Analytical Study of Rectifier Circuit for Wireless Power Transfer Systems

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***Abstract* - In this paper, we develop an analytical model of a rectifier circuit that is used in wireless power transfer (WPT) systems, considering the switching timing, ON/OFF resistance, and load characteristics of the rectifier. The model enables us to estimate the performance of the rectifier and optimum load resistance that maximizes the output power and the power efficiency. The model can be extended into Y parameter, which is useful when we consider to develop multi-stage rectifier circuit. By comparing the derived model with the circuit simulation, we confirmed that the performance of the rectifier can be estimated with high accuracy.**

***Index Terms* — Rectifier, Load characteristic, Switch resistance, Switching timing.**

1. Introduction

A highly efficient rectifier circuit design is strongly required to develop wireless power transfer (WPT) systems. The output voltage and power efficiency of the rectifier change according to load conditions. There is an optimum load resistance that maximizes the output power and efficiency. However, it is quite difficult to estimate the optimum load resistance because the optimum load condition varies with various design parameters such as an output resistance of input voltage source, switching timing, and switch resistance. The optimum load resistance is derived in [2]. However, switching timing and switch resistance have not been considered enough.

In this paper, we study the load characteristics of the rectifier circuit, considering switching timing and switch resistance. We model the rectifier into Y parameter to simplify the analysis. The model gives us more realistic insight into the rectifier design. Our model can estimate the optimum resistance that shows maximum output power and power efficiency.

2. Analytical model of the rectifier

The characteristics of a rectifier is investigated. Figures 1 (a) and (b) show a schematic of the rectifier and control signal, respectively. In response to the control signal, SW1 and SW2 turns ON alternately. The rectifier rectifies the input voltage $v_{IN}(t)$ and the charge is stored on the output capacitor C_L. By repeating the operation, we obtain DC output voltage $v_{OUT}(t)$, which has a finite ripple voltage.

The characteristics of the rectifier circuit can be analyzed by solving circuit equations. As shown in Fig. 1 (b), we can define ON and OFF states as follows: (i) SW1 and SW2 are OFF when $0 < t < t_1$, $T/2 - t_1 < t < T/2 + t_1$, and $T - t_1 < t < T$, where T is a period of the control signal. (ii) SW1 and SW2 are ON when $t_1 < t < T/2 - t_1$ and $T/2 + t_1 < t < T - t_1$. Note that, ON condition and t_1 are different with different type of rectifiers and can be summarized as shown in Tab. I. The circuit equations of each state are given by

$$\text{(i)} \qquad C_L R_L r'_{OFF} \cdot \frac{di_{RL,OFF}(t)}{dt} + (R_L + r'_{OFF}) i_{RL,OFF}(t) = 0$$

$$\text{(ii)} \qquad \alpha v_{IN}(t) = C_L R_L r'_{ON} \cdot \frac{di_{RL,ON}(t)}{dt} + (R_L + r'_{ON}) i_{RL,ON}(t)$$

where $i_{RL,FF}(t)$, $i_{RL,ON}(t)$, r'_{OFF} and r'_{ON} are the load current and equivalent resistance at OFF and ON condition.

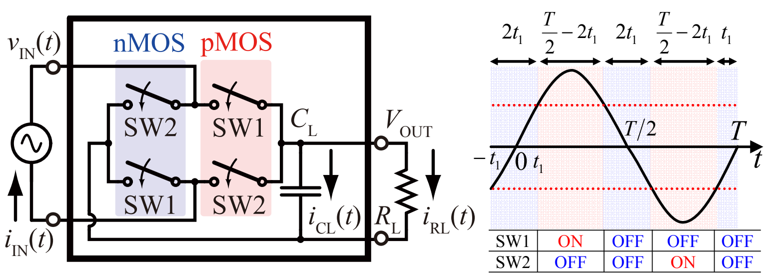
The capacitor current $i_{CL}(t)$, or the charge and discharge voltage of C_L, can be calculated using $i_{RL}(t)$. In a steady state condition, because the total amount of charge and discharge current are equal, the following equation is established,

$$\int_{-t_1}^{t_1} i_{CL,OFF}(t)dt + \int_{t_1}^{\frac{T}{2}-t_1} i_{CL,ON}(t)dt = 0.$$

