POS1-28

Voltage and Current Wave Behaviour on Transmission-Line Network With Cross-Junction Discontinuities

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1. Introduction

A simple time-domain numerical analysis of transmission line networks with cross-junction discontinuities is presented. The numerical method bases on the modified central difference technique, in which the internal boundary treatment introduced the time-domain scattering matrix is carried out for the cross junction of the internal line discontinuity. The voltage and current wave propagation on the whole transmission line network model of the lattice-line hybrid are demonstrated to clarify the microwave circuit operation and the time domain reflectometry.

The efficient simulation technique has been required in the system design of transmission line network models, such as planar type microwave circuits and antennas including various line discontinuities. It is very important to analyze the behaviour of the propagation signal over the transmission lines in the planar circuit systems, so it has to clarify the circuit operation and the reflection characteristic in terms of the time-domain analysis. Many analytical and numerical techniques for obtaining input/output responses of the planar type microwave circuits have been described in [1]. Then, the three-dimensional electromagnetic field analysis simulators, such as the finite element method, the method of Moment, and the FD-TD method [2], have also reported in many researches. However, their approaches relatively take much more CPU time and memory requirements for obtaining the time-domain solutions of the voltage and current waves propagation on the transmission line networks with the various line discontinuities, such as branch-line couplers, hybrid rings and so on. Here, we apply the modified central difference (MCD) method with the boundary treatments combining the time-domain scattering matrix formulation to analyze the transmission line network including cross-junction discontinuities. By using this new time-domain simulation technique, the signal propagation, reflection and the standing waves of the lattice-line hybrid can be represented for various excitation wave forms. Namely, it is shown that from the simulated results, the behaviour of the propagation signals and reflections for the voltage and current waves are easily clarified by visualizing the time-domain solutions.

The aim of this paper is to present a simple simulation technique for visualising signal propagation on the transmission line network with cross-junction discontinuities. It is show that this numerical approach is useful to analyze the transient responses and the transition of the voltage and current distributions on the whole transmission line network model, e.g. the lattice-line hybrid directional coupler. The behaviour of the signal propagation and reflection is demonstrated to confirm the circuit operation including multiple reflections caused by each of the junction. This method can be used for educational and engineering purposes sufficiently.

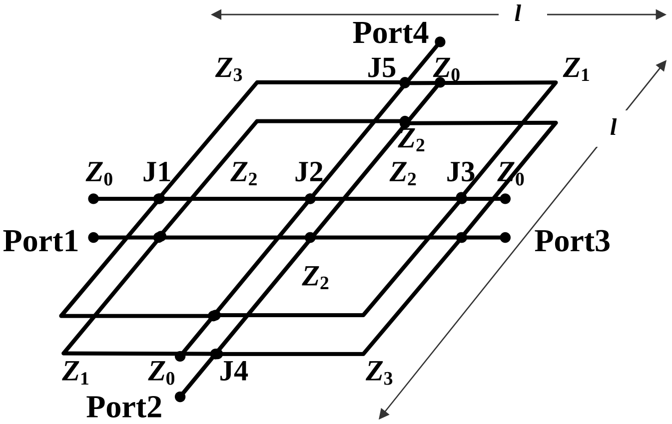
2. Modelling

Let consider a transmission-line network model with cross-junctions as shown in Fig.1. With the assumption of TEM transmission line, namely a typical two-wire transmission line, the time domain transmission-line equations can be generally written by following form:

$$\frac{\partial}{\partial x}\begin{bmatrix} V(x,t) \\ I(x,t) \end{bmatrix} + \begin{bmatrix} 0 & L_n \\ C_n & 0 \end{bmatrix} \frac{\partial}{\partial t}\begin{bmatrix} V(x,t) \\ I(x,t) \end{bmatrix} = -\begin{bmatrix} 0 & R_n \\ G_n & 0 \end{bmatrix}\begin{bmatrix} V(x,t) \\ I(x,t) \end{bmatrix} \qquad (1)$$

ISBN: 978-4-88552-223-9 C3055©IEICE

where, $V(x,t)$, $I(x,t)$ are the line voltage and current at any time t and at distance x, respectively. The primary parameters L_n, C_n, R_n, and G_n denote inductance, capacitance, resistance, and conductance per unit length of each line, respectively. According to the above assumption, the characteristic impedance $Z_{0n}=\sqrt{L_n/C_n}$ and phase velocity $v_p=1/\sqrt{L_nC_n}$ are the secondary line constants.



