# Simple Design of Null-fill for Linear Array

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***Abstract* – This paper proposes the simple design method to reduce amplitude ripples of sidelobe peak revel in equispaced *N*-elements linear arrays by the amplitude slope of 1/*N*. This design is based on Schelkunoff and Elliott method to move roots of outside of unit circle in complex plane. The amplitude ripples are reduced less than 1dB by the proposed method, which is greatly improved rather than the conversational method.**

***Index Terms* — Null-fill, Schelkunoff, Elliott, linear array, sidelobe ripple**

## 1. Introduction

Recently, the number of small cell is increasing to improve throughput and frequency utilization efficiency of cellular systems. In small cell coverage area, direct paths from base station are dominant under the line of sight environment, and necessary to fill nulls in vertical plane pattern of base station antennas.

Then, Rodriguez and Bjorn Lindmark [1, 2] introduced a method for the design of null-fill antenna of a linear array. The method introduces that null fill in the pattern can be created by moving some of the Schelkunoff [3] roots inside or outside the unit circle by a fixed ratio the amplitude distribution. In this work, an extension of the Elliott method is used in order to reduce null revel with corresponding to sidelobe peak revel in equispaced *N*-elements linear arrays by linear amplitude distribution whose gradient equals 1/*N*.

## 2. Description of method

Consider an equispaced linear array of $N$ elements along the $z$ axis, with $d$ the element spacing and $I_n$ the excitation of the $n$-th element. Then the array factor is given by (1)

$$f(\theta)=\sum_{n=0}^{N-1} I_n e^{in(\beta d\cos\theta+\phi_0)}=\sum_{n=0}^{N-1} I_n e^{jn\psi}=I_N\prod_{n=1}^{N-1}(w-w_n) \quad (1)$$

where $\psi = kd\cos(\theta)$, $k$ wavenumber, $w = e^{j\psi}$, and $w_n$ are the roots of the array factor polynomial. Then, we can fill nulls by moving some of the roots inside or outside the unit circle by a fixed ratio , rewriting the array factor by (2)

$$f(\theta)=\prod_{n=1}^{N-1}\left(w-(1+\varepsilon_n)w_n\right) \quad (2)$$

where $\varepsilon_n$ are real [4].

In the conventional method, all the $\varepsilon_n$ are same for the simplicity in the formulation, however we try to change to reduce the ripples of sidelobe level. It is easy to obtain by decreasing gradually the excitation amplitude of the $n$-th element.After examining various value of $\varepsilon$, we conclude that linear amplitude distribution whose gradient equals 1/*N* is the best one to reduce the ripples of sidelove levels of uniform array..

## 3. Numerical examples

In order to fill nulls, we change $\varepsilon$ from 0.05 to 0.2 using a 16-elements equispaced linear array for $d$ = 0.67$\lambda$. In this parametric study, directivity patterns are shown in Fig. 1 and their magnitude distributions are shown in Fig. 2. It is found that the larger $\varepsilon$ decreases null ripples with the drawback of sideloe level increase.

In the next step, we examine other amplitudes as shown in Fig. 3, such as, linear, log, cos, and the obtained directivity patterns are shown in Fig. 4. This results show the linear amplitude slope suppresses null ripple less than 1 dB without increasing the sidelobe level. In terms of $w$ plane, the linear case can fill nulls, because its roots do not exist on the unit circle as shown in Fig. 5.

To compare null depth by the proposed method with conventional one, Fig. 6 shows radiation pattern of three cases. The moving roots method (case 1) shows that the angle of null correspond to uniform array and reduce null revel below 5 dB. On the other hand, the proposed linear slope method (case 2) shows that the angle of null is different with the pattern without null filling and the ripple is suppressed in low level less than 1 dB.

## 4. Conclusion

In equispaced *N*-elements linear arrays, an extension of the Orchard-Elliott method whose amplitude distribution is given by linear with gradient 1/*N*, providing the reduction of null revel below 1 dB without degradation of array directivity in uniform array. As future task, this proposed method is used for base station array antennas actually.

## References

[1] B. Lindmark, "Analysis of Pattern Null-fill in Linear Arrays," 2013EuCAP, pp. 1457-1461, Apr. 2013.
[2] J.A. Rodriguez, "Synthesis of shaped beam antenna patterns with null-filling in the sidelobe region," IET Electronics Lett., Vol. 33, No. 24, pp. 2004-2005, Nov.1997.
[3] S. A. Schelkunoff, "A Mathematical Theory of Linear Arrays," The Bell System Technical Journal, Vol. 22, pp. 80-107, Jan. 1943.
[4] M. Kijima, "Relation between Array Excitation Distribution and Radiation Pattern Null Depth," IEICE Technical Report AP89-35, 1989.

