

Application of Meta-film Surface Impedance to Equivalent Transmission Line Model of Meta-Surface for Scattering Analysis

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Abstract – The basic design and analysis of scattering properties of meta-surfaces are usually performed by a transmission line model. The main idea is the decomposition of a periodic structure into a frequency selective surface(FSS) or a meta-film and dielectric slab as a spacing medium between the FSS and a ground plane. Hence, how precisely obtain the scattering property of the FSS is one of essential points. This paper modifies a surface impedance expression of the FSS composed of square patch array so as to enable us to apply for general case, and incorporates it into the equivalent transmission line model of the meta-surface. The effectiveness of the proposed model is numerically confirmed by comparing with the FDTD calculation.

Index Terms —meta-surface, meta-film, transmission line model, FDTD method

1. Introduction

Many kinds of meta-surfaces including an electromagnetic band gap(EBG) structure have been developed in the fields of antennas and propagation, microwave and optical devices and electromagnetic compatibilities[1]. Electromagnetic properties of the meta-surface for both a scattering of plane wave and a dispersion of surface waves are usually analyzed by the transmission line model, and by numerical techniques such as the finite difference time domain(FDTD) method when more accurate results are needed[1]~[3]

In the case of scattering analysis discussed in this paper, the meta-surface is decomposed into a section of planar meta-film and the dielectric slab between the meta-film and the ground plane. The latter can precisely be modeled by a basic transmission line terminated by a short line. Hence, the modeling of the meta-film influences the entire accuracy as a result. Recently, the reflection and transmission properties of the meta-film have been investigated comprehensively, and the accurate surface impedance expression of square patch array has been derived for both TE and TM incident plane waves under the condition that the gap between the patches is very small compared with the period[4]. In this paper, we modify this surface impedance so as to remove this limitation, and incorporate it into the equivalent transmission line model of the meta-surface composed of the square patch array on the dielectric substrate backed by a perfect electric conductor(PEC). The accuracy of the proposed model is numerically tested by comparing with the FDTD calculation.

2. Modification of Surface Impedance and Equivalent Transmission Line Model

The geometry of the meta-surface considered here is shown in Fig. 1. The meta-surface is composed by the square patch array and the dielectric substrate backed by the PEC. The period of the patches is T, and the gap is w as shown in Fig.1(b). The thickness and the relative permittivity of the substrate are h and ϵ_r , respectively. The plane wave incidences obliquely to the meta-surface with the incident angle θ (TE polarization in Fig.1(b)).

Considering that the patch array is placed on a dielectric interface, the original surface impedance expressions of the meta-film in the vacuum[4] are modified as follows.

$$Z_{gc}^{TE} = -j \frac{\eta_{eff}}{2\alpha(1 - \frac{k_0^2 \sin^2 \theta}{k_{eff}^2})}, \quad Z_{gc}^{TM} = -j \frac{\eta_{eff}}{2\alpha} \quad (1)$$

where α is given by

$$\alpha = \frac{k_{eff}}{2} \frac{N\alpha_E}{1 - \frac{N\alpha_E}{4R}} \quad (2)$$

and where k_0 is a wavenumber in vacuum, $k_{eff} = k_0 \sqrt{\epsilon_{eff}}$, $\eta_{eff} = \eta_0 / \sqrt{\epsilon_{eff}}$ is an effective wave impedance, η_0 is the wave impedance in vacuum and $\epsilon_{eff} = (\epsilon_r + 1)/2$. The coefficients N and R are given by $N = 1/T^2$ and $R = 0.6956T$. Another coefficients α_E is called an electric polarizability and originally given by $\alpha_E = 1.02(T-w)^3$ in [5]. However, eq.(1) was derived essentially under the condition of $w \ll T$, we checked the accuracy of eq.(1) using the FDTD method. Fig.3 shows the error of a phase of impedance as a function of x when we put $\alpha_E = x(T-w)^3$ for a normal incidence. It is found that $x = 1.15$ the best value.

Characteristic impedances of the transmission line in the dielectric substrate for TE and TM waves are given by

$$Z_{ds}^{TE} = j\omega\mu \frac{\tan(\beta h)}{\beta}, \quad Z_{ds}^{TM} = j\omega\mu \frac{\tan(\beta h)}{\beta} \cos^2 \theta' \quad (3)$$

where $\beta = \sqrt{k_0^2 \epsilon_r - k_0^2 \sin^2 \theta}$ is the wavenumber in the substrate. $\theta' = \arcsin(\sin \theta / \sqrt{\epsilon_r})$ is calculated from the

snell's law. Using the above parameters, the equivalent transmission line model of the meta-surface for scattering analysis can be illustrated as Fig.3.

