# Asymptotic Analysis of a Wearable Device Attached to the Human Body by Using Sommerfeld integral

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## **1. Introduction**

As cellular phones, personal digital assistants (PDAs), digital video cameras, pocket video games, and other information and communication devices become smaller and more widespread, such appliances have begun to occupy a personal space in daily life and the opportunities to use such small computers have been increased. In the current of high technologies, the investigation for the miniaturization of such devices and the improvement of a communicating environment will lead the actualization of a ubiquitous computing society [1]. At present, the technology of telecommunications is developing rapidly, but the exchange data directly among such personal devices is so restrictive yet. The connection of such devices with a cable is clearly impractical because various kinds of cable are necessary and the physical inconvenience should be endured. Therefore, some sort of short-range wireless technology is required. The concept for networking such personal devices has been proposed as Personal Area Networks (PANs) which uses the human body as a transmission channel [2]. In this reference, the earth ground, which includes all conductors and dielectrics in the close environment to a PAN device, was introduced for the return path between a transmitter and a receiver in the network. The earth ground was applied to the modelling of the wearable transmitter which transmits a signal in the environment of PANs, and the necessity of the earth ground was established by an equivalent circuit model [3].

A Wearable equipment, which uses the human body as a transmission channel, has been focused on the attention because prominent advantages in relation to a security and a convenience can be expected in the living parts like an auto login system, an electric money or a direct exchange of data. Such a wearable device forms a transmission channel when the user wearing the wearable equipment touches the receiving electrode of a receiver. This communication system is created by the electromagnetic wave generated from the device itself and the coupled components with the human body. Up to now, the transmission mechanism of a wearable device was researched only by the numerical calculation, such as the finite-difference time-domain (FDTD) method, and it was verified by an experimental method [3].

In this paper, in order to investigate the fundamental propagating principle of a wearable device attached to the human body in the view of electromagnetics, a simplified theoretical analysis is suggested with two assumptions on which the human body consists of a single tissue and makes an infinitely planar structure being able to approximate to the upper body. Firstly, the representation of cylindrical wave, which can be converted from the spherical wave of a point source by the Sommerfeld identity of Green's function, is used to derive the exact field formulation [4]. The analysis modes of waves are determined in accordance with the directions of source currents. In addition, in order to analyze the influence of the human body as a lossy medium, the reflection and transmission coefficients by the electrical characteristics of the human body according to an operating frequency are separately calculated. The derivation of such coefficients uses virtual perfect magnetic conductor (PMC) and perfect electric conductor (PEC) walls for the pertinent application of transmission-line theory. The exact field formulation is performed by asymptotic expansions of Fourier-type integrals and the geometrical optics (GO) approximation [5]. Finally, the contribution of surface or leaky wave is considered with the location of poles calculated by each reflection or transmission coefficient [6]. The results of this full-wave analysis can be calculated by the method of moment (MoM) solution.

#### 2. Structure for Analysis

A wearable device, which has to propagate a power through the surface of the human body, requires that two electrodes named as signal and GND electrodes are adhered to the human body like shown in figure 1. The requisition of two electrodes was verified by an equivalent circuit models including the impedance characteristic of the human body [3]. Figure 2 shows the planar structure of figure 1 at y = 0 plane. In this case, a wearable device can be approximately represented to a finite bent wire. To simplify the analysis, a finite bent wire is analyzed as a wearable device under the infinitely planar structure of the human body, which consists of a single tissue, in *xy*-plane. The difference between two devices in figures 1 and 2 is only the overall current distribution caused by the gross area of a device.



Fig. 1. Structure of a wearable device [3].



Fig. 2. Planar structure for analysis of a wearable device (y = 0 plane).

### **3. Exact Field Formulation**

The finite bent wire can be considered as several segments separated by an appropriately small size. Each segment of the wire can be treated as an infinitesimal dipole. The whole wire should be analyzed by dividing it into two classes. One is the vertical electric dipole (VED) component to the planar human body and another is the horizontal electric dipole (HED). When the human body is located infinitely on *xy*-plane and the wire is placed like in figure 1, VED corresponds to the analysis of  $TE_z$  and  $TM_z$  modes, and HED to the  $TE_x$  and  $TM_x$  modes.

