

# Resonance Frequency Calculation of Microstrip Structure Located on Cylindrical Surface Using Hybrid Technique

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**Abstract** – This paper presents the analysis of open microstrip structures placed on dielectric coated, conducting cylindrical surfaces. The numerical analysis of investigated structures is based on expansion of electric and magnetic fields into cylindrical function series. The boundary problem is solved with the use of hybrid technique combining a finite difference technique and a method of moments, which takes into account the arbitrary shape of microstrip patch. Several numerical examples are presented. The results are verified by comparing them with the ones obtained from commercial software.

**Index Terms** — Complex resonance frequency, Conformal structure, Cylindrical microstrip structure, FDFD method, Method of moments.

## 1. Introduction

The conformal microstrip structures, due to the aerodynamic or hydrodynamic reasons, find many practical applications in airplanes, spacecraft, speedboats and other high-speed vehicles. The most popular application of these structures are in antenna technique for the construction of conformal antennas. Additionally, the conformal antennas have larger angular coverage, wider operation band and lower RCS compared to planar antennas. Conformal structures are commonly analyzed with the use of the mode matching method [1], method of moments (MoM) [2]–[4], the finite element method often combined with other approaches [5], or the finite-difference time-domain (FDTD) method [6]. In the case of large structures high frequency approaches based on the asymptotic techniques are utilized [7]. In this article the problem of determining the complex resonant frequency of the microstrip structures with arbitrary shape, located on dielectric coated, conducting cylindrical surface is considered. In order to solve the problem, the fullwave approach based on the combination of the method of moments and the finite difference technique in cylindrical coordinates is utilized.

## 2. Formulation of the Problem

The schematic view of the investigated structure is illustrated in Fig. 1. Considering a single dielectric layer substrate, the structure can be divided into two regions: region 1 - dielectric layer with a relative permittivity  $\epsilon_r$  covering a metallic cylinder and region 2 - outside of the

structure. Microstrip patch is located at the interface between regions 1 and 2. The  $z$  components of electric  $E$  and magnetic  $H$  fields take the form:

$$P_z^{(\kappa)} = \int_{-\infty}^{\infty} \sum_{m=-M}^M (H_m^{(1)}(\xi_\kappa) A_{m,\kappa}^F + J_m(\xi_\kappa) B_{m,\kappa}^F) \times e^{j(k_z z + m\phi)} / 2\pi dk_z \quad (1)$$

where  $F = \{E, H\}$ ,  $A_{m,\kappa}^F$  and  $B_{m,\kappa}^F$  are unknown coefficients,  $J_m(\cdot)$  and  $H_m^{(1)}(\cdot)$  are Bessel and Hankel functions, respectively, of order  $m$ ,  $\xi_\kappa = k_{\kappa\rho}$ ,  $k_{\kappa\rho} = \text{sqrt}(\omega^2 \mu_0 \epsilon_0 - k_z^2)$ , for  $\kappa = \{1, 2\}$  denoting the region. Because we do not consider any incident field from the outside region the coefficients  $B_{m,2}^E = 0$  and  $B_{m,2}^H = 0$ .

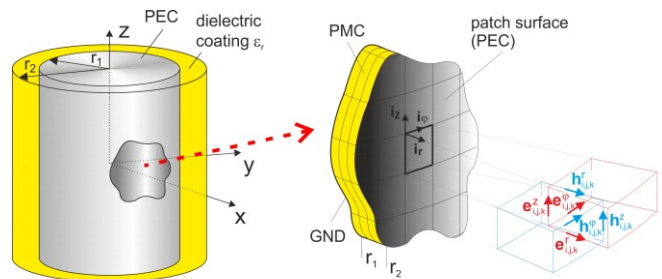


Fig. 1. General configuration of the investigated structure and the cavity model approach for current basis functions evaluation using FDFD method.

In order to determine the resonance frequency the boundary conditions must be satisfied for the tangential field components at the metallic surface of the cylindrical core ( $E_t^{(1)} = 0$  at  $\rho = r_1$ ) and between the regions at  $\rho = r_2$ :

$$E_t^{(2)} - E_t^{(1)} = 0, \quad \mathbf{i}_a \times (\mathbf{H}_t^{(2)} - \mathbf{H}_t^{(1)}) = \mathbf{J}, \quad (2)$$

where  $\mathbf{J}$  is an unknown current distribution on the surface of microstrip patch and  $\mathbf{i}_a$  denotes a normal vector to the patch surface. In order to solve the above problem, the patch current should be expanded in a series of current basic functions:

