POS2-10

Vertical transmission from catenaries

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1. Introduction

Rhombic or delta antennas are frequently used, e.g. for ionospheric sounding, because of their large gains. Normally, in such antennas, the suspended wire forms a catenary that substantially changes the transmitted electric field. It is therefore important to investigate the vertical transmission from catenaries in order to obtain a better antenna design, suitable for real conditions. We analyze the performance of resistance-terminated traveling-wave aerials, with real shapes and heights. Analytical formulas for the electric field transmission in vertical direction are deduced for such aerials with one and two catenaries. A perfect, flat ground for the reflection is assumed, and the attenuation along the wire is neglected. It is shown that aerial catenaries produce shallower gaps in the electric field intensity compared to straight-wire constructions. Representative calculations are presented for the ionosonde transmit antenna at the RMI Geophysical Centre in Dourbes, Belgium [1].

2. Catenary characteristics

Let's consider the characteristics of a catenary (Fig.1, left). The suspended wire forms an arc TAB with point T being at the lowest end. In Cartesian coordinates: $y = D\,Ch(x/D)$ with $D = OT$. Also, $s = D\,tg\alpha$ and $ds = D\,d(tg\alpha)$, where s is the arc length from point T and α is the angle between the tangent and the horizontal axis.

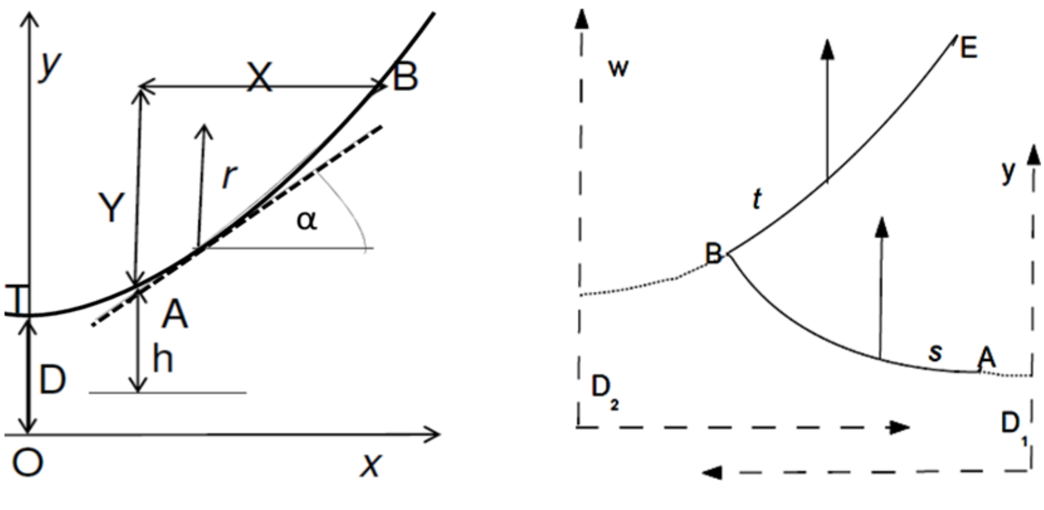

Figure 1: Catenary properties for one (left) and two (right) wires

From geometric considerations: $s = D\,Sh(x/D)$, $y - s = D\exp(-x/D)$, $y + s = D\exp(x/D)$, and $y = D/\cos\alpha = D\sqrt{tg^2\alpha + 1}$ for (-π/2 < α< π/2). By introducing a new variable $q = \sqrt{tg^2\alpha + 1} - tg\alpha$, obviously $y - s = D\left(\sqrt{tg^2\alpha + 1} - tg\alpha\right) = Dq$. Also, $tg\alpha = (1 - q^2)/2q$, $dtg\alpha = -(1 - q^2)dq/2q^2$, $\cos\alpha = 2q/(1 + q^2)$, and $\cos\alpha ds = -Ddq/q = Dqd(1/q)$. To find D , y_A , q_A , and q_B from the known values $x_B - x_A = X$, $y_B - y_A = Y$, and $s_B - s_A = L$, we use hyperbolic functions [3].

3. Vertical transmission using antennas with one and two catenaries

Assuming the electric current $I_s = I_A \exp[-jk(s - s_A)]$ is without attenuation, the element of the direct electric field at a distance R from a wire of length L (at an angle $\pi/2 - \alpha$ from the vertical) is: $dE_s = 30\, j\, k\, I_s \cos\alpha \exp(-jkr)\, ds / r$, with $F = 30 I_A / R \exp(-jkR)$ and $r = R - y + y_A$. Considering the newly introduced variable q and the relation $\cos\alpha\, ds = -D\, dq / q$, we obtain for the (direct) electric field:

$$E_d = \int_A^B dE_s = -j\, D\, F \exp(-j\, k\, D\, q_A)$$

Having $z = k\, D\, q$, $\exp(-j\, k\, D\, q_A) = \exp(-z_A)$, $G_d = j\, k\, F\, D \exp(-j\, z_A)$, and after some transformations [2], we obtain:

$$E_d = -G_d \int_A^B \exp(jz)/z dz = -G_d \{C_i(z_B) - C_i(z_A) + j[S_i(z_B) - S_i(z_A)]\} = -G_d [C_i(z) + jS_i(z)]_A^B$$

where C_i and S_i are respectively the cosine and sine integrals [3].