From this equation, output current I_{OUT} can be obtained when the load capacitance C_L is infinity. The calculated output voltage V_{OUT}, input power P_{IN}, output power P_{OUT} and the power efficiency η are summarized in Tab. II. To simplify the equations, β and ϕ are used as shown in Tab. II. The input power is calculated by

$$P_{IN} = \frac{1}{T/2}\left\{\int_{-t_1}^{t_1} v_{IN}(t) \cdot i_{IN,OFF}(t)dt + \int_{t_1}^{\frac{T}{2}-t_1} v_{IN}(t) \cdot i_{IN,ON}(t)dt\right\},$$

where $i_{IN}(t)$ is the input current.



(a) Schematic. (b) Control signal.

Fig. 1 Rectifier.

Table I.
ON condition and t_1 of a rectifier.

Type	ON condition	Expression of ωt_1
Diode bridge	$V_{IN}\lvert\sin\omega t\rvert \geq V_{OUT} + 2V_{TH}$	$t_1 = \frac{1}{\omega}\sin^{-1}\frac{V_{OUT} + 2V_{TH}}{V_{IN}}$
Cross couple [3]	$V_{IN}\lvert\sin\omega t\rvert \geq V_{TH}$	$t_1 = \frac{1}{\omega}\sin^{-1}\frac{V_{TH}}{V_{IN}}$
Gate biasing [3]	$V_{IN}\lvert\sin\omega t\rvert \geq V_{OUT}$	$t_1 = \frac{1}{\omega}\sin^{-1}\frac{V_{OUT}}{V_{IN}}$

*V_{TH} : Threshold voltage

Table II. Model of the rectifier.

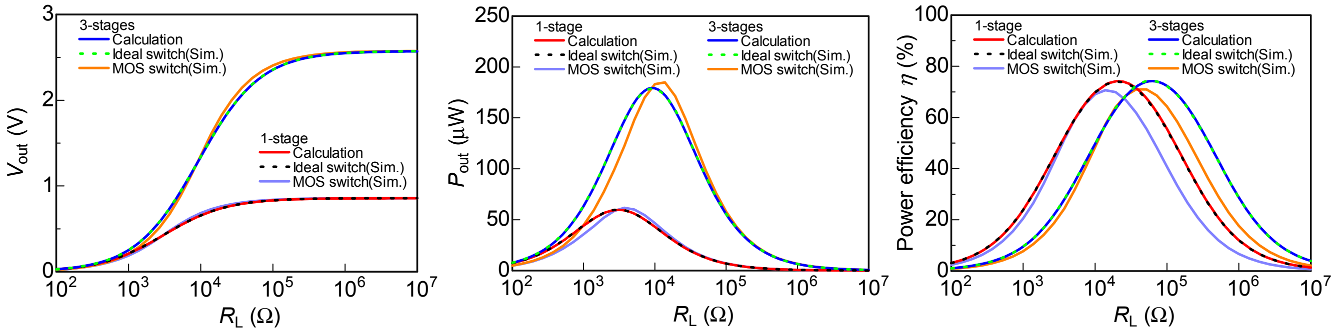
Parameter	Formula	
r'_{ON}	$r_{p,ON} // r_{p,OFF} + r_{n,ON} // r_{n,OFF}$	
r'_{OFF}	$(r_{p,OFF} + r_{n,OFF})/2$	
α	$\frac{r_{p,OFF} r_{n,OFF} - r_{p,ON} r_{n,ON}}{(r_{p,ON} + r_{p,OFF})(r_{n,ON} + r_{n,OFF})}$	
ϕ	$\pi - (1 - r'_{ON}/r'_{OFF}) \cdot 2\omega t_1$	
β	$\frac{(r_{p,ON} + r_{n,OFF})(r_{n,ON} + r_{p,OFF})}{(r_{p,ON} + r_{p,OFF})(r_{n,ON} + r_{n,OFF})}(\pi - 2\omega t_1 + \sin 2\omega t_1) + \frac{r'_{ON}}{r_{p,OFF} // r_{n,OFF}}(2\omega t_1 - \sin 2\omega t$	
V_{OUT}	$2\alpha V_{IN} \cos\omega t_1 / (\phi + \pi r'_{ON}/R_L)$	
P_{IN}	$\frac{V_{IN}^2}{2\pi r'_{ON}}\left[\beta - \frac{8\alpha^2 \cos^2 \omega t_1}{\phi + \pi r'_{ON}/R_L}\right]$	
P_{OUT}	V_{OUT}^2/R_L	
η	P_{OUT}/P_{IN}	
Max P_{OUT}	$\alpha^2 V_{IN}^2 \cos^2 \omega t_1 / (\pi \phi r'_{ON})$	
	Optimum R_L	$R_{OUT} = \pi r'_{ON}/\phi$
Max η	$\left(\frac{2\sqrt{2}\alpha \cos \omega t_1}{\sqrt{\beta\phi} + \sqrt{\beta\phi - 8\alpha^2 \cos^2 \omega t_1}}\right)^2$	
	Optimum R_L	$\pi \beta r'_{ON} / \sqrt{\beta\phi(\beta\phi - 8\alpha^2 \cos^2 \omega t_1)}$

