

# Exact Matching Approach with Circuit Element Ohmic Loss

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**Abstract** – Impedance matching is very important technique to improve the efficiency of transmitting or receiving system not only for wireless communication system but also for wireless power transfer(WPT) system. However, most lumped matching circuits have been designed by using reactive components such as inductors and capacitors without consideration of their ohmic losses. An exact approach to design the lumped matching circuit at the presence of the ohmic loss is proposed in this paper and will be confirmed using various examples. Moreover, the effect of the matching circuit’s ohmic loss and mismatching will be investigated.

**Index Terms** — Matching Circuit, Ohmic Loss, Efficiency, Wireless Power Transfer, Q-factor.

## 1. Introduction

Impedance matching circuit is an important circuit to reduce the undesired reflection between two unmatched impedances and improve the efficiency of transmitting or receiving system not only for wireless communication system but also for wireless power transfer(WPT) system. Traditional impedance matching circuit can be lumped parameter circuit and distributed parameter circuit [1]. L-section matching, T-section and  $\pi$ -section are lumped parameter circuits. Among lumped parameter circuits, L-section matching is a typical one and uses two reactive elements to match arbitrary impedance to arbitrary desired impedance. Usually, these reactive components are calculated without consideration of their ohmic loss.

In [2], the effect of the ohmic losses from inductors and capacitors of L-section matching circuit on the efficiency of WPT system has been calculated. The results show that the efficiency of WPT system is reduced greatly due to the ohmic loss from the matching components. In [3], a design method for perfect matching circuit even at the presence of the ohmic loss was proposed for T-section matching, however unfortunately, the Q-factor for T-section circuit was not completely one because the ohmic loss was defined as  $Q = X/R$  or  $Q = B/G$ , rather than  $Q = |X|/R$  or  $Q = |B|/G$ .

In this paper, firstly a quite simple approach to obtain L-section matching circuit will be previewed in session 2, and then an exact approach to obtain L-section matching circuit with its lossy element will be proposed in session 3. The former one will be called as non-lossy matching approach(NLMA) and the later one will be called as lossy matching approach(LMA) in this paper.

In session 4, some examples will be introduced to confirm the effective of our proposed approach and to compare the efficiency of matching circuit with and without loss.

## 2. Matching circuit without loss

To explain the matching method easily but without loss of generality, one desired impedance  $Z_s$  which usually represents the source side is assumed as only having real part  $Z_s = R_s$ , and the other side of the impedance is  $Z_l$  which is a complex one with real part  $R_l$  and imaginary part  $X_l$ . The matching circuit is designed to change  $Z_l$  to  $Z_l^{mat}$  which is equal to  $Z_s^*$ , that is

$$Z_s = (Z_l^{mat})^* \Rightarrow R_s = Z_l^{mat}$$

The above equation is also called as matching condition. Still without loss of generality, the design method only for L-section matching circuit will be focused in this paper. Certainly, the proposed method will be easily to be extended to T-section and  $\pi$ -section matching circuits. L-section matching circuit with one shunt component and one series component can be divided into two types as shown in Fig.1.

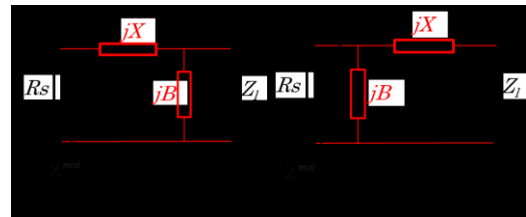


Fig. 1. L-section matching circuit

BX type is used for the case of  $G_s > G_l$  as shown in Fig.1(a), where a shunt susceptance  $B$  is firstly connected to  $Z_l$ . Here,  $G_s = 1/R_s$ ,  $G_l$  and  $B_l$  are the conductance and susceptance of  $Z_l$ , they are

$$G_l = \frac{R_l}{R_l^2 + X_l^2}; B_l = \frac{-X_l}{R_l^2 + X_l^2} \quad (1)$$

While XB type as shown in Fig.1(b) is used for the case of  $R_s > R_l$  where a series component  $X$  is connected firstly to  $Z_l$ . In the following description, the susceptance  $B$  is used as shunt component and reactance  $X$  is used as series component. Both  $B$  and  $X$  can be inductor or capacitor. Plus  $B$  implies a capacitor and negative  $B$  implies an inductor. On the contrary, plus  $X$  means an inductor while negative  $X$  means a capacitor.

In case of  $G_s > G_l$ , a susceptance  $B$  and reactance  $X$  are obtained by the matching condition exactly as

$$B = -B_l \pm \sqrt{G_l G_s - G_l^2}; X = \pm \frac{\sqrt{G_l G_s - G_l^2}}{G_l G_s} \quad (2)$$

In the same way, for the case of  $R_s > R_l$ ,  $B$  and  $X$  can be obtained by the matching condition exactly as

$$X = -X_l \pm \sqrt{R_l R_s - R_l^2}; B = \pm \frac{\sqrt{R_l R_s - R_l^2}}{R_l R_s} \quad (3)$$

### 3. Matching circuit with loss

Now, an exact approach to design an L-section matching circuit with ohmic loss will be discussed. Let Q-factors  $Q_B$  and  $Q_X$  represent the ohmic losses in  $B$  and  $X$ , and have

$$Q_B = \frac{|B|}{G}, Q_X = \frac{|X|}{R} \quad (4)$$

where  $R$  represents the ohmic loss of  $X$ , while  $G$  represents the ohmic loss of  $B$ .