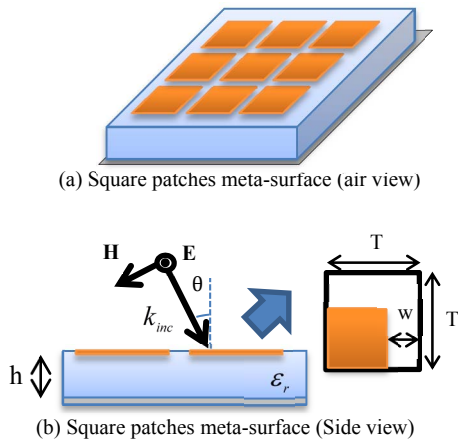


Fig. 1 Analysis model

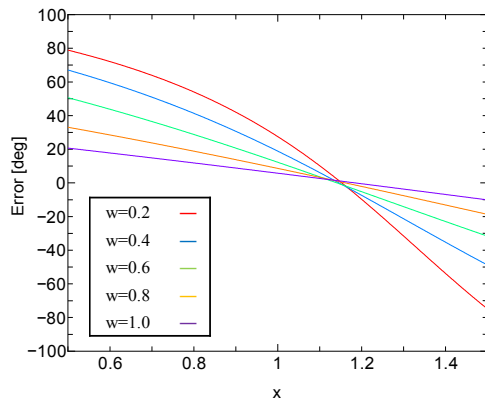


Fig. 2 Phase error of surface impedance (T=2.0mm)

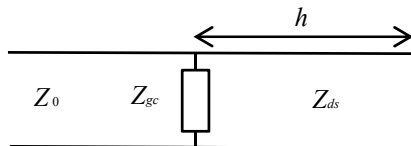


Fig. 3 Equivalent transmission line model

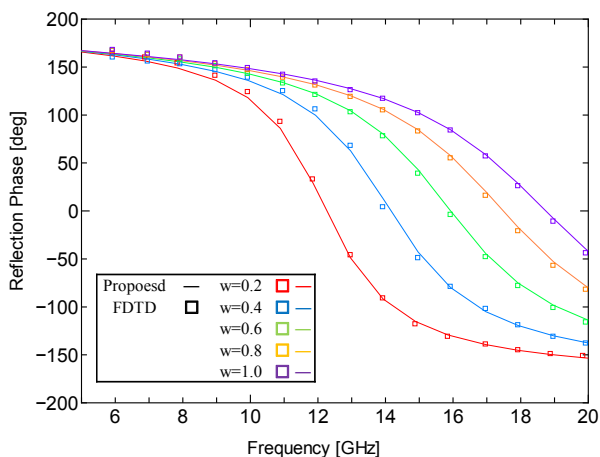


Fig. 4 The reflection phase diagram for TE incidence, angle of incidence is 60°

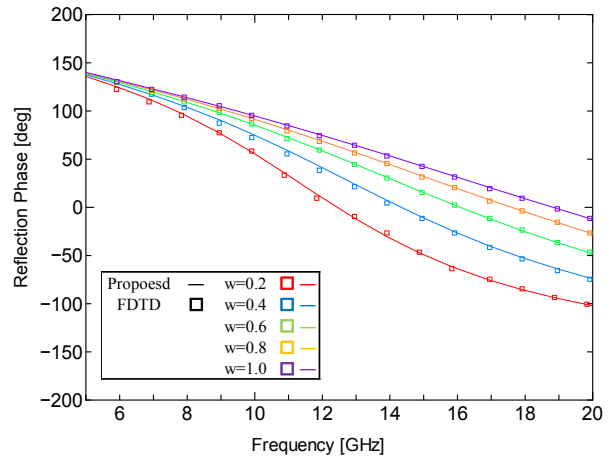


Fig. 5 The reflection phase diagram for TM incidence, angle of incidence is 60°

3. Analysis results

In order to confirm the accuracy of the proposed model, we have calculated frequency characteristics of the reflection coefficients for TE and TM wave incidences, and compared with the FDTD. We set T=2mm and h=2mm and $\epsilon_r = 4.0$. The incident angle for both polarizations is 60 degrees.

Fig.4 and 5 indicates the reflection coefficient diagram when the gap between the patches w is changed. It is found that the all of the results of the proposed model agrees excellently well with the FDTD. The calculation time of the proposed model is surely negligible.

4. Conclusion

In this paper, the surface impedances of the periodic square patch array placed on the dielectric interface have been derived for TE and TM plane wave incidences, and applied to the calculation of the meta-surface scattering problem. It has been demonstrated numerically that the proposed model is considerably accurate for wide frequency range.

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