POS1-49

On Arrangement of Scattering Points in Jakes' Model for Generating i.i.d. Time-Varying MIMO Channels

Hiroshi Nishimoto, Toshihiko Nishimura, Takeo Ohgane, and Yasutaka Ogawa
Graduate School of Information Science and Technology, Hokkaido University
Kita 14, Nishi 9, Kita-ku, Sapporo 060–0814, Japan
{hn,nishim,ohgane,ogawa}@ist.hokudai.ac.jp

Abstract

For simulating i.i.d. time-varying MIMO channels using multiple Jakes' rings, it is desired to generate channels having stable statistics with fewer scatterers. In the conventional Jakes' model, its statistical fading property may depend on the initial phase set assigned to scattering points. In this paper, we present simple and effective conditions on arrangement of scattering points for stable fading properties. The results show that the proposed arrangement provides higher statistical stability of time-varying channels than the conventional one does.

1. Introduction

Recently, high data-rate service with high mobility has been one of the growing demands for future wireless communications. The multiple-input multiple-output (MIMO) system is already the core technology for some standards to achieve such high data speeds [1]. Thus, there are many opportunities to use a time-varying MIMO channel model in performance evaluations of MIMO systems [2].

Jakes' model has been extensively used for simulating time-varying Rayleigh fading with U-shaped power spectrum [3], [4]. The model can be simply applied to MIMO channels. When each element of a MIMO channel matrix independently obeys the model, i.e., by using multiple scattering rings, we can obtain independent and identically distributed (i.i.d.) time-varying MIMO channels, theoretically. However, statistical validity is achieved with large enough number of scatterers in the rings. Thus, decreasing the scattering points without consideration on their arrangement may lead statistics fluctuation depending on the initial phase at each point, as will be shown later.

In this paper, we establish simple and effective conditions on arrangement of scattering points in MIMO Jakes' model for sufficient stability of statistics.

2. Simplified Jakes' Model

We consider a narrow-band MIMO system equipped with N_{tx} transmit (TX) antennas and N_{rx} receive (RX) antennas. It is assumed that the kth RX antenna, which is surrounded by a scattering ring R_{kl} with M scattering points, moves with a velocity v as illustrated in Fig. 1. A channel $h_{kl}(t)$ from the lth TX antenna to the kth RX antenna, which is an element of the kth row and lth column in the MIMO channel matrix and is affected only by the corresponding scattering ring R_{kl}, is assumed to be time-varying. We define x- and y-axes as the moving direction and its orthogonal one, respectively. In a baseband system, the channel $h_{kl}(t)$ is represented as

$$h_{kl}(t) = \sum_{m=1}^{M} a_{kl,m}\, e^{j\left\{2\pi f_D\left(\cos\theta_{kl,m}\right)t+\phi_{kl,m}\right\}}, \tag{1}$$

where $a_{kl,m}$, $\theta_{kl,m}$, and $\phi_{kl,m}$ are a received amplitude, an angle of arrival, and an initial phase of the mth scattered wave component, respectively, and f_D is the maximum Doppler frequency. The Doppler shift caused by the mth scatterer is $f_D\cos\theta_{kl,m}$. Both $\theta_{kl,m}$ and $-\theta_{kl,m}$ contribute to the same Doppler shift because of $\cos\theta_{kl,m} = \cos(-\theta_{kl,m})$. Thus, in Jakes' model, it is known that scattering points should not be arranged symmetrically to the x-axis. Here, we define that all M points are distributed only in the range

ISBN: 978-4-88552-223-9 C3055©IEICE

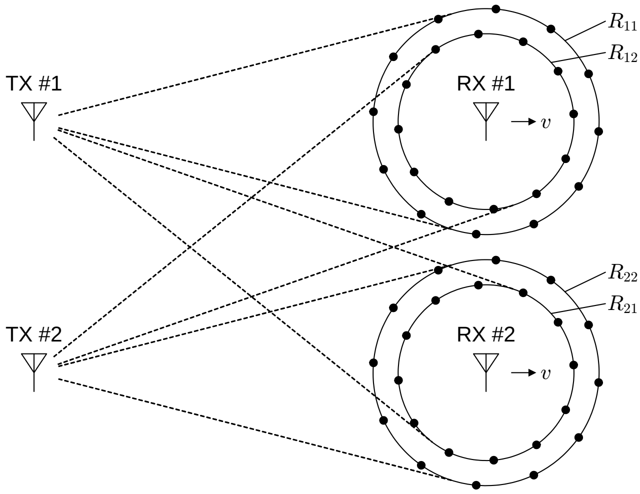


Fig. 1: Concept of an i.i.d. time-varying MIMO channel model using multiple Jakes' rings ($N_{\rm tx} = N_{\rm rx} = 2$).