Sommerfeld identity of Green's function written in Eq. (1) is available to solve a 3-dimensional problem in a planar structure. It is the representation of a spherical wave into cylindrical waves, which can be expressible with Hankel function, in  $\rho$  direction and a plane wave in z direction over all wave number  $k_{\rho}$  [4]. The exact electric field formulation satisfying the inhomogeneous Helmholtz equation can be expanded by using the formulation of *TE* and *TM* modes in rectangular coordinates.

$$G(\mathbf{r}-\mathbf{r'}) = \frac{1}{4\pi} \frac{e^{-jk_{z_{0,1}}|\mathbf{r}-\mathbf{r'}|}}{|\mathbf{r}-\mathbf{r'}|} = \frac{1}{4\pi} \left( \frac{1}{j_2} \int_{-\infty}^{\infty} \frac{k_{\rho}}{k_{z_{0,1}}} H_0^{(2)} (k_{\rho}|\rho - \rho'|) e^{-jk_{z_{0,1}}|z-z'|} dk_{\rho} \right)$$
(1)

The wave numbers of z direction in the region 1 or 3, and region 2 are, respectively,

$$k_{z0} = \sqrt{k_0^2 - k_{\rho}^2}$$
,  $k_{z1} = \sqrt{k_1^2 - k_{\rho}^2}$ . (2)

The field formulation has to contain the reflection coefficient by the human body. In order to solve the reflection coefficient, regions 2a and 2b in figure 2 are first treated separately with a perfect magnetic conductor (PMC) or a perfect electric conductor (PEC) at z = d [5]. Two solutions of reflection coefficient are denoted as  $\Gamma_{PMC}$  and  $\Gamma_{PEC}$ , respectively. According to the image theory, when a source is located at (x', z') on the left of z = d, an image source is created at (x', 2d - z'). If two solutions are subtracted or added, the general solutions of reflection coefficient with the location of a source can be solved. If a source is located in region 1 or 2a,  $\Gamma^{VED}$  and  $\Gamma^{HED}$  are

$$\Gamma^{VED,HED} = \left(\Gamma^{VED,HED}_{PMC} + \Gamma^{VED,HED}_{PMC}\right)/2 , \qquad (3-a)$$

and if a source is in region 2b or 3,  $\Gamma^{VED}$  and  $\Gamma^{HED}$  are

$$\Gamma^{VED} = \left(\Gamma^{VED}_{PEC} - \Gamma^{VED}_{PMC}\right)/2 \quad , \quad \Gamma^{HED} = \left(\Gamma^{HED}_{PMC} - \Gamma^{HED}_{PEC}\right)/2 \quad . \tag{3-b}$$

The reflection coefficients of the PMC and PEC in the case of VED or HED are in terms of *TM* and *TE* reflection coefficients

$$\Gamma_{PMC}^{VED,HED_{TM}} = \frac{jk_{z1}\cot(k_{z1}d) + \varepsilon k_{z0}}{jk_{z1}\cot(k_{z1}d) - \varepsilon k_{z0}}, \quad \Gamma_{PEC}^{VED,HED_{TM}} = \frac{jk_{z1}\tan(k_{z1}d) - \varepsilon k_{z0}}{jk_{z1}\tan(k_{z1}d) + \varepsilon k_{z0}}$$

$$\Gamma_{PMC}^{VED,HED_{TE}} = \frac{j\mu k_{z0}\cot(k_{z1}d) + k_{z1}}{j\mu k_{z0}\cot(k_{z1}d) - k_{z1}}, \quad \Gamma_{PEC}^{VED,HED_{TE}} = \frac{j\mu k_{z0}\tan(k_{z1}d) - k_{z1}}{j\mu k_{z0}\tan(k_{z1}d) + k_{z1}}$$
(4)

where  $\varepsilon = (\varepsilon_1 \varepsilon_0 - j\sigma/\omega)/\varepsilon_0$ ,  $\mu = 1$  on the environment of surrounding the free space. The transmission coefficients to region 2 from region 1 are

$$T_{21\_PMC}^{VED,HED\_TM} = \frac{j2k_{z1}\cos[k_{z1}(d-z)]}{jk_{z1}\cos(k_{z1}d) - \epsilon k_{z0}\sin(k_{z1}d)} , \quad T_{21\_PEC}^{VED,HED\_TM} = \frac{j2k_{z1}\sin[k_{z1}(d-z)]}{jk_{z1}\sin(k_{z1}d) + \epsilon k_{z0}\cos(k_{z1}d)}$$
(5)  
$$T_{21\_PMC}^{VED,HED\_TE} = \frac{j2\mu k_{z0}\cos[k_{z1}(d-z)]}{j\mu k_{z0}\cos(k_{z1}d) - k_{z1}\sin(k_{z1}d)} , \quad T_{21\_PEC}^{VED,HED\_TE} = \frac{j2\mu k_{z0}\sin[k_{z1}(d-z)]}{j\mu k_{z0}\sin(k_{z1}d) + k_{z1}\cos(k_{z1}d)}.$$

When a source is in region 3 and region 1 is observed, the transmission coefficient expressed as  $T_{13}$  is identical to the reflection coefficient in the case of a source in region 3.