For the case of  $G_s > G_l$ , BX L-section matching will be used. From the matching condition the following equations with unknowns  $B$  and  $X$  will be established,

$$|X|/Q_X + \frac{G_l + |B|/Q_B}{(G_l + |B|/Q_B)^2 + (B_l + B)^2} = R_s \quad (5)$$

$$X = \frac{B_l + B}{(G_l + |B|/Q_B)^2 + (B_l + B)^2} \quad (6)$$

then,  $B$  and  $X$  can be obtained exactly by solving equations (5)(6) very easily.

Similarly, for case of  $R_s > R_l$ , XB L-type will be used, the following equations with unknowns  $B$  and  $X$  are obtained from the matching condition,

$$|B|/Q_B + \frac{R_l + |X|/Q_X}{(R_l + |X|/Q_X)^2 + (X_l + X)^2} = G_s \quad (7)$$

$$B = \frac{X_l + X}{(R_l + |X|/Q_X)^2 + (X_l + X)^2} \quad (8)$$

Again,  $B$  and  $X$  can be obtained exactly and easily by solving the equations (7)(8).

### 4. Results

To confirm the proposed method,  $R_s$  is set to  $50\Omega$  and  $Z_l$  is  $100+j50[\Omega]$ , or  $1+j6[\Omega]$ . The element parameters of each matching circuit calculated by NLMA are listed in Table I. When  $Z_l = 100 + j50[\Omega]$ , BX L-section matching circuit is used due to  $G_s > G_l$ . When  $Z_l = 1 + j6[\Omega]$ , satisfying  $R_s > R_l$ , then XB L-section matching circuit is used. The synthesized impedance  $Z_l^{\text{mat}}$  is almost equal to  $R_s$ , representing  $Z_l$  is getting matched to  $R_s$  by using the designed matching method as presented in session 2.

Table I Results

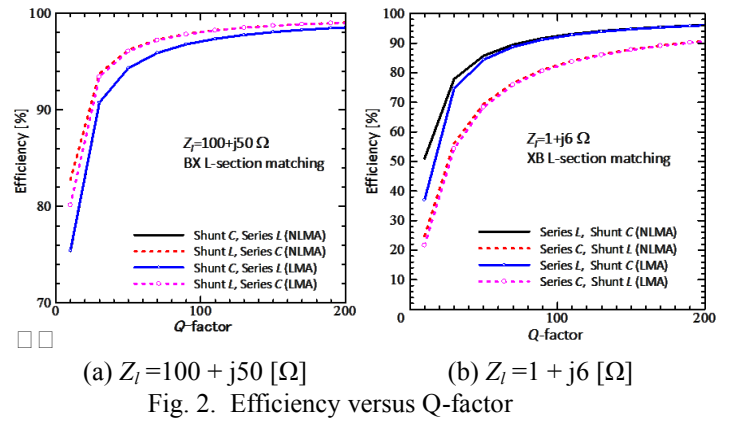
$Z_l[\Omega]$	Without ohmic loss			With ohmic loss( $Q_B=Q_X=100$ )			
				NLMA		LMA(Proposed)	
	$B[S]$	$X[\Omega]$	$Z_l^{\text{mat}}$	$Z_l^{\text{mat}}[\Omega]$	$B[S]$	$X[\Omega]$	$Z_l^{\text{mat}}[\Omega]$
$100+j50$	0.01	61.20	50.0	57.1+j7.98	0.02	52.00	50.00
	-0.01	-61.20	50.0	56.7-j3.47	-0.01	-63.20	50.7-j7.2
$1+j6$	0.14	1.00	50.0	27.8-j0.45	0.09	5.53	50.00
	-0.14	-13.00	50.0	17.1+j3.35	-0.07	-20.80	48.7+j5.9

Also, the effect of the ohmic loss of  $B$  and  $X$  on the synthesized impedance is illustrated in Table I.  $Z_l^{\text{mat}}$  by using NLMA and that by using LMA are compared where  $Q_B = Q_X = 100$ . As seen, all  $Z_l^{\text{mat}}$  got by using NLMA are not matched to  $R_s$ , but all  $Z_l^{\text{mat}}$  by using proposed lossy matching approach are approaching to  $R_s$ , representing proposed approach is effective method to design a matching circuit even at the presence of ohmic loss.

Furthermore, the efficiencies versus Q-factor with L-section matching circuit for  $Z_l = 100 + j50[\Omega]$  and  $1 + j6[\Omega]$  are plotted in Fig. 2. The efficiency  $\eta$  is defined as  $\eta = P_l / P_a$ , where  $P_l$  denotes the power consumed at  $Z_l$  and  $P_a$  is available maximum power for the load,  $V^2/(4R_s)$ ,  $V$  is the voltage of the source generator.

For the case of  $100+j50[\Omega]$ , two BX L-sections are possible used as the matching circuits. The efficiencies of shunt  $L$  and series  $C$  are better than shunt  $C$  and series  $L$  by using both NLMA and LMA.

For the case of  $1+j6[\Omega]$ , two XB L-sections are possible realized and the efficiencies of series  $L$  and shunt  $L$  are better than those of series  $L$  and shunt  $C$  by using both NLMA and LMA. Unfortunately, the efficiencies using lossy matching circuit are slightly lower than those calculated by using non lossy approach. It perhaps results from the unreasonable Q-factor and it will be discussed in detail in our next ongoing paper.



### 5. Conclusions

An exact approach for designing lumped matching circuit with ohmic has been proposed in this paper. The proposed approach has been confirmed effective for any two difference pair impedances to be conjugated matched. The effects of ohmic loss and mismatching on the circuit efficiency have been investigated as well.

### Acknowledgment

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### References

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