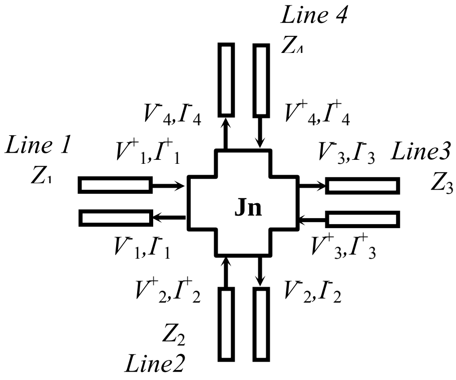


Fig. 1. Transmission line network with cross-junction discontinuities.   Fig. 2. A cross junction.

For the voltages and currents of the transmission lines, to solve the above partial differential equation numerically, the modified central difference approximation [3] is successfully applied to Equation (1). The iterative equations are given by as follows.

$$\begin{bmatrix} V_{i,j+1} \\ I_{i,j+1} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1-\Delta x G_n Z_{0n} & Z_{0n} \\ 1/Z_{0n} & 1-\Delta x R_n/Z_{0n} \end{bmatrix}\begin{bmatrix} V_{i-1,j} \\ I_{i-1,j} \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1-\Delta x G_n Z_{0n} & -Z_{0n} \\ -1/Z_{0n} & 1-\Delta x R_n/Z_{0n} \end{bmatrix}\begin{bmatrix} V_{i+1,j} \\ I_{i+1,j} \end{bmatrix} \quad (2)$$

where the suffices $i,j,i+1,i-1$, etc. denote the position at $i\Delta x \rightarrow i$ $(0< i\leq l)$, $j\Delta t \rightarrow j$ $(\Delta t=\Delta x/v_p)$ on the $(x\text{-}t)$plane. l is the line length. By using this iterative equation, the numerical solutions of the voltage and current on the whole transmission lines are obtained at the same time. Here, the initial conditions are given by as follows: $V(x,t=0)=0$, $I(x,t=0)=0$. Then, the boundary conditions at each port k (k=1 to 4) of the lattice-line hybrid coupler are also given by as follows: $V_{p1}(t)=e(t)Z_0/(Z_G+Z_0)$, $I_{p1}(t)=V_{p1}(t)/Z_0$, $V_{p2}(t)=0$, $I_{p2}(t)=0$, $V_{p3}(t)=0$, $I_{p3}(t)=0$, $V_{p4}(t)=0$, $I_{p4}(t)=0$, where all ports are with matched load. Also, at each time step, the boundary treatments for each cross-junction of the internal discontinuities have to be carried out as shown in the next paragraph. The voltage and current solutions of the lattice-line hybrid are obtained for various waveform excitations.

For the internal boundary treatment of a cross-junction discontinuity as shown in Fig.2, the voltages and currents at the boundary are related with the incident, reflected and transmitted quantities which are represented with a time-domain scattering matrix form as follows.

$$S=\begin{bmatrix} \Gamma_1 & T_{12} & T_{13} & T_{14} \\ T_{21} & \Gamma_2 & T_{23} & T_{24} \\ T_{31} & T_{32} & \Gamma_3 & T_{34} \\ T_{41} & T_{42} & T_{43} & \Gamma_4 \end{bmatrix}=\begin{bmatrix} \Gamma_1 & 1+\Gamma_2 & 1+\Gamma_3 & 1+\Gamma_4 \\ 1+\Gamma_1 & \Gamma_2 & 1+\Gamma_3 & 1+\Gamma_4 \\ 1+\Gamma_1 & 1+\Gamma_2 & \Gamma_3 & 1+\Gamma_4 \\ 1+\Gamma_1 & 1+\Gamma_2 & 1+\Gamma_3 & \Gamma_4 \end{bmatrix} \quad (3)$$

Where, suffices i and j (=1 to m) denote the line numbers, m=4. The reflection coefficients of each connected line at the junction can be given by

$$\Gamma_i = \frac{1 - Z_i \sum_{j\neq i}^{m} \frac{1}{Z_j}}{1 + Z_i \sum_{j\neq i}^{m} \frac{1}{Z_j}} \quad (4)$$

Then, for the incident voltages $\boldsymbol{V}^+=[V^+_1,V^+_2,V^+_3,V^+_4]^T$ and currents $\boldsymbol{I}^+=[I^+_1,I^+_2,I^+_3,I^+_4]^T$ from the connected transmission lines at a junction, according to signal flow graph analysis, the following equations (5) and (6) are used to calculate the scattering quantities. Namely, the reflected and transmitted voltages $\boldsymbol{V}^-=[V^-_1, V^-_2,V^-_3,V^-_4]^T$ and currents $\boldsymbol{I}^-=[I^-_1,I^-_2,I^-_3,I^-_4]^T$ are obtained.