If the suitable application of reflection or transmission coefficient and the consideration of a phase shift in z direction are involved in the integrand of the Sommerfeld integral of Eq. (1), exact field formulations in all regions can be derived.

#### 4. Asymptotic Expansions

There are largely two types for the asymptotic expansion of Fourier-type integrals except for uniform expansions. If the method of stationary phase and Sommerfeld identity are used to the Sommerfeld integrals, exact field formulations can be approximated as the function of the solution of scalar wave equation like Eq. (1) in spherical coordinates. Furthermore, the saddle-point method (or the steepest descent method) and GO make the integrals to represent in clear and concise expressions. The stationary phase point  $k_{\rho s}$  of the method of stationary phase is  $k_0 \sin \theta$  which is the same as the steepest descent point of the saddle-point method.

Figure 3 is the source coordinates used for the saddle-point evaluation of incidence, reflection, transmission from region 1 to region 2 ( $T_{21}$ ), and transmission from region 3 to region 1 ( $T_{13}$ ). When a source is at (x', z'), its image for reflection is placed at (x', -z'). For the transmission case of  $T_{13}$ , a field at (x, z) is due to a virtual source at (x', 2d - z'), and for the case of  $T_{21}$ , the distance to the boundary face between the free space and the human body should be only considered in the field formulation. Reciprocity property can be applied in the relation between  $T_{13}$  and  $T_{31}$  because of the symmetrical environment with the axis of symmetry at z = d.

$$G_{13}(x,z;x',z') = G_{31}(x',z';x,z)$$
(6)

The asymptotic expansions of the Sommerfeld integral with complex  $k_{\rho}$  in Eq. (1) make the variable of integrand change to complex w. Figure 4 shows the original integral path  $P_s$  and asymptotic integral path  $P_A$  in complex w plane. The transformation of the variable of integrand aids to establish the contribution of poles, which is like the path  $P_P$  in figure 4, due to each reflection or transmission coefficient as well as the simplification of the integral. The location of poles is determined by the electric characteristics ( $\varepsilon_1$  and  $\sigma$ ) and the thickness (2*d*) of the human body.

$$A^{2} - B^{2} = (\mu \varepsilon - 1)(k_{0}d)^{2}$$
<sup>(7)</sup>

$$j\mu B = -A \cot A \left[ \Gamma_{PEC}^{TE} \text{ and } T_{21\_PEC}^{TE} \text{ poles} \right], \quad j\mu B = +A \tan A \left[ \Gamma_{PMC}^{TE} \text{ and } T_{21\_PMC}^{TE} \text{ poles} \right]$$

$$j\epsilon B = +A \tan A \left[ \Gamma_{PEC}^{TM} \text{ and } T_{21\_PEC}^{TM} \text{ poles} \right], \quad j\epsilon B = -A \cot A \left[ \Gamma_{PMC}^{TM} \text{ and } T_{21\_PMC}^{TM} \text{ poles} \right]$$
(8)

where  $A = k_{z2} d$ ,  $B = k_{z1} d$ . The poles are found from the intersection of (7) and (8). The pole contribution at any pole detours  $P_P$  can be calculated by the residue theorem taking a clockwise detour. By the saddle-point method, the steepest descent point  $\theta$  having a real value physically represents the angle between the spherical polar distance of an observation point and x = x' axis:  $\theta_R$ ,  $\theta_{T13}$ , or  $\theta_{T21}$  in figure 3. Only when a physical angle  $\theta$  is larger than the saddle-point ( $\theta_P$ ) through a pole, the interpretation of surface or leaky wave is added to the overall analysis [6].

For reference, at 2.45 GHz, all saddle-points ( $\theta_P$ ) through the calculated poles shown in figure 4 are under  $-\pi/2$  or beyond  $+\pi/2$ . However, in figure 3, all kinds of physical angle relative to reflection and transmission coefficients have the range of  $-\pi/2$  to  $+\pi/2$ . Therefore, the consideration about surface or leaky wave is not necessary at 2.45 GHz because a valid pole in the analysis does not exist.



Fig. 3. Source coordinates for saddle-point evaluation Fig. 4. Integral paths in complex *w* plane of the incident, reflected, and transmitted fields. and location of poles at 2.45 GHz.

#### **5.** Conclusion

In the environment of the infinitely planar human body, this paper developed the exact field formulations from a point source by using the Sommerfeld integral, and the simplified analysis was suggested by asymptotic expansions. In the application of the saddle-point evaluation, the possibility, which the contribution of surface or leaky wave pole influences on the whole analysis, was considered also.

Now, the results from this analysis are calculating. Additionally, the investigation about the existence of surface or leaky wave according to an operating frequency is in progress too. An improved wearable device will be proposed on the basis of an analyzed propagating principle as a wearable device.

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