Table III. Calculation and simulation conditions.

Parameter	Value	Parameter	Value
Type	Cross couple	ωt_1	$\sin^{-1}(V_{TH}/V_{IN})$
V_{IN}	1 V	V_{TH}	0.59 V
$W_{p/n}$	13 / 4.5 μm	$L_{p/n}$	0.35 μm
$r_{p,ON}/r_{n,ON}$	980 / 860	$r_{p,OFF}/r_{n,OFF}$	3.1 / 3.0 M
f	1 kHz	C_L	1 mF
R_L	100 – 10M	Number of stages	1, 3

Tabel IV. Comparsion of the results.

	1-stage			3-stages		
	Calc.	Ideal switch (Sim.)	MOS switch (Sim.)	Calc.	Ideal switch (Sim.)	MOS switch (Sim.)
Max P_{OUT} (μW)	60		62	180		185
Optimum R_L (kΩ)	3.1		3.7	9.2		12
Max η (%)	74		72	74		71
Optimum R_L (kΩ)	21		15	62		46



(a) Output voltage V_{OUT}. (b) Output power P_{OUT}. (c) Power efficiency η.

Fig. 2. Load characteristics' comparison of calculation and ideal/MOS switch simulation results.

Using the derived equations, we can estimate the optimum load resistance that shows the maximum power and efficiency. The optimum output resistance of the rectifier is given by $R_{OUT} = \pi r'_{ON}/\phi$ because the output power is maximized when the output resistance of rectifier equals to the load resistance.

The rectifier can be modeled using Y parameter by converting the equations shown in Tab. II. Y parameter is expressed as,

$$\begin{bmatrix} I_{IN} \\ -I_{OUT} \end{bmatrix} = \frac{1}{2\pi r'_{ON}} \begin{bmatrix} \beta & -4\alpha\cos\omega t_1 \\ -4\alpha\cos\omega t_1 & 2\phi \end{bmatrix} \begin{bmatrix} V_{IN} \\ V_{OUT} \end{bmatrix}.$$

By using the Y parameter, it is easy to develop multi-stage rectifier circuit. The output resistance of n-stages rectifier is given by $nR_{OUT} = \pi n r'_{ON}/\phi$.

3. Calculation and simulation results

We evaluated our model accuracy by comparing the results using circuit simulations. We developed 1- and 3-stage cross couple rectifiers. In the circuit simulations, a rectifier with ideal switches and one with MOS switches were considered. Table III summarizes calculation and simulation conditions.

Figure 2 shows results of the output voltage, output power, and efficiency. The simulated results with ideal switches agreed with the calculated results. However, the results with MOS switches slightly changed. This was because of the parasitic loss. The results are summarized in Tab. IV. The optimum load resistance that shows the maximum output power of 3-stages rectifier increased three times higher than that of 1-stage rectifier as discussed in Sect. 2. Thus, our models are useful to estimate the performance of the rectifier circuit.

4. Conclusion

In this paper, we developed an analytical model of the rectifier circuit that is useful for the design of wireless power transfer systems, considering switching timing and switch resistance. The model enables us to estimate the performance of the rectifier and the optimum load resistance that maximizes the output power and the power efficiency. The accuracy of our model was compared with the circuit simulation and showed that our proposed model can estimate the performance of the rectifier with high accuracy.

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