Approximate Field Continuity Conditions for Thin Anisotropic Conductive Layer

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Abstract - **In this paper the approximate field continuity conditions are developed for a very thin anisotropic conductive layer. In order to relate the tangential components of electric and magnetic fields on both sides of the investigated layer the transfer matrix is derived using transmission line approximation. The transfer matrix allows to examine the effect of electromagnetic field scattering on the anisotropic thin layer. Numerical results concerning circular waveguide with magnetized thin semiconductor plate are presented to validate the proposed continuity conditions.**

Index Terms — **Semiconductor, graphene, approximate field continuity conditions.**

1. Introduction

The magnetically biased thin plates of material in which free carriers interact with alternating electric fields are characterized by a conductivity tensor. This material property results mainly from Hall effect appearing in gyroelectric plasma, magnetized semiconductors [1]-[3] and 2D or volumetric graphene [4]-[6]. In these gyroelectric materials the nonreciprocal effects occur in the frequencies from microwave up to optical range. The Faraday and field displacement effects [1],[3] were observed in waveguides loaded with thin plate of high mobility semiconductor or graphene biased by longitudinal or transverse magnetic field.

Commonly, the surface impedance continuity conditions specified for a thin semiconductor, graphene or metal slab are applied in the analysis of these structures [4], [5], [7]. The tensor element expressions can be derived from semi-classical and quantum-mechanical approaches and depend on different material parameters and operation frequency.

In this contribution we propose the novel continuity conditions derived for thin slab with anisotropic conductivity. This condition is represented in a form of a transfer matrix relating the tangential components of electric and magnetic fields on both sides of this slab. The transfer matrix is found using the transmission line approximation for the propagation of plane wave in the considered thin layer. The proposed model allows us to include in the analysis the thickness of the layer. Moreover, it can be used in discrete methods e.g. finite difference technique, where such layers are assumed to have a few meshcells thickness. The proposed field continuity conditions are applied for the circular waveguide comprising semiconductor plate which is magnetized in the direction of propagation. In this arrangement the influence of semiconductor thickness on the polarization properties of TE11 mode is examined. The obtained numerical results are verified using HFSS.

2. Approximate Continuity Conditions

Let us consider a thin material slab in Let xy-plane with parameters $\varepsilon = \varepsilon_0$, $\mu = \mu_0$ and conductivity given in the dyadic form $\boldsymbol{\sigma} = \sigma \mathbf{I}_t + \sigma_a \mathbf{J}_t$, where $\mathbf{I}_t = \mathbf{a}_x\mathbf{a}_x + \mathbf{a}_y\mathbf{a}_y$ and $\mathbf{J}_t = \mathbf{a}_y\mathbf{a}_x - \mathbf{a}_x\mathbf{a}_y$. The σ is longitudinal conductivity parallel to the electric field and σ_a is Hall conductivity perpendicular to the electric field and resulting from magnetic field applied along z-axis. Examples of such slabs are semiconductor films, graphene sheets and isotropic and anisotropic metal foils. We assume that the slab is originated at z_1 and has thickness d which is small compared to the wavelength. The waves inside the material propagate only perpendicularly to the slab surface. In this case the transverse electric field defined as $\mathbf{E}_t = E_x\mathbf{a}_x + E_y\mathbf{a}_y$ must satisfy the following wave equation:

$$\nabla_z^2 \mathbf{E}_t + k_0^2 \{(1 - j\chi)\mathbf{E}_t + j\chi_a (\mathbf{a}_z \times \mathbf{E}_t)\} = 0\,, \tag{1}$$

where k_0 and η_0 are wave number and intrinsic impedance in a free space, respectively, $\chi=\eta_0\sigma_v/k_0$, $\chi_a=\eta_0\sigma_{va}/k_0$, σ_v and σ_{va} are in (S/m). The relation between transverse magnetic and electric field components is written as:

$$\mathbf{a}_z \times \nabla_z \mathbf{E}_t = -jk_0\eta_0 \mathbf{H}_t\,. \tag{2}$$

Assuming z-dependence of field as exp($-jkz$) and projection of (1) and (2) on the coordinate axis, we may find the expressions describing the variation of the electric and magnetic fields along the z-axis inside the slab:

$$\mathbf{F}(z) = \mathbf{Q}\mathbf{D}(z)\mathbf{A}\,, \tag{3}$$

where $\mathbf{F} = [E_x, E_y, H_y, H_x]^T$ is a vector of tangential field components at z_i-plane and $\mathbf{A} = [A_1, A_2, A_3, A_4]^T$ is a vector of unknown field coefficients. In equation (3) matrix $\mathbf{Q}$ = [1,1,1,1;-j,-j,j,j;Y_1,-Y_1,Y_2,-Y_2;jY_1,–jY_1,-jY_2,jY_2] and matrix $\mathbf{D}$ = diag{exp($-jk_1\delta$), exp($jk_1\delta$), exp($-jk_2\delta$), exp($jk_2\delta$))}, where $\delta = z-z_1$, $k_n = k_0[(1-j\chi)-\alpha_n\chi_a]^{1/2}$, $\alpha_n=(-1)^n$, $Y_n = k_n/(k_0\eta_0)$ and n = 1,2. Using equation (3) we define the fields at interfaces z_1 and $z_2=z_1+d$ bounding the slab which can be related with the use of transfer matrix as follows:

