)}.$$
(9)

Here, calculating the mean of (9) for statistical evaluation, we obtain

$$E(\Delta f) = \langle E'(\Delta f) \rangle$$
  
=  $\left\langle \frac{\mathbf{h}^{H}(f)\mathbf{h}(f + \Delta f)}{\mathbf{h}^{H}(f)\mathbf{h}(f)} \right\rangle.$  (10)

In the remainder of this paper, we consider  $E(\Delta f)$  given by (10) as the degradation coefficient of the signal detection due to the frequency difference.

#### 3. **Considerations on the Frequency Correlation**

Here, we numerically investigate the frequency correlation and the degradation coefficient of the signal detection due to the frequency difference for distributed antenna systems in an exponential delay profile case. We assume that the number of Remote Antenna Units (RAUs) [4] is 2 or 7 as shown in Figs. 2 and 3, respectively, and that 16 multipaths with the same time interval arrive at each RAU from a Mobile Terminal (MT). We also assume that the average power of each path decays successively by 1 dB and the delay spread is 500 ns. In this case, the time interval between the adjacent multipaths is 115.2 ns. Moreover, the average power of each multipath attenuates in proportion to the 4th power of the distance between a RAU and a MT, and each multipath experiences independent Rayleigh fading. We ignore the shadowing effect. A MT is located at one of the positions (250 m, 0 m), (50 m, 0 m) or (10 m, 0 m) in either case as shown in Figs. 2 and 3.

Figures 4 and 5 show the amplitudes of the frequency correlation defined by (7) and the degradation coefficient of the signal detection defined by (10) when the number of RAUs is 2 and 7, respectively. We generated independent 100,000 fading conditions using random numbers, and obtained the averaged values. As seen from these figures, the frequency correlation and the degradation coefficient have a periodicity of 8.68 MHz depending on the MT position. This is because the time interval between the adjacent multipaths is 115.2 ns. Moreover, in the case of 2 RAUs, the frequency correlation has ripples with the period of 3 MHz when the MT is located at (50 m, 0 m). In this case, since we have a delay difference due to the path difference of 100 m for the RAUs, the delay difference causes the periodicity of 3 MHz. As for the case of the MT position of (10 m, 0 m), the peak value at 8.68 MHz has a much lower value compared to the other MT positions. The path difference for the RAUs is 20 m, and this causes the periodicity of 15 MHz and the deep dip at 7.5 MHz. Thus, the peak level at 8.68 MHz has a





Figure 4: Frequency correlation  $|C(\Delta f)|$  and degradation coefficient  $|E(\Delta f)|$ ; 2 RAU case.

Figure 5: Frequency correlation  $|C(\Delta f)|$  and degradation coefficient  $|E(\Delta f)|$ ; 7 RAU case.

low value. These are the behavior stemming from the distributed antenna system. In the case where the MT position is (250 m, 0 m), we see the similar behavior to the conventional frequency correlation for a single antenna [3]. It is easily seen that the RAU at (-500 m, 0 m) is far apart from the MT at (250 m, 0 m), and that the signal received at the RAU at (500 m, 0 m) is dominant. Therefore, we can see only the periodicity due to the exponential delay profile fading. On the other hand, when the number of the RAUs is 7, all the frequency correlations almost coincide with each other except for the case where the MT position is (250 m, 0 m). For the MT position of (250 m, 0 m), we can see small ripples caused by the distributed antenna system. Furthermore, it is seen that the degradation coefficient of the signal detection shows almost the same behavior as that of the frequency correlation regardless of the MT position and the number of RAUs.

### 4. Conclusions

In this paper, we have investigated the proposed frequency correlation for distributed antenna systems in the exponential delay profile case. It has been clarified that the frequency correlation for the 2 RAU case varies depending on the MT position. On the other hand, in the case of 7 RAUs, it has been shown that all the frequency correlations almost coincide with each other. Moreover, it has been shown that the frequency correlation and the degradation coefficient of the signal detection show almost the same behavior. Therefore, we can evaluate the degradation of the signal detection using the proposed frequency correlation.

## References

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