Extension of Two-Level Nested Array with Larger Aperture and More Degrees of Freedom

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Abstract - **This paper presents a novel array configuration which improves the accuracy for high-resolution direction of arrival (DOA) estimation using the concept of Khatri-Rao (KR) product. We extended the concept of two-level nested array and found a novel array configuration which has larger array aperture and more degree of freedom compared with that of the two-level nested array. The performance of the proposed array is evaluated through computer simulation.**

1. Introduction

Direction-of-arrival (DOA) estimation plays an important role in radar, sonar, and indoor and outdoor wireless communications. High resolution DOA estimation methods using sensor arrays have been studied in the last three decades and have attracted much attention [1], [2]. The well-known methods, MUSIC and ESPRIT are based on the eigenvalue decomposition of a sample covariance matrix of an array input.

Recently the DOA estimation using the concept of Khatri-Rao (KR) product [3] has been proposed which can identify the sources more than the number of array elements. The nested array [4] is also an efficient array configuration, and there is a closed-form expression even that is not a Minimum Redundancy Array (MRA) [5]. However, most of the other efficient arrays also have closed-form expressions.

In this paper, we propose a novel array configuration which can be regarded as an extension of two-level nested array with the concept of KR product. This array also has a closed-form expression of array geometry and has larger aperture and more Degree Of Freedom (DOF) than those of two-level nested array. The performance of the proposed array is evaluated through computer simulation.

2. Preliminaries

(1) Signal Model

Assume that L far-field incident signals are received by an M-element linear array in an additive white Gaussian noise (AWGN) environment, where the signals and noises are statistically independent. The array input vector $\boldsymbol{x}(t)$ can be written as $\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t)$ where $\boldsymbol{A}$, $\boldsymbol{s}(t)$, and $\boldsymbol{n}(t)$ denote the array steering matrix, incident signal vector, and noise vector, respectively.

(2) DOA estimation using the concept of KR product [3]

We assume uncorrelated sources so that the source autocorrelation matrix of becomes diagonal. The covariance matrix $\boldsymbol{R}_{xx} = \mathrm{E}[\boldsymbol{x}(t)\boldsymbol{x}^H(t)]$ is then vectorized as $\boldsymbol{z} = \mathrm{vec}(\boldsymbol{R}_{xx}) = (\boldsymbol{A}^* \odot \boldsymbol{A})\boldsymbol{p} + \mathrm{vec}(\boldsymbol{R}_N)$, where $*$, $\odot$ and $\mathrm{vec}(\cdot)$ respectively denote the conjugation, KR product and the vectorization, and $\boldsymbol{p}$ consists of the diagonal element of $\boldsymbol{S}$ [3].

The distinct rows of $\boldsymbol{A}^* \odot \boldsymbol{A}$ behave like the manifold of a longer array whose sensor locations are given by the distinct values in the set $\{d_i - d_j,\ i, j = 0, 1, \cdots, M-1\}$ where d_i denotes the position vector of the ith sensor of the original array. This array is precisely the difference co-array of the original array. Hence we can apply DOA estimation to the vectorized data $\boldsymbol{z}$ and work with the difference co-array instead of the original array.

(3) Two-level Nested Array [4]

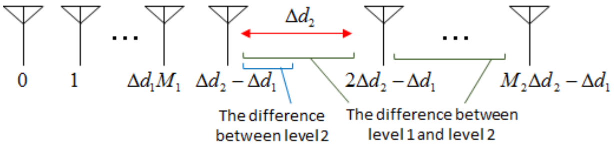
Two-level nested array is basically a concatenation of two Uniformed Linear Arrays (ULAs), where the first ULA has M_1 elements with the interval Δd_1 and the second ULA has M_2 elements with the interval Δd_2 where $\Delta d_2 = (M_1 + 1)\Delta d_1$. More precisely, it is a linear array with the sensors locations given by the union of the sets $S_1 = \{(m_1 - 1)\Delta d_1, m_1 = 1, \cdots, M_1\}$ and $S_2 = \{m_2\Delta d_2 - \Delta d_1, m_2 = 1, \cdots, M_2\}$ as shown in Fig. 1(a).

3. Proposed Array Configuration

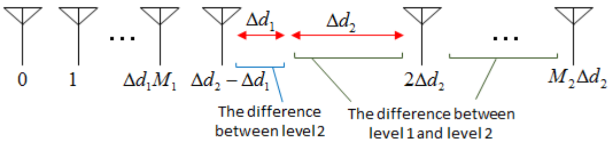
Two-level nested array has more degree of freedom than the ULA with same number of elements, and there is a closed-form expression of array geometry while there is not for MRA [5]. However, the others arrays also have a closed-form expression of array geometry and therefore two-level nested array is not the array which has the maximum DOF.

We extend the concept of two-level nested array by shifting all the level 2 elements to have Δd_1-more interval as shown in Fig. 1 (b), where M_{p1}, M_{p2} denote the number of elements in each level in the proposed array. The location of the proposed array is represented as

$$\Delta d_1 = M_{p1}\Delta d_1,$$
$$S_1 = \{(m_1 - 1)\Delta d_1\},$$
$$S_2 = \{(m_2 - 1)\Delta d_2\} = \{(m_2 + 1)\,(M_{p1} + 1)\Delta d_1\},$$



(a) two-level nested array



(b) proposed array

Fig. 1. M sensors array.

