An Autocorrelated Inverse Method for Nakagami-*m* Envelope Simulation

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***Abstract* – the autocorrelation characteristic of Nakagami-*m* fading channel simulated by the original Inverse Method agree with the theoretical value poor because of the loss of phase information. An efficient method to improve the autocorrelation characteristic of the Inverse Method is presented adding the lost phase information of Rayleigh sequence during inverse transform process by symbol correction. Simulation results show that the autocorrelation of proposed method is much better than the original method.**

***Index Terms* — Nakagami-*m* fading, Inverse Method, Autocorrelation.**

1. Introduction

Nakagami distribution can be a good description of channel fading. It can describe fading channel, with different values of *m*, from slight to serious including Rayleigh fading, Gaussian distribution, and Rice distribution [1]. Nakagami-*m* channel becomes fast fading and autocorrelated when the the Doppler shift is considered. Scholars have presented several time domain autocorrelation envelope simulation methods for autocorrelated Nakagami-*m* fading channel including the Inverse Method, the Brute Force Method and the Rank Matching Method [2-4]. Inverse Method is an efficient method to simulate arbitrary values of the fading parameter *m*, and can greatly reduce the simulation time compare to the other models. Unfortunately, the autocorrelation result still differ large compare to the theoretical result. Therefore, this paper proposed a modified inverse method to improve the autocorrelation characteristic of the Nakagami-*m* envelope simulation.

2. Stastical property of Nakagami-*m* Fading Process

Envelope PDF of autocorrelation properties Nakagami-*m* distribution is given as follows [2]:

$$P(r) = \frac{2m^m r^{2m-1}}{\Gamma(m)\Omega^m} e^{-\frac{mr^2}{\Omega}}, r \geq 0 \qquad (1)$$

$$\rho_R(\tau) = \frac{1 - F_1(-0.5, -0.5; 1; J_0(2\pi f_d \tau))}{1 - \frac{4}{\pi}} \qquad (2)$$

Where $\Omega = E[r^2]$ is the average power, $m = \Omega^2 / E\{[r^2 - \Omega^2]^2\}$ is the Nakagami distribution shape factor, and $\Gamma(.)$ is the gamma function. J_0 (.) is the zero-order Bessel function of the first kind, f_d is the maximum Doppler frequency, andτrepresents time. Where $F_1(.,.,.,.)$ is the hypergeometric function defined in [5].

3. The Improved Autocorrelated Inverse Method

Autocorrelation means that the sequence order of the simulation random process must meet specific order. The Nakagami-*m* envelope simulated by the inverse method introduces certain autocorrelation property from autocorrelated Rayleigh sequences [2], which has a specific relationship with the first-order Bessel function.

The simulation model put autocorrlated Rayleigh random sequences into the cumulative distribution (CDF) function directly and give birth a uniform distribution [0, 1.], and the output value of the cumulative distribution function become wholly positive, while in fact the Rayleigh random sequence before the cumulative and inverse operation has the existence of negative values, the negative phase information missed during the cumulative and inverse process. This leads to autocorrelation characteristic simulated match the theoretical value bad. We present an autocorrelated inverse method to improve the autocorrelation by amending the lost Rayleigh phase information through symbol correction as illustrated in Fig.1.

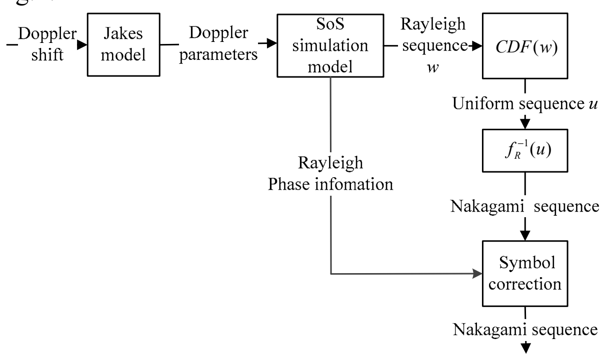

Fig. 1. Simulation process of the proposed method

The specific steps are as follows:

Step 1: Calculate Jakes Doppler coefficient, discrete Doppler frequency and Doppler phase parameters using arbitrary one of the following classical methods: MED, MSEM, MEA, MCM, LPNM, MEDS and JM.

