PLANE WAVE SCATTERING BY A CLUSTER OF IDENTICAL PARTICLES

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1. ABSTRACT

A set of volume integral equations is used to the electromagnetic scattering by a cluster of identical particles in free space. Each particle is assumed to be homogeneous and anisotropic. By means of the plane wave angular spectrum expansion of electromagnetic field inside a homogeneous scatterer, novel entire domain basis functions,which are solutions to the wave equation inside the scatterer, are given and employed as a set of common basis functions. Namely, the internal electric field of all scatterers are represented by a set of common basis functions with different expansion coefficients. A Galerkin method in the Fourier transform spectral domain, which gives numerically stable solutions, is applied to convert the volume integral equation to a linear system of equations. Instead of fast Fourier transform, the relevant Fourier transforms are calculated by a numerical quadrature. The final linear system of equations for N scatterer can be solved by a N^2 recursive algorithm.

2. INTRODUCTION

In recent years, there has been a growing interest in the study of scattering from a cluster of homogeneous particles[1-7]. Many practical applications call for the analysis of clustering effect from a cluster of N identical scatterers[1-4]. Also, in the Monte Carlo simulations of the effective properties of random discrete scatterers, one actually calculates the scattering from a finite number of particles[5-7]. It is noted that in many practical configurations, while the particles themselves are not overlapping, their circumscribed spheres of particles do overlap[4,7]. Therefore,the powerful T-matrix method and recursive T-matrix algorithm seem not tractable yet to be applicable to the scattering of a cluster with overlapping circumscribed spheres since T-matrix formulation is based on the scattering field representation outside the circumscribed sphere of each scatterer.

This article follows up our recent paper[8],which deals with the scattering by a single scatterer. Here we address the more complex scattering geometry of N identical scatterers following the essential feature of [8]. A system of coupled volume integral equations for internal electric fields of particles are taken as the starting point. Because the particles are identical, a common set of entire domain basis functions are employed to represent the internal electric fields of different particles by means of different sets of expansion coefficients. The spectral domain Galerkin method[9-12] are applied to solve the volume integral equations. The reduced linear system of equations for N scatterers is of special right-hand-side terms and can be solved by a N^2 recursive algorithm[13]. The approach of this paper is also strongly related to the method widely used in the analysis of finite array of rectangular conducting patches[11,12]. Instead of the mathematical detail, the main emphasis is given to the idea. Throughout this paper, time factor is $\exp(j\omega t)$.

3. FORMULATION

Consider scattered fields generated by N homogeneous scatterer located at $\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N$ illuminated by a incident plane wave $\boldsymbol{E}_{inc}(\boldsymbol{r})$

$$\boldsymbol{E}_{inc}(\boldsymbol{r}) = \boldsymbol{E}_0 \exp[-jk_0\hat{k}(\theta_{inc}, \phi_{inc})] \tag{1}$$

where E_0 is a constant vector. Here and hereafter, we define

$$\hat{k}(\theta,\phi) = \cos\phi\sin\theta\hat{x} + \sin\phi\sin\theta\hat{y} + \cos\theta\hat{z} \tag{2}$$

with $\hat{x}$, $\hat{y}$, and $\hat{z}$ being unit vectors in a rectangular coordinate system. While the circumscribed spheres of particles may overlap the particles are not overlapping. The scatterers are located in free space with constant permittivity ε_0, magnetic permeability μ_0 ,and wavenumber $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$. Assume each homogeneous and anisotropic object with identical constant tensor permittivity $\varepsilon_0\underline{\varepsilon}$ and permeability $\mu_0\underline{I}$, where $\underline{I}$ is the unit dyad. Furthermore, we assume that the N particles are of identical volume V and identical surface S. Here we suppose that the identical particles may have different orientation. The source induced in the lth homogeneous dielectric scatterer is polarization current J^l

$$J^l(r) = j\omega\varepsilon_0[\underline{\varepsilon} - \underline{I}] \bullet E^l(r) \tag{3}$$

The total electric field can be described by the following volume integral equation

$$E(r) = E_{inc}(r) + \int \underline{G}_0(r,r') \bullet \sum_{l=1}^{N} k_0^2(\underline{\varepsilon} - \underline{I}) \bullet E^l(r')dr' \tag{4}$$

where

$$\underline{G}_0(r,r') = (\underline{I} + \frac{1}{k_0^2}\nabla\nabla)\frac{\exp[-jk_0|r-r'|]}{4\pi|r-r'|} \tag{5}$$

is the dyadic Green's function in free space. In order to proceed with the solution of (2), let the observation point r in (2) is restricted inside each homogeneous anisotropic scatterer respectively, a set of coupled integral equations is obtained for the unknown electric field $E^l(r)$ inside the lth scatterer. Following the method used in the references[14-16], we can express the electric field inside the homogeneous and anisotropic scatterer as