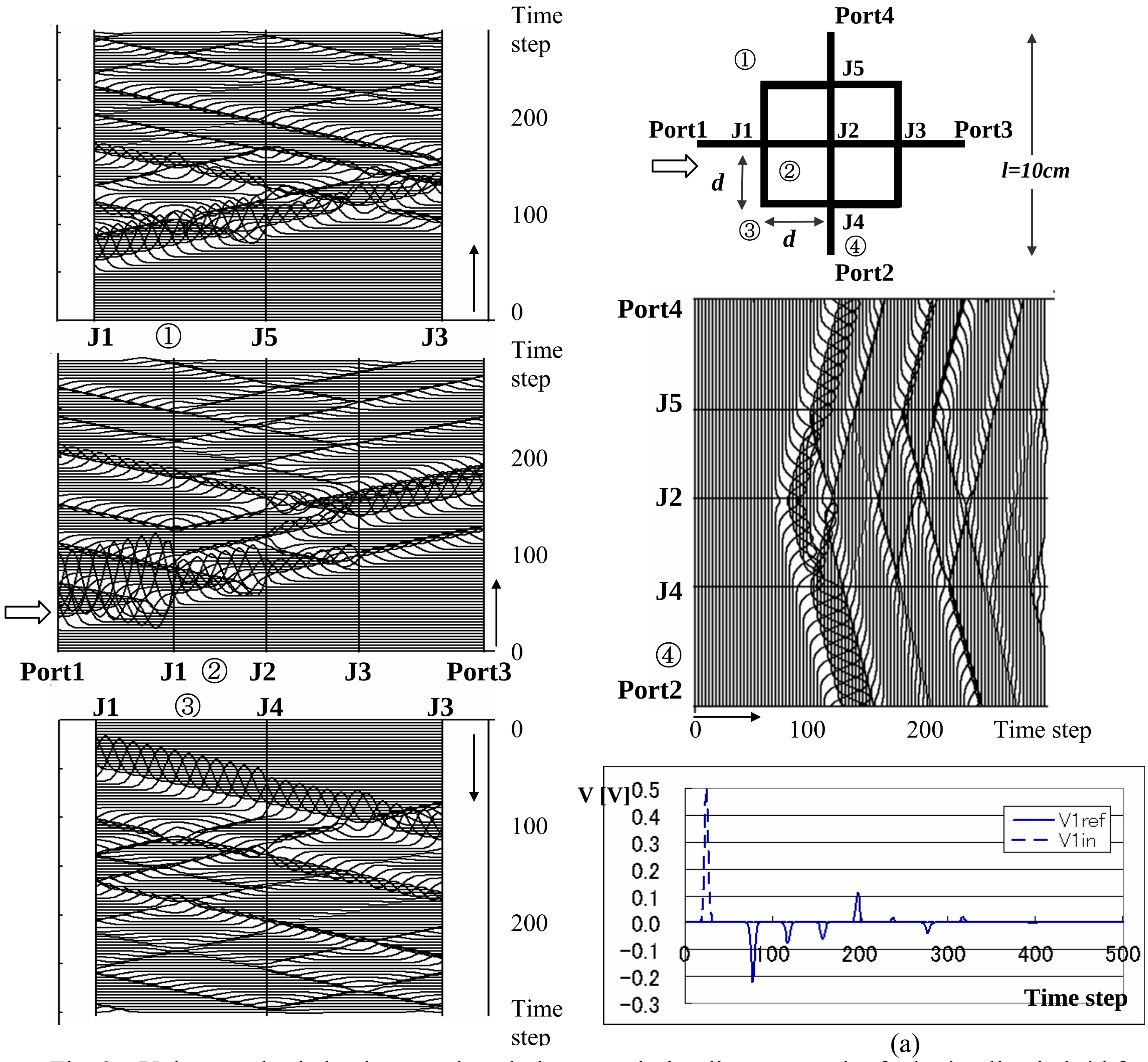
$$\boldsymbol{V}^- = \boldsymbol{S}\boldsymbol{V}^+ \qquad (5)$$

$$\boldsymbol{I}^- = \boldsymbol{Z}^{-1}\boldsymbol{S}\boldsymbol{Z}\boldsymbol{I}^+ \qquad (6)$$

Where, $\boldsymbol{Z}$ is defined by the diagonal impedance matrix constructed by the characteristic impedances of the connected transmission lines.