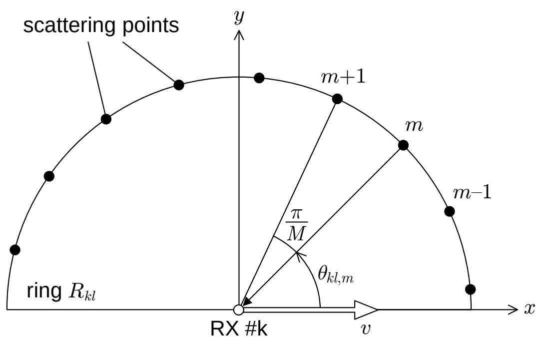


Fig. 2: Simplified Jakes' model.

of $0 \leqq \theta_{kl,m} \leqq \pi$ with equal interval π/M to facilitate the following discussion, as illustrated in Fig. 2. Although, strictly speaking, each ring becomes a semicircle under the above angular condition, we still refer to it as ring hereinafter. In addition, we assume $a_{kl,m} = a = 1/\sqrt{M}$ regardless of k, l, and m for the sake of simplicity. Under the assumption, we obtain an ergodic channel power $E\left[|h_{kl}(t)|^2\right] = 1$.

3. Intra-Ring Condition

First, we consider only the scattering ring R_{kl}. When an arbitrary pair of scattering points m and m' locates in y-axis symmetry, i.e., $\theta_{kl,m} = \pi - \theta_{kl,m'}$, their absolute values of Doppler shift components are the same, i.e., $\left|f_D \cos\theta_{kl,m}\right| = \left|f_D \cos(\pi - \theta_{kl,m'})\right|$. In this case, the channel may have exceptional characteristics depending on the initial phase. For example, let us consider the case of $\phi_{kl,m} = \phi_{kl,m'} = 0$. A superposed wave composed of these two scattered waves is expressed as $a\, e^{j2\pi f_D(\cos\theta_{kl,m})t} + a\, e^{-j2\pi f_D(\cos\theta_{kl,m})t} = 2a\cos\{2\pi f_D(\cos\theta_{kl,m})t\}$. This means that the superposed wave component does not have its imaginary part, and that the amplitude becomes twice. For another example, in the case of $\phi_{kl,m} = 0$ and $\phi_{kl,m'} = \pi$, it does not have its real part because it is expressed as $j2a\sin\{2\pi f_D(\cos\theta_{kl,m})\, t\}$. Thus, the y-axis symmetric arrangement of scattering points may cause instability on statistical fading properties. To avoid such phenomena, the following condition on arrangement of scattering points should be satisfied

$$\theta_{kl,m} \neq \pi - \theta_{kl,m'} \qquad \text{for } 1 \leqq m, m' \leqq M. \tag{2}$$

We define the above condition as the intra-ring condition in the paper.

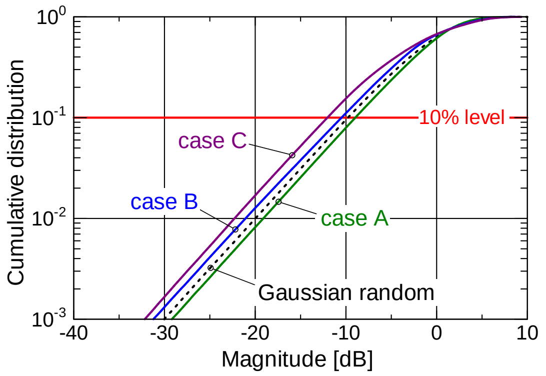


Fig. 3: CDF examples of magnitudes for SISO cases.