$$\begin{aligned} E(r) &= \sum_{n=1}^{2}\int_0^{2\pi}\int_0^{\pi} d\phi_k \sin\theta_k d\theta_k C_n(\theta_k,\phi_k)E_n(\theta_k,\phi_k)\exp[-jk_n(\theta_k,\phi_k)\cdot r] \\ &= \sum_{n=1}^{2}\sum_{s=0}^{S}\sum_{t=0}^{T} C_{nst}E_n(\theta_{kt},\phi_{ks})\exp[-jk_n(\theta_{kt},\phi_{ks})\cdot r] \end{aligned} \tag{6}$$

$$E_n(\theta_k,\phi_k) = E_{nx}(\theta_k,\phi_k)\hat{x} + E_{ny}(\theta_k,\phi_k)\hat{y} + E_{nz}(\theta_k,\phi_k)\hat{z} \tag{7}$$

$$k_n(\theta_k,\phi_k) = k_n(\theta_k,\phi_k)\hat{k}(\theta_k,\phi_k) \tag{8}$$

$$\hat{k}(\theta_k,\phi_k) = \cos\phi_k\sin\theta_k\hat{x} + \sin\phi_k\sin\theta_k\hat{y} + \cos\theta_k\hat{z} \tag{9}$$

$$C_{nst} = \frac{2\pi^2}{ST}W_sW_tk_n(\theta_{kt},\phi_{ks})C_n(\theta_{kt},\phi_{ks})\sin\theta_{kt} \tag{10}$$

$$\theta_{kt} = \frac{t\pi}{T}, \phi_{ks} = \frac{s2\pi}{S} \tag{11}$$

where $C_n(\theta_k,\phi_k)$,$k_n(\theta_k,\phi_k)$ and $E_n(\theta_k,\phi_k)$ are the undetermined amplitude, wave number and wavevector direction of the nth eigenwave in an unbounded homogeneous anisotropic medium[14-16], respectively; T and S, and W_t and W_s are the node numbers, and weight coefficients of numerical quadratures with respect to θ_k and ϕ_k, respectively. Moreover, the multi-index nst can be denoted by a single index m and therefore (8) can be rewritten as

$$E(r) = \sum C_mE_m(r) \tag{12}$$

Because the assumed scatterers are identical, we can use the above common set of basis functions to expand the internal electric fields of every scatterer in the scatterer coordinate system as follows:

$$E^l(r) = \sum C_m^lE_m(r-r^l) = \sum C_m^lE_m(r)\exp[jk_m\cdot r^l] \tag{13}$$

Taking into account the Fourier expansion of scalar Green's function in a spherical coordinate system

$$\frac{\exp[-jk_0|r-r'|]}{4\pi|r-r'|} = \frac{1}{(2\pi)^3}\int_0^{\infty}\int_0^{2\pi}\int_0^{\pi} k^2 dk d\phi_k \sin\theta_k d\theta_k \frac{\exp[-jk\hat{k}\cdot(r-r')]}{k^2-k_0^2} \tag{14}$$

:multiplying both side of (2) by the following vector $[F^{l'}_{m'}]^*(r)$(* denotes the complex conjugate)

$$F^{l'}_{m'}(r) = (\underline{\varepsilon}-\underline{I})\bullet E^{l'}_{m'}(r), m' = n's't', n' = 1,2; s' = 0,\ldots,S; t' = 0,\ldots,T \tag{15}$$

where the explicit expression of $E^{l'}_{m'}(r)$ is given in (12); using the Parseval theorem in the spectral domain approach[7-12], we finally obtain

$$[\underline{Z}^{ll'}_{M\times M}]_{N\times N}[\underline{C}^{l}_{M\times 1}]_{N\times 1} = [\underline{V}^{l'}_{M\times 1}]_{N\times 1} \tag{16}$$

where

$$\begin{aligned} V^{l'}_{m'} &= \int_V [F^{l}_{m'}]^*(r)\bullet E_{inc}(r)dr \\ &= V_{m'}[\exp(jk_{m'}\cdot r_{l'})]^* = S^{l'0}V_{m'} = S^{l'0}\int_V F^*_{m'}(r)\bullet E_{inc}(r)dr \qquad (17) \\ [\underline{V}^{l'}_{M\times 1}] &= \underline{S}^{l'0}\bullet[\underline{V}_{M\times 1}] = S^{l'0}\underline{I}\bullet[\underline{V}_{M\times 1}] \qquad (18) \\ Z^{ll'}_{mm'} &= \int_0^{2\pi}\int_0^{\pi} d\phi_k \sin\theta_k d\theta_k \hat{F}^*_{m'}(\theta_k,\phi_k)\frac{\omega\varepsilon_0 k_0}{8\pi^2} \\ &\bullet[\underline{I}-\hat{k}\hat{k}]\bullet\hat{F}_m(\theta_k,\phi_k)\exp[j(k_m\cdot r_l](\exp[jk_{m'}\cdot r_{l'}])^*; l\neq l' \qquad (19) \\ Z^{ll}_{mm'} &= \int_0^{2\pi}\int_0^{\pi} d\phi_k \sin\theta_k d\theta_k \hat{F}^*_{m'}(\theta_k,\phi_k)\frac{\omega\varepsilon_0 k_0}{8\pi^2} \\ &\bullet[\underline{I}-\hat{k}\hat{k}]\bullet\hat{F}_m(\theta_k,\phi_k)+\frac{1}{3}\int_V F^*_{m'}(r)\bullet F_m(r)dr \qquad (20) \end{aligned}$$