Step 2: Obtain a Rayleigh envelope sample sequences *w* with the specified phase properties and autocorrelation by SOS (sum of sinusoids) simulator with parameters generated in step 1.

Step 3: Obtain uniformly distributed random sequence *u* with autocorrelation of Rayleigh sequences by substituting the Rayleigh random sequences into the cumulative distribution function.

Step 4: Obtain the original sequence x_0 by putting the uniformly distributed random sequence u into the inverse CDF function of the Nakagami-*m*.

$$x_0 = F_{Nak}^{-1}(u) = \int_0^u \frac{2m^m t^{2m-1}}{\Gamma(\mathrm{m})\Omega^m} e^{-(\frac{m}{\Omega})t^2} dt \approx \eta + \frac{a_1\eta + a_2\eta^2 + a_3\eta^3}{1 + b_1\eta + b_2\eta^2} \quad (3)$$

Where η is an ancillary variable defined as

$$\eta = \left(\sqrt{\ln\frac{1}{1-u}}\right)^{\frac{1}{m}} \quad (4)$$

a_1, a_2, a_3, b_1 and b_2 are coefficients chosen to minimize the approximation error as detained in literature [2].

Step 5: Obtain Power matched Nakagami-*m* sequences x for the original sequence x_0

$$x = \sqrt{\frac{\Omega}{m}}.x_0 \quad (5)$$

Step 6: Obtain the final Nakagami-*m* random sequences through symbol correction from Rayleigh envelope sample sequences w.

$$x_i = \begin{cases} x_i & w_i \geq 0 \\ -x_i & w_i < 0 \end{cases} \quad (6)$$

4. Simulation Results Analysis

MEDS method is used to calculate model parameters in step 1. Comparison of the envelop PDF and the envelope autocorrelation property of the proposed Nakagami-*m* simulation method with theoretical values are given in Fig. 2 and Fig.3 respectively when Ω=1, *m*=1.5, 6. It can be seen that the simulation results of PDF agree with theoretical value and the original method well under distinguished *m*. The envelope autocorrelation of the proposed method is much better than the original model. A minor error still exists versus the theoretical values lies that autocorrelation in our simulation is introduced from Rayleigh autocorrelation approximation for there is no measured autocorrelation data of Nakagami-*m* directly.

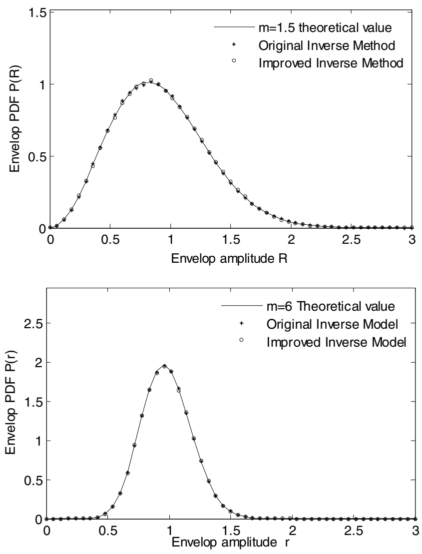

Fig. 2. PDF of Nakagami-*m* distribution with *m* =1.5, 6. Ω=1.

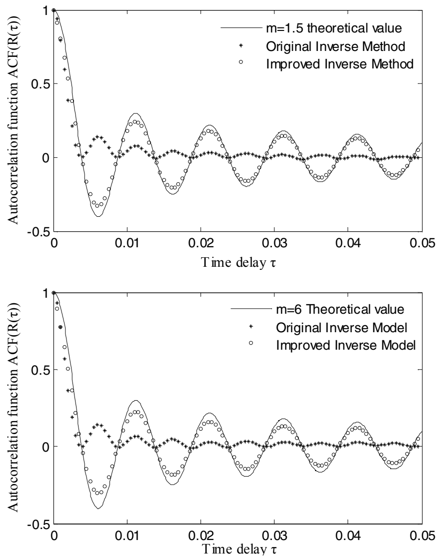

Fig. 3. Autocorrelation property of Nakagami-*m* distribution with *m* =1.5, 6. Ω=1.

5. Conclusion

This proposed autocorrelated Inverse method improve the autocorrelation of the of Nakagami-*m* envelope simulation amending phase information loss during CDF and inverse operation. Simulation results show that the envelope autocorrelation of the improved model fit the theoretical values much better compare to the original model.

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