Reflection from the ground should also be considered. Assuming a perfect flat ground, and taking into account that $r = R + y - y_A + 2h$, we obtain:

$$E_r = j\, k\, F \exp[jk(y_A + s_A - 2h)] \int_A^B \cos\alpha \exp[-jk(y + s)]\, ds$$

But $y + s = D/q$ and $\cos\alpha\, ds = D\, q\, d(1/q)$, so having also $u = k\, D/q$, $g = 4\pi h / c$, and $G_r = j\, k\, F\, D \exp[\, j(u_A - g\, f)\,]$, then

$$E_r = G_r \int_A^B \exp(-ju)/u\, du = G_r [C_i(u) - jS_i(u)]_A^B .$$

Thus, the total field at a large distance from the wire in vertical direction becomes:

$$E_t = E_d + E_r = -G_d [C_i(z) + jS_i(z)]_A^B + G_r [C_i(u) - jS_i(u)]_A^B .$$

Let's now consider two catenaries (Fig.1, right). The parameters are different for each catenary and the relative phase generated by two catenaries must also be considered. This relative phase involves three factors; note that (t,w) replaces (s,y) for the upper wire BE and $L_1 = s_A - s_B$.

First, the relative position of the second wire is against the current, which means there is a factor (-1) in front of the second wire's contribution.

Second, the current is given by: $I_t = I_B \exp[-jk(t - t_B)] = I_A \exp[-jk(L_1 - t_B)] \exp(-jkt)$.

Third, the (vertically) travelled distance is $r = R - H_1 - w + w_B$ in upward direction and $r = R + H_1 + 2h + w - w_B$ downward. Thus, the direct contribution has an added phase factor:

$$(-1)\exp[-jk(R + w_B + L_1 - H_1 - t_B + t - w)] = (-1)\exp[-jk(R + L_1 - H_1 + w_B - t_B)] \exp[-jk(t - w)]$$

and the reflected contribution has:

$$-\exp[-jk(R - w_B + L_1 - t_B + H_1 + 2h + t + w)] = -\exp[-jk(R + L_1 + H_1 - w_B - t_B + 2h)] \exp[-jk(t + w)]$$

The terms G_d and G_r (in front of the integrals) for the second wire (using D_2 for the second catenary instead of D) become:

$$G_{2d} = jkFD_2 \exp[-jk(D_2 q_{2B} + L_1 - H_1)] = jkFD_2 \exp(-jz_{2B}) \exp(-jkM_1 f), \quad M_1 = 2\pi(L_1 - H_1)/c$$

and

$$G_{2r} = -jkFD_2 \exp(ju_{2B} - jgf) \exp(-jN_1 f), \quad N_1 = 2\pi(H_1 + L_1)/c$$

So, for the total field we have:

$$E_t = \{-G_{1d}[C_i(z_1) + jS_i(z_1)] + G_{1r}[C_i(u_1) - jS_i(u_1)]\}_A^B$$
$$+ \{G_{2d}[C_i(z_2) + jS_i(z_2)] - G_{2r}[C_i(u_2) - jS_i(u_2)]\}_B^E$$

For a rhombic antenna with catenaries, we have: $L_1 = L_2$, $D_1 = D_2$, $z_1 = z_2$, $u_1 = u_2$. Therefore:

$$E_t = 2\left\{[-G_{1d} + G_{2d}][C_i(z_1) + jS_i(z_1)]_A^B + [G_{1r} - G_{2r}][C_i(u_1) - jS_i(u_1)]_A^B\right\} =$$

$$= 2kFD\left\{\exp(-jz_A)[1-\exp(-jMf)][C_i(z) + S_i(z)]_A^B + \exp(ju_B - jgf)[1-\exp(-jNf)][C_i(u) - jS_i(u)]_A^B\right\}$$

As in the case of two-wire rectilinear rhombic aerial [2], we also have here the factors due to phase reference at point A: direct, $1-\exp(-jMf)$, and reflected, $1-\exp(-jNf)$.

4. Results and Discussion

To demonstrate the catenary influence, computation results for the direct, reflected, and total field are presented (Fig.2) for a wire AB with a length of 42.6m, inclined at 26°, and raised at h=3m. This is the case for the ionosonde transmit antenna in Dourbes [1]. The curve “t1” is for a rectilinear wire, “t2” is for a catenary with added length of 0.02m, “t3” for one with added length of 0.07m, “t4” for added length of 0.12m, and “t5” for added 0.42m, everything else unchanged. All values are computed at a 10 km height above the phase reference A, with a current of 1A.