where

$$F_m(r) = (\underline{\varepsilon}-\underline{I})\bullet E_m(r), m = nst, n = 1,2; s = 0,\ldots,S; t = 0,\ldots,T \tag{21}$$

$$\hat{F}_m(\theta_k,\phi_k) = \int_V F_m(r)\exp[jk_0\hat{k}\cdot r]dr \tag{22}$$

Note that $Z^{ll'}_{mm'}$ and $Z^{ll}_{mm'}$ are of special form[11]. Consequently, only one set of Fourier transform $\hat{F}_m(\theta_k,\phi_k)$ of one scatterer[8.15] is needed to compute the above integrals for various combination of indices l and l'. In other words, we can set up a data file for Fourier transform $\hat{F}_m(\theta_k,\phi_k)$ of one scatterer and call the data to produce the matrix elements of N scatterers. In particular, for a rectangular dielectric parallelepiped[9], a finite circular cylinder and a sphere, the Fourier transform (19) can be analytically calculated. In addition, for an arbitrary volume V, all the volume integrals can be reduced to surface integrals by virtue of the second Green's identity. Due to the special right-hand-side terms, we do not solve the (16) by inversing the full matrix directly, which would need N^3 float operation and larger storage. Instead, we introduce a configuration dependent (or $\underline{S}^{l'0}$ dependent) block diagonal matrix as the inverse matrix based on the representation theory of linear transform(the readers may be referred to [13] for detail) [13] for detail.). Explicitly, we may let

$$\underline{C}^{l} = \underline{Y}^{l(N)}\bullet\underline{S}^{l0}\bullet\underline{V} \tag{23}$$

where $\underline{Y}^{l(N)}$ represents the Y matrix of the lth scatterer in the N scatterer configuration, which can be recursively determined as follows[13]:

$$\underline{Y}^{N+1(N+1)}\bullet\underline{S}^{N+1,0} = [\underline{I}-\underline{Y}^{N+1(1)}\bullet\sum_{i=1}^{N}\underline{Z}^{N+1,i}\bullet\underline{Y}^{i(N)}\bullet\underline{Z}^{i(N+1)}]^{-1}$$

$$\bullet \underline{Y}^{N+1(1)} \bullet [\underline{S}^{N+1,0} + \sum_{i=1}^{N} \underline{Z}^{N+1,i} \bullet \underline{Y}^{i(N)} \bullet \underline{S}^{i0}] \tag{24}$$

$$\underline{Y}^{i(N+1)} \bullet \underline{S}^{i0} = \underline{Y}^{i(N)} \bullet [\underline{S}^{i0} + \underline{Z}^{i(N+1)} \bullet \underline{Y}^{N+1(N+1)}] \bullet \underline{S}^{N+1.0} \tag{25}$$

4. CONCLUDING REMARKS

The proposed approach makes a full use of the homogeneity of a scatterer and the convolution characteristic of the volume integral equation. Probably, this work can be viewed as a minor extension of spectral domain approach for finite array of rectangular conducting patches to a cluster of N scatterers. The great advantage of our approach is that the fourier integrals are with respect to finite regions.Therefore, instead of the fast Fourier transform, we will use the novel numerical quadrature applied in [15] to evaluate all the closed surface integrals appearing in this article. This procedure totally avoids the so called aliasing problem[8] arising from the application of fast Fourier transform. As it has already proved for some specific geometry[17.18], a successful selection of plane wave basis functions can lead to a converging rate much higher than that of using subdomain basis functions. Another advantage of the present approach is that no addition theorem is involved in the analysis of scattering by N scatterers. In fact,after obtaining the internal electric field, we can easily get the relevant physical quantities following the method of [2]. So we believe that this paper makes complements of current approaches.especially for the complex scattering geometry with overlappingcircumscribed spheres. Following the procedure of [13], this approach including the matrix inversing is equally suitable for arbitrary incident field. And except for the matrix inversing,this approach can be applied to the scattering by N different scatterers provided that each scatterer is homogeneous. The method proposed here can also be applied to the elastic wave scattering[14]. The numerical implementation of present approach is in consideration and will be reported later.

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