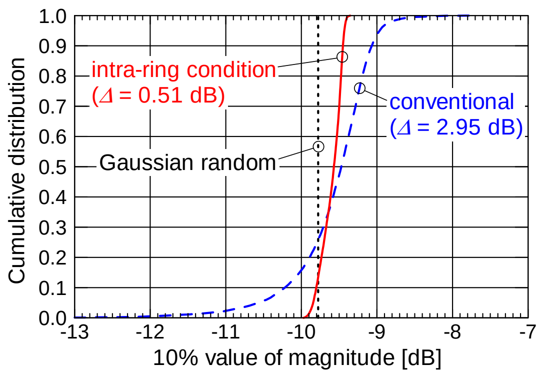


Fig. 4: 10%-value CDFs of magnitudes for SISO cases.

4. Inter-Ring Condition

When using multiple scattering rings, arrangement of scattering points across the rings should be considered. At first, we assume a time-varying $N_{\rm rx} \times N_{\rm tx}$ MIMO channel $\boldsymbol{H}(t)$ based on $N_{\rm tx}N_{\rm rx}$ scattering rings, where all the rings have the same scattering-point structure. The channel $\boldsymbol{H}(t)$ can be decomposed into M channel components $\boldsymbol{H}_1(t), \ldots, \boldsymbol{H}_M(t)$, where the mth component $\boldsymbol{H}_m(t)$ is composed of mth scattered waves in all the $N_{\rm tx}N_{\rm rx}$ rings, i.e., $h_{kl,m}(t)$ for $k = 1, \ldots, N_{\rm rx}$ and $l = 1, \ldots, N_{\rm tx}$. In $\boldsymbol{H}_m(t)$, all the angles of arrival of wave components are the same, i.e., $\theta_{kl,m} = \theta_m$ regardless of antenna indices k and l, due to the same arrangement. Hence, $\boldsymbol{H}_m(t)$ can be expressed as

$$\boldsymbol{H}_m(t) = a\, e^{j2\pi f_D(\cos\theta_m)t}\boldsymbol{\Phi}_m, \tag{3}$$

where $\boldsymbol{\Phi}_m$ represents an $N_{\rm rx} \times N_{\rm tx}$ initial phase matrix in which an element of the kth row and lth column is $\phi_{kl,m}$. The above equation implies that regularity of the matrix $\boldsymbol{H}_m(t)$ depends on the given initial phase set $\boldsymbol{\Phi}_m$. In a case of $\phi_{11,m} = \phi_{12,m} = \cdots = \phi_{N_{\rm rx}N_{\rm tx},m}$, it is obvious that the matrix is singular, i.e., $\mathrm{rank}\,[\boldsymbol{H}_m(t)] = 1$, regardless of time. Of course this is an over-simplified example, and the actual channel matrix $\boldsymbol{H}(t)$ is superposed by M channel components so that its regularity will be maintained. It is conjectured, however, that such singular matrix components may cause an unstable property of $\boldsymbol{H}(t)$. The simplest way to avoid this is to satisfy the following condition on arrangement of scattering points across rings defined as the inter-ring condition

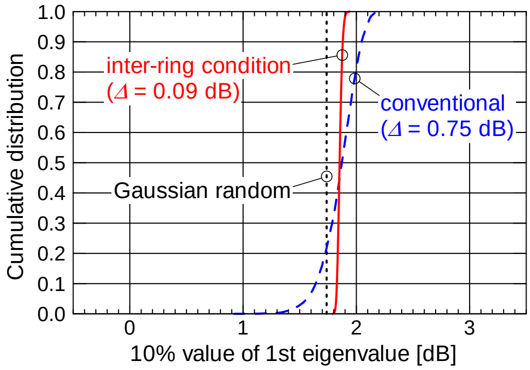
$$\theta_{kl,m} \neq \theta_{k'l',m'} \qquad \text{for}\ \ 1 \leqq k, k' \leqq N_{\rm rx},\ 1 \leqq l, l' \leqq N_{\rm tx},\ 1 \leqq m, m' \leqq M, \tag{4}$$

except $m = m'$ in the case of $k = k'$ and $l = l'$.