3. Numerical Examples

We show that the proposed simulation technique is useful to understand the operation of transmission line networks in the time domain. First, as a numerical example of the pulse reflection (echo from cross-junction discontinuities), the voltage responses on the transmission line network with the cross junctions for Gaussian pulse excitation are shown in Fig.3. It is found that the multiple reflections reached from each cross junction Jn are observed in detail. The design parameters: Z_0=50Ω,Z_1=81Ω,Z_2=50Ω,Z_3=50Ω, d=2cm, $v_p=3\times10^8$m/s,Δt=3.3ps,Δx=1mm, were used.



(a)

Fig. 3. Voltage pulse behaviour on the whole transmission lines network of a lattice-line hybrid for 300 time steps. (a) Incident and reflected pulses extracted from port 1 for 500 time steps.

Fig.4 shows a result of the voltage transient response of a 3-dB lattice-line hybrid directional coupler for a sinusoidal excitation ($e(t)$=sin($2\pi ft$)). The design parameters of the directional coupler: Z_0=50Ω, Z_1=81Ω, Z_2=50Ω, Z_3=50Ω, d=2cm, l=10cm, v_p=3x10^8m/s, Δt=3.3ps, Δx=1mm, were used. The transient phenomena of multiple reflections on the transmission line network causing by the cross-junction discontinuities are demonstrated for 800 time steps, and they can be represented by the dynamic expression. It can be seen that the isolation at port 2 and the equal power sprit to port 3 and port 4 are observed after around 3 periods.

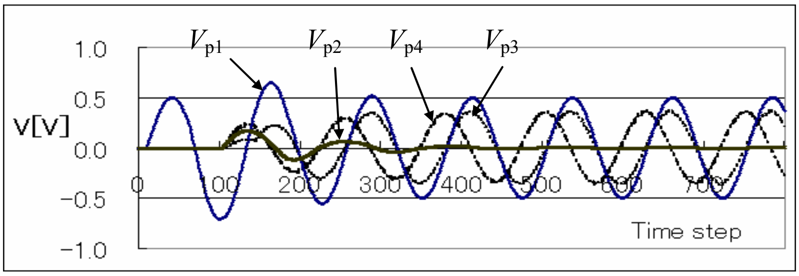


Fig. 4. Input/output responses of a 3-dB lattice-line hybrid directional coupler.

Next, Fig.5 shows the transition of the voltage and current distributions on the 3-dB lattice-line hybrid directional coupler for an operation frequency f =2.42GHz in the steady state region. The standing waves on the whole transmission lines and power isolation at port 2 are observed.

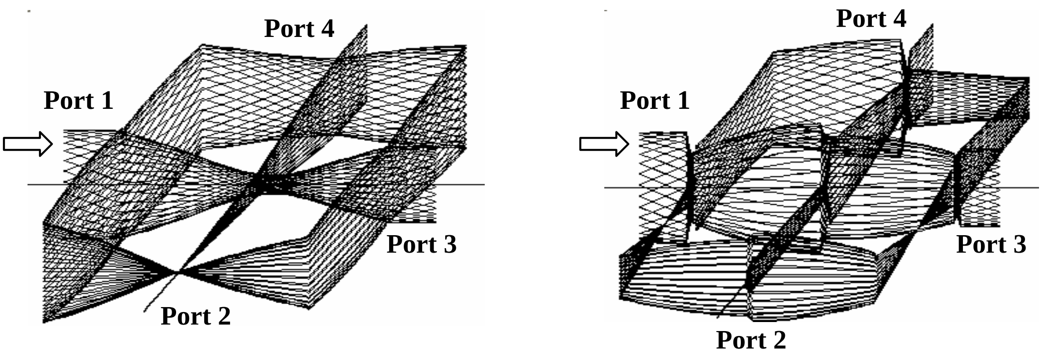


Fig. 5. Transition of instantaneous voltage and current distributions of the lattice-line hybrid directional coupler in the steady state region after 600 time steps: (a) voltage and (b) current standing waves.

4. Conclusion

The time domain analysis using the new modified central difference method has been presented for the transmission-line network model with cross-junction discontinuities. The voltage and current wave behaviours on the lattice-line hybrid have been demonstrated for representing the circuit operation, reflection characteristics, and standing waves. As a checking tool, this simulation technique is useful to test the signal propagation and reflection characteristics of the microwave circuits constructed by transmission line networks. This method is sufficiently applicable to analysis of the cases of taking account the transmission-line loss.

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