$$\mathbf{J} = \sum_{p=1}^P a_p \mathbf{J}_p \quad (3)$$

where  $\mathbf{J}_p$  are the appropriate current basis functions and  $a_p$  are unknown expansion coefficients of the expansion. The most common choice of the basis functions is the cavity-model functions with or without the edge-singularity condition for the tangential component of the surface current at the patch edges. For simple cases such as rectangular patches the basis functions have analytical form and can be found in [2]–[4]. For the complex shapes of the patch, in order to determine the field distribution in the cavity we utilize the finite-difference frequency-domain (FDFD) method which for the case of spherical antennas was described in [8]. Next, we take into account that the scalar product of the electric field and current on the patch is zero, and apply MoM with the current basis functions as testing functions, obtaining a set of homogeneous equations [2]. Nontrivial solutions (complex resonance frequency) can exist if the determinant of this set equals zero.

### 3. Numerical Results

Two examples of microstrip patch structure have been analyzed. First structure is composed of a rectangular patch with two cut slits placed on a dielectric substrate with relative permittivity  $\epsilon_r=2.32$  and thickness  $h=r_2-r_1=2\text{mm}$ , covering a metallic cylindrical core of radius  $r_1=50\text{mm}$ . The dimensions of the patch are  $26\text{mm} \times 30\text{mm}/r_2$  and the slit width is  $2\text{mm}/r_2$ . The obtained resonant frequencies for  $\text{TM}_{10}$  and  $\text{TM}_{01}$  modes as a function of slit length are shown in Fig. 2. It can be seen that the increase of the length of the slits causes a decrease in the resonant frequency of  $\text{TM}_{10}$  mode and does not affect  $\text{TM}_{01}$  mode.

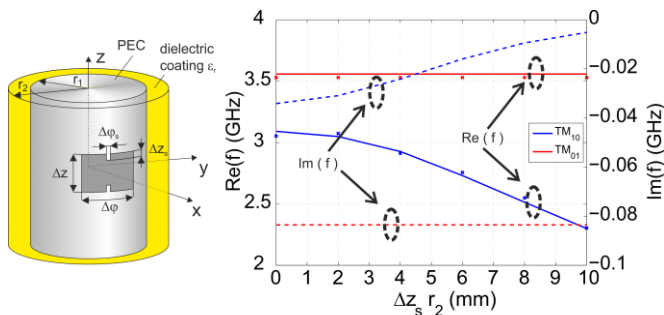


Fig. 2. The geometry of the rectangular-cylindrical structure and resonant frequencies of  $\text{TM}_{10}$  and  $\text{TM}_{01}$  modes in function of slot lengths. Solid and dashed lines - this method; Crosses - HFSS.

Second structure is a composed of an elliptical patch of dimensions  $20.8\text{mm} \times 37.5\text{mm}/r_2$  placed on a dielectric substrate with relative permittivity  $\epsilon_r=2.32$  and thickness  $h=r_2-r_1$ , covering a metallic cylindrical core of radius  $r_1=50\text{mm}$ . The obtained resonant frequencies for  $\text{TM}_{10}$  and  $\text{TM}_{01}$  modes as a function of substrate height are presented in Fig. 3. It can be seen that the resonant frequencies of both modes decrease with the increase of substrate thickness.

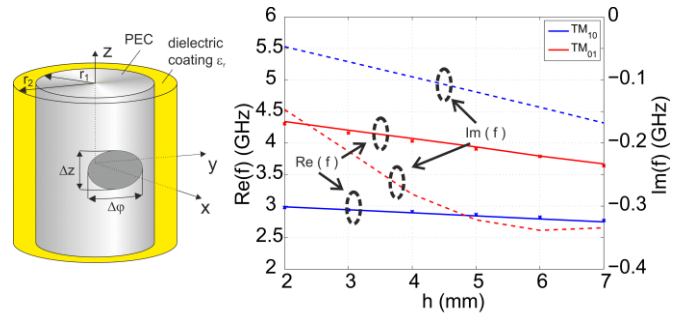


Fig. 3. The geometry of the elliptical-cylindrical structure and resonant frequencies of  $\text{TM}_{10}$  and  $\text{TM}_{01}$  modes in function of substrate thickness  $h$ . Solid and dashed lines - this method; Crosses - HFSS.

### 4. Conclusion

The procedure for calculating the complex resonance frequencies of arbitrary shape microstrip structure placed on dielectric coated conducting cylinder was proposed. A hybrid technique utilizing a method of moment and a finite difference frequency domain technique was employed. Examples of microstrip patches with complex geometries were analyzed and the obtained results were verified by comparing them with calculations of commercial software. A good agreement was achieved proving the correctness of the presented approach.

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