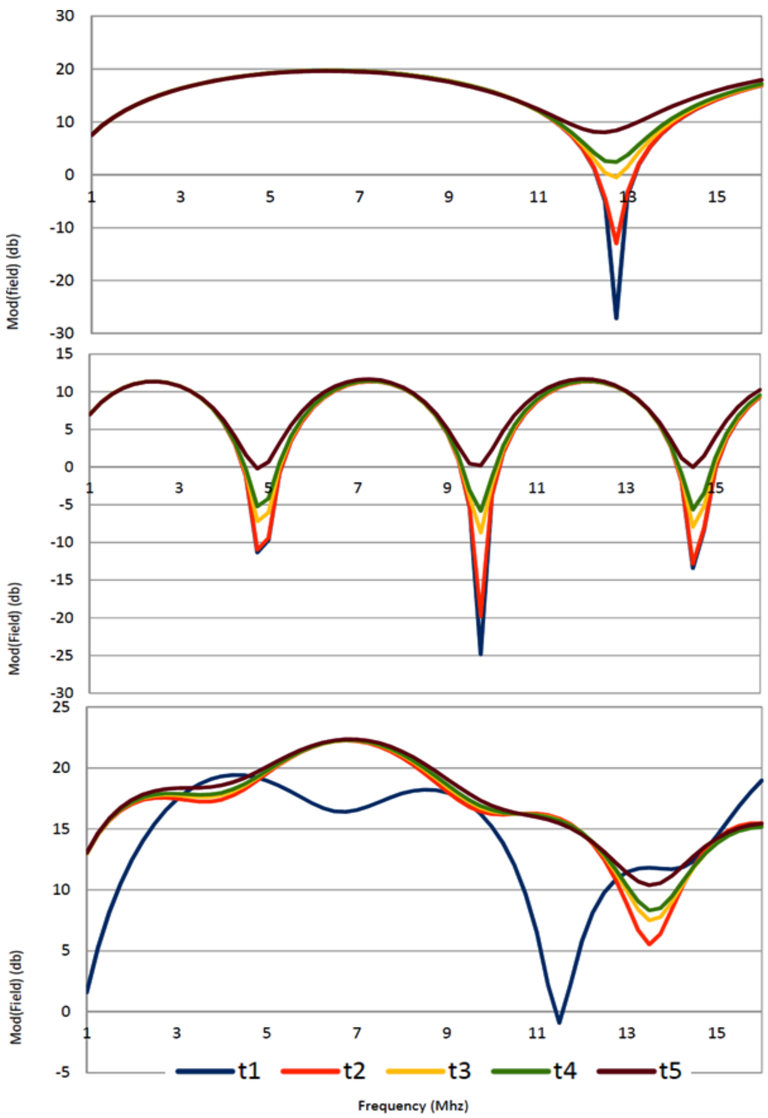

Figure 2: Electric field (modulus) for direct (top panel), reflected (middle), and total (bottom) transmission from a wire

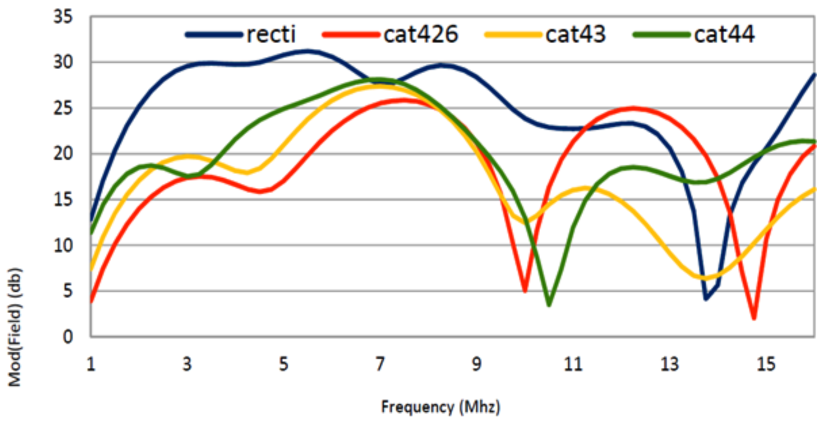

Figure 3: Total field (modulus) for a rhombic and catenaries

As another example (Fig.3), the case of two rectilinear wires (rhombic antenna) is compared with catenaries having the same extreme points. The curve "recti" is for rectilinear wires, "cat426" is for a catenary with added length of 0.2m, "cat43" is for one with added length of 1.4m, etc. Again, all values are computed at 10 km height above the phase reference, with a current of 1A.

In conclusion, it has been shown that consideration for the real shape of the wire is very important for optimization of the vertical transmission. The results show an improved bandwidth as constructive interferences act to fill some gaps. The search for an optimal solution leading to a shape as flat as possible can be facilitated by computing the following ratio. Having the discrete modulus values of the total E field corresponding to the set of computed frequencies (e.g. 1.0, 1.5, 2.0, ... , 16.0 MHz), one can compute the mean and the variance then make the ratio of the variance over the mean. The lowest value of this ratio indicates the best choice; decision can be made also by visually examining the plotted discrete values. We remind that the results here were obtained with the following assumptions: a) the current is only phased by the travel in the wire and not attenuated, b) the terminal impedance is perfectly matched, and c) the ground reflection is presumed perfect.

Acknowledgements

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References

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