$$\mathbf{F}(z_i) = \mathbf{T}\mathbf{F}(z_{i-1})\,, \tag{4}$$

where $\mathbf{T} = \mathbf{QD}(z_i)\mathbf{Q}^{-1}$ and its elements can be simplified to closed form expressions given as:

$$T_{11} = T_{22} = T_{33} = T_{44} = \frac{\left(\cos(k_1 d) + \cos(k_2 d)\right)}{2}$$

$$T_{12} = -T_{21} = -T_{34} = T_{43} = \frac{j\left(\cos(k_1 d) - \cos(k_2 d)\right)}{2}$$

$$T_{13} = -T_{24} = -j\,\frac{k_0\eta_0}{2}\left(\frac{\sin(k_1 d)}{k_1} + \frac{\sin(k_2 d)}{k_2}\right)$$

$$T_{14} = T_{23} = \frac{k_0\eta_0}{2}\left(\frac{\sin(k_2 d)}{k_2} - \frac{\sin(k_1 d)}{k_1}\right).$$

$$T_{32} = T_{41} = -\frac{j}{2k_0\eta_0}\left(k_1 \sin(k_1 d) + k_2 \sin(k_2 d)\right)$$

$$T_{32} = T_{41} = \frac{1}{2k_0\eta_0}\left(k_1 \sin(k_1 d) - k_2 \sin(k_2 d)\right)$$

The proposed continuity conditions simplify to the ones presented in [7] for thin isotropic (σ_a=0) conductor.

3. Numerical Results

The transfer matrix is utilized to examine the polarization properties of TE_{11} mode in circular waveguide containing a thin plate of semiconductor (see Fig. 1). In the analysis two orthogonal TE_{11} modes oriented along *x*- and *y*-axis have been assumed.

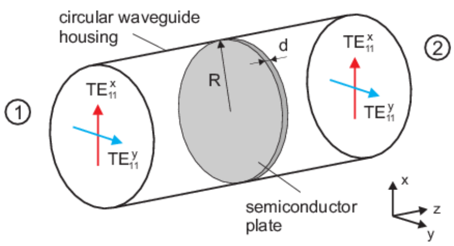

Fig. 1. Investigated structure of circular waveguide containing thin semiconductor plate.

The calculated scattering parameters at f_0 = 100GHz for the investigated structure as a function of semiconductor plate thickness are presented in Fig. 2(a). It can be noticed that when the structure is excited with TE_{11}^x mode we observe the coupling to the orthogonal mode TE_{11}^y caused by semiconductor plate. As a result the polarization plane rotation is observed for TE_{11}^x mode exciting the structure (see Fig. 2(b)). The angle of polarization plane rotation increases with the increase of the thickness of semiconductor plate. The obtained results are in good agreement with HFSS.

4. Conclusion

In this paper the approximate boundary conditions are derived for thin anisotropic conductive layer. This boundary conditions are defined by transfer matrix describing the relation between tangential components of electric and magnetic fields on both sides of the investigated layer. The derived in this paper transfer matrix for anisotropic conductors can be used in the analysis of variety of structures

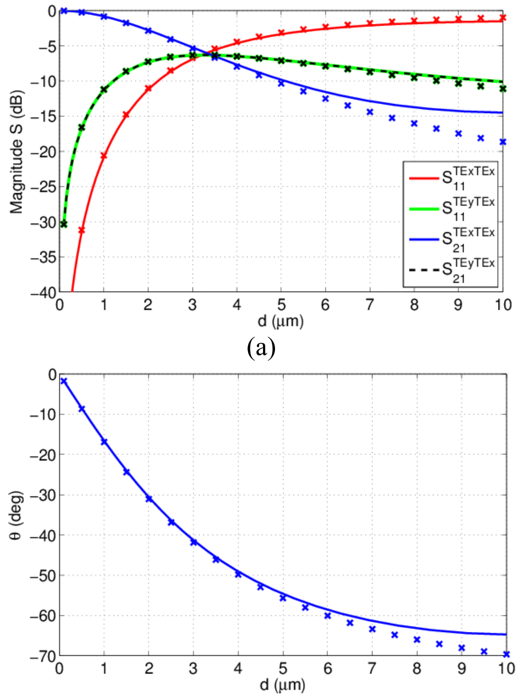

Fig. 2. Results for structure from Fig. 1 excited with TE_{11}-mode as a function of semiconductor plate thickness: a) scattering parameters and (b) polarization plane rotation angle (solid line - our method, crosses - HFSS). Parameters: f_0=100GHz, R=1mm, σ_v=22.37-i14.03S/m, σ_{va}=907.35-i0.692S/m.

containing semiconductor or graphene layers. The example of such structure is the one investigated in this paper where Faraday rotation is observed. The obtained results are verified with the use of HFSS.

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