TABLE I
The Difference Co-array

u_1	u_2	the difference co-array
M_{p1}	S_1	$\{0, 1, 2, \cdots, M_{p1}\}$
S_2	S_2	$\{M_{p1} + 1\}$
S_2	S_1	$\{M_{p1} + 2, \cdots, (M_{p1} + 1)(M_{p2} + 1) - 1\}$

where $M_{p1} = M_1 + 1$, $M_{p2} = M_2 - 1$, $m_1 = 1, \cdots, M_{p1}$ and $m_2 = 1, \cdots, M_{p2}$. The proposed array can create the virtual ULA from $-(M_{p1} + 1)(M_{p2} + 1) + 1$ to $(M_{p1} + 1)(M_{p2} + 1) - 1$ after the extension in the case of $\Delta d_1 = 1$ for simplicity.

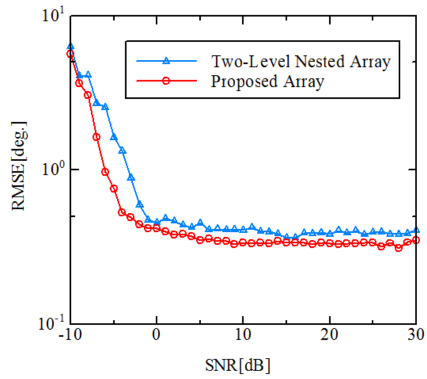
Now we explain the reason why the proposed array has ULA after the extension. Let S denote the set of sensor positions in the proposed array that is given by $S = \{0, 1, \cdots, M_{p1} - 1, M_{p1}\} \cap \{(M_{p1} + 1)(n + 1) - 1\}$, where $n = 1, 2, \cdots, M_{p2}$. The difference set of the proposed array is denoted as $S_{aug} = \{u_1 - u_2\}$, $u_1, u_2 \in S$.

Table I shows the difference co-array in some cases. For example, in the case of $u_1 = M_{p1}$ and u_2 is an arbitrary value in S_1, the position of the difference co-array becomes $\{0, 1, 2, \cdots, M_{p1}\}$. The sum of their difference co-arrays makes a difference co-array on the positive (right) side in the proposed array, and we can make the other co-array on the negative (left) side by the same approach. As a whole, this difference co-array forms a ULA from $-(M_{p1} + 1)(M_{p2} + 1) + 1$ to $(M_{p1} + 1)(M_{p2} + 1) - 1$.

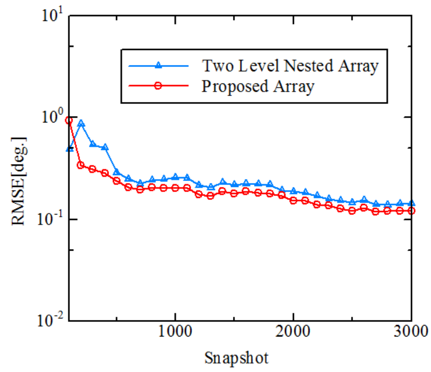
4. Simulation

The DOA estimation accuracy of the proposed array is evaluated through computer simulation and compared with the accuracy of the method of [4]. We consider the case of $M = 6$ array elements and $L = 6$ uncorrelated incident signals. The array locations are {0, 1, 2, 3, 7, 11} for the two-level nested array, and {0, 1, 2, 3, 8, 12} for the proposed array. The six DOAs are randomly given between [–60, 60] where the DOAs have difference at least 5 degree.

Figure 2 shows the behavior of RMSE as a function of SNR and the number of snapshots. The number of snapshots is set to 200 in Fig.2(a) and the SNR is set to 10 dB in Fig.2(b). We can see from Fig. 2 that the proposed method



(a) Behavior of RMSE as function of SNR



(a) Behavior of RMSE as function of snapshot

Fig. 2. 6 sensors array.

achieves better performance than the two-level nested array due to the larger array aperture.

Also we confirmed that the proposed array have more DOF; the proposed array could estimate DOAs of up to 12 sources while the two-level nested array could do up to 11 sources.

5. Conclusion

This paper proposed an extended version of two-level nested array, which has more degree of freedom.

Acknowledgment

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References

[1] C. A. Balanis, P. I. Ioannides, "Introduction to Smart Antennas," *Morgan & Claypool.*, 2007.

[2] T. E. Tuncer, B. Friedlander, "Classical and Modern Direction-of-Arrival Estimation," *Academic Press*, 2009.

[3] W. -K. Ma, T.-H. Hsieh, and C.-Y. Chi, "DOA estimation of quasistationary signals via Khatri-Rao subspace," *in Proc. Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, 2009, pp. 2165-2168, Apr. 2009.

[4] P. Pal and P. P. Vaidyanathan, "Nested Arrays: A Novel Approach to Array Processing With Enhanced Degrees of Freedom," *IEEE Trans. Signal Processing*, vol. 58, no. 8, pp. 4167-4181, Aug. 2010.

[5] C. S. Ruf., "Numerical annealing of low-redundancy linear arrays," *IEEE Trans. Antennas Propag.*, vol. 41, no. 1, pp. 85-90, Jan. 1993.