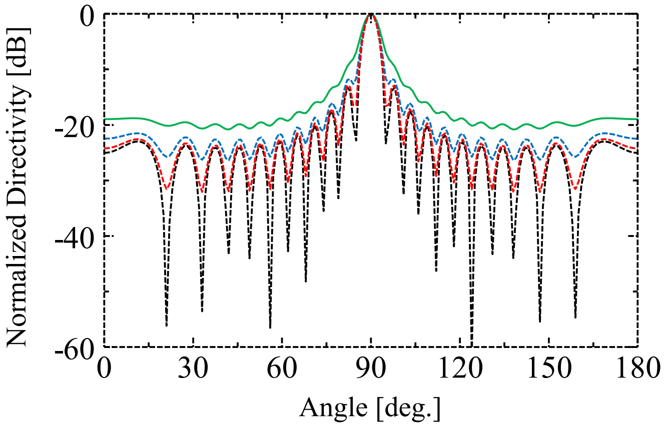


Fig.1 Directivity changing $\varepsilon$

Uniform 0.05 0.1 0.2

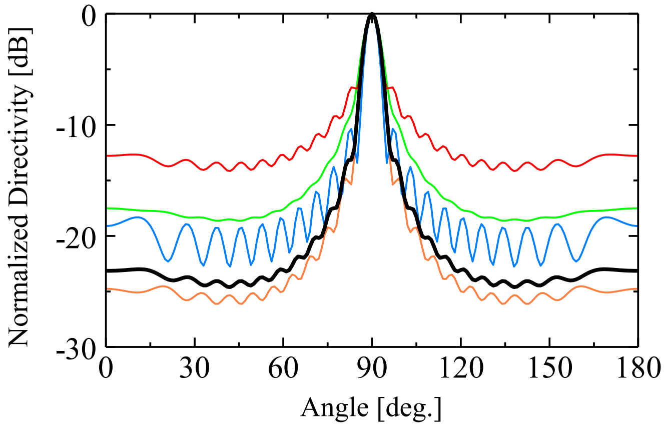


Fig.4 Directivity changing magnitude distribution

$-\frac{1}{N}x+1$ $\frac{1}{x}$ $\frac{1}{\sqrt{x}}$ $\log_{10}\frac{N}{x}$ $\cos\left(\frac{\pi}{2N}x\right)$

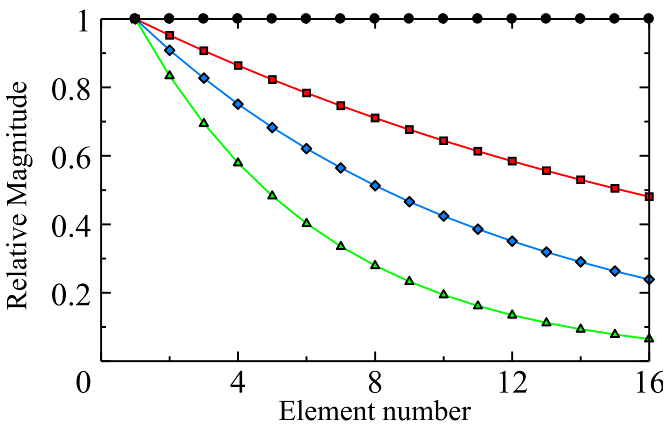


Fig.2 Magnitude distribution changing $\varepsilon$

Uniform 0.05 0.1 0.2

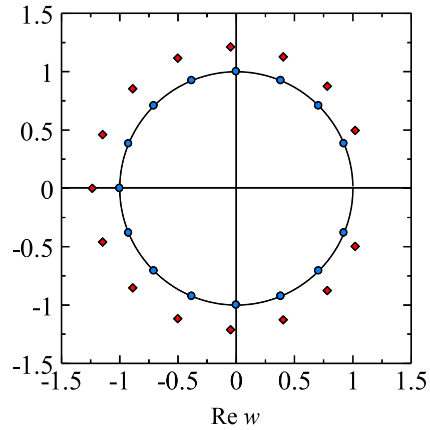


Fig.5 Roots of linear amplitude in $w$ plane

Uniform Linear

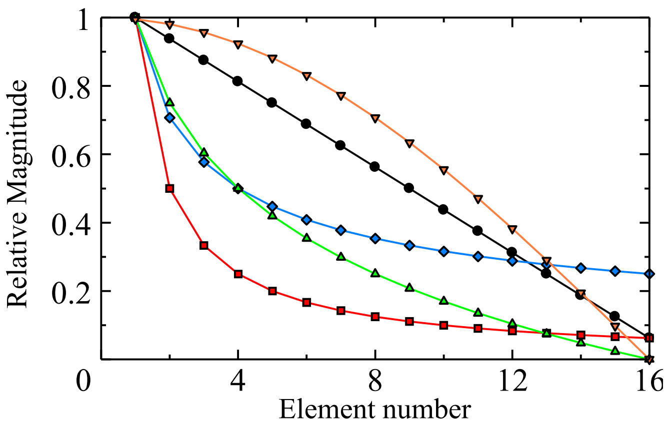


Fig.3 Magnitude distribution

$-\frac{1}{N}x+1$ $\frac{1}{x}$ $\frac{1}{\sqrt{x}}$ $\log_{10}\frac{N}{x}$ $\cos\left(\frac{\pi}{2N}x\right)$

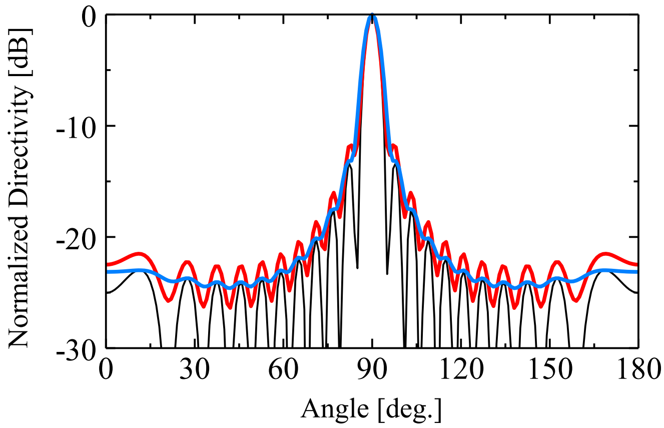


Fig.6 Comparison case 1 with case 2 in directivity

Uniform

Case 1 : moving roots of uniform outside 0.1 from unit circle

Case 2 : linear magnitude whose gradient equals $1/N$