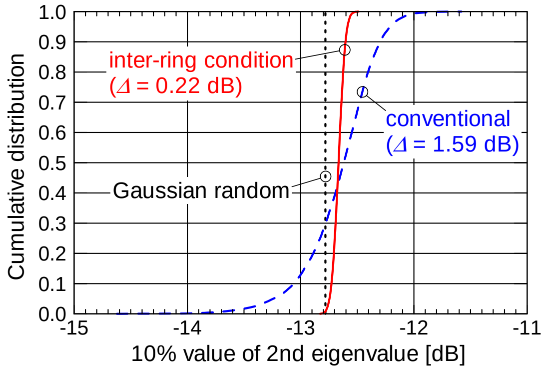
5. Numerical Analysis

We simulated time-varying channels based on Jakes' model for single-input single-output (SISO) and 2×2 MIMO cases to evaluate the intra- and inter-ring conditions. We set $M = 12$ for each scattering ring and prepared 10,000 initial phase sets randomly given. For each initial phase set, we generated time-varying channels based on (1) and captured 1,000,000 snapshots to obtain its cumulative distribution function (CDF) of magnitudes and eigenvalues for SISO and MIMO cases, respectively. Figure 3 demonstrates three examples of CDFs for SISO channels yielded by different initial phase sets, where all the three Jakes' rings had the same scattering-point structure without the intra-ring condition, i.e., y-axis symmetric structure. For comparison, we also show the CDF obtained by complex Gaussian random process. It it seen in Fig. 3 that the three CDFs do not correspond to that of the Gaussian random process case and depend on given initial phases. To evaluate the fluctuation, we observed an additional CDF of 10% values (hereinafter we refer to it as 10%-value CDF) for each scattering-point arrangement. Also, we measured a value spread $\mathit{\Delta}$ defined as difference between 1% and 99% values in the 10%-value CDF.

We first evaluate the intra-ring condition for SISO channels by using 10%-value CDFs of the magnitudes and their value spreads shown in Fig. 4. Here, "conventional" denotes a case of the y-axis symmetric arrangement of scattering points. As a reference, we also show a 10% value for the Gaussian random



(a) First eigenvalue



(b) Second eigenvalue

Fig. 5: 10%-value CDFs of first and second eigenvalues for 2×2 MIMO cases.

process case. It is clear that the conventional arrangement yields various fading properties depending on the initial phase setting. On the other hand, the arrangement under the intra-ring condition provides more stable fading properties than the conventional arrangement does. The value spread is effectively reduced to 0.51 dB from 2.95 dB with the intra-ring condition. We confirmed that, improvement of the stability under the intra-ring condition is slight even though M increases, and that $M > 30$ is necessary for the conventional arrangement to achieve the same stability as the arrangement under the intra-ring condition.

Next, we evaluate the inter-ring condition. Figure 5 presents 10%-value CDFs of the first and second eigenvalues for 2×2 MIMO channels, where both arrangement types "inter-ring condition" and "conventional" are subject to the intra-ring condition. We can see from the CDFs for the conventional arrangement that a common arrangement over all the scattering rings causes dependent properties upon the initial phase setting even if under the intra-ring condition. Furthermore, the difference is larger for the second eigenvalues. In contrast, the arrangement under the inter-ring condition gives much more stable eigenvalue properties. With the condition, the value spread for the first eigenvalues is reduced to 0.09 dB from 0.75 dB, and that for the second eigenvalues is significantly reduced to 0.22 dB from 1.59 dB.

6. Conclusions

We have established simple and effective conditions on scattering-point arrangement in Jakes' model for stable fading simulation. We confirmed that, for a single scattering ring, the intra-ring condition is effective for obtaining stable fading property in the aspect of statistics regardless of the initial phase setting. Moreover, it was shown that arrangement under the inter-ring condition provides fading properties robust to the given initial phases in a multiple-ring case such as 2×2 MIMO channels. It should be noted that the proposed inter-ring condition can be effective not only for MIMO flat fading channels but also for other fading channels, e.g., SISO channels with a few delay paths.

Acknowledgement

This work was supported in part by a Research Fellowship for Young Scientists from the Japan Society for the Promotion of Science.

References

[1] A. Ghosh, D.R. Wolter, J.G. Andrews, and R. Chen, "Broadband wireless access with WiMax/802.16: current performance benchmarks and future potential," IEEE Commun. Mag., vol.43, no.2, pp.129–136, Feb. 2005.

[2] B.H. Phu, Y. Ogawa, T. Ohgane, and T. Nishimura, "Channel extrapolation techniques for E-SDM systems in time-varying fading environments," IEICE Trans. Commun., vol.E89-B, no.11, pp.3083–3092, Nov. 2006.

[3] W.C. Jakes, Jr., Microwave Mobile Communications, Wiley, New York, 1974.

[4] P. Dent, G.E. Bottomley, and T. Croft, "Jakes fading model revisited," Electron. Lett., vol.29, no.13, pp.1162–1163, June 1993.