ANALYSIS OF A PULSED PLANE WAVE SCATTERED FROM A TWO-LAYERED DIELECTRIC CYLINDER USING FD-TD METHOD

Takato KUDOU and Hideyuki ASOH
Dept. of Electrical and Electronic Eng., Oita University
700 Dannoharu, Oita 870-11, Japan

1. Introduction

In electromagnetic inverse scattering problems, such as diffraction tomography or underground survey, it is required to extract target informations involved in the scattered wave in order to visualize the target image. Because scattering characteristics are closely concerned with the frequency of the incident wave, it is obvious that the scattering analysis using pulsed waves is advantageous to obtain wide-range informations of the target in comparison with the analysis using a monochromatic wave. For this reason, it is important to analyze transient scattering phenomena by various shapes, and the result is expected to give a clue to the inverse problems mentioned above. Recently the phenomena have been systematically discussed in detail in both two and three-dimensional problems[1]–[3].

On the other hand, according as computer techniques are rapidly improved, numerical methods for solving Maxwell's curl-equations directly have become of a matter of great concern. The FD-TD (Finite Difference Time Domain) method proposed by K. S. Yee[4] is one of the methods and has been already applied to various scattering problems[5]. This method is also expected to be useful for the inverse scattering problems.

In this paper we apply the FD-TD method to the analysis of a pulsed plane wave scattered from a two-layered dielectric cylinder and investigate back-scattering responses when the radius of the inner layer of the cylinder is changed. We use an E-polarized Gaussian pulse as the incident wave and a PML (Perfectly Matched Layer) technique[6] for the absorption of the scattered wave on boundaries of an analysis domain.

2. Formulation

Let us consider a pulsed wave scattering by a two-layered dielectric cylinder located in vacuum as shown in Fig.1. In this figure, ε_0 and μ_0 are, respectively, the permittivity and the permeability of vacuum. Parameters ε_1 and ε_2 are permittivities of each layer in the cylinder, a is the radius of the cylinder, and b is the radius of the inner layer of the cylinder. Coordinates (x_c, y_c) and (x_o, y_o) denote the center of the cylinder and the observation point for the scattered wave, respectively. The analysis domain is a rectangle of the length L and the width W, and it is surrounded by PML media of thickness d. The PML media are finally ended by perfectly conducting conditions.

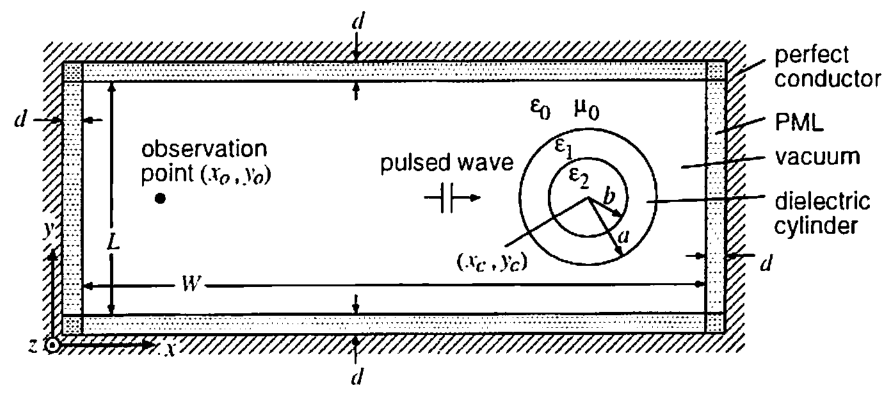


Fig.1. Analysis model.

When we quantize space, letting $x = i\,\Delta u$ and $y = j\,\Delta u$, and time, letting $t = n\,\Delta t$, each electromagnetic field component is specified by the i, j and n indices, and Maxwell's curl-equations are approximately replaced by difference equations. Introducing two-dimensional Yee cell[4] shown in Fig.2, the field components H_x^s, H_y^s and E_z^s of the scattered wave in the analysis domain are calculated by the following equations[5]

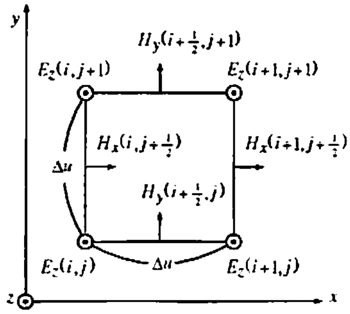


Fig.2. Two-dimensional Yee cell.

$$H_x^s(i,j+\frac{1}{2})^{n-\frac{1}{2}} = H_x^s(i,j+\frac{1}{2})^{(n-1)-\frac{1}{2}} - \frac{\Delta t}{\mu_0 \Delta u}\left\{E_z^s(i,j+1)^{n-1} - E_z^s(i,j)^{n-1}\right\}, \quad (1)$$

$$H_y^s(i+\frac{1}{2},j)^{n-\frac{1}{2}} = H_y^s(i+\frac{1}{2},j)^{(n-1)-\frac{1}{2}} + \frac{\Delta t}{\mu_0 \Delta u}\left\{E_z^s(i+1,j)^{n-1} - E_z^s(i,j)^{n-1}\right\}, \quad (2)$$

$$E_z^s(i,j)^n = E_z^s(i,j)^{n-1} - \frac{\{\varepsilon(i,j)-\varepsilon_0\}\Delta t}{\varepsilon(i,j)}\dot{E}_z^{inc}(i,j)^n - \frac{\Delta t}{\varepsilon \Delta u}\left\{H_x^s(i,j+\frac{1}{2})^{n-\frac{1}{2}} - H_x^s(i,j-\frac{1}{2})^{n-\frac{1}{2}} - H_y^s(i+\frac{1}{2},j)^{n-\frac{1}{2}} + H_y^s(i-\frac{1}{2},j)^{n-\frac{1}{2}}\right\}, \quad (3)$$

where $\dot{E}_z^{inc}$ and ε denote the time derivative of the electric field of the incident wave and the permittivity of the point specified by the i and j indices, respectively. In the two-dimensional FD-TD method, the space increment Δu and the time increment Δt have to satisfy the following Courant stability condition

$$\Delta t \leq \frac{\Delta u}{c\sqrt{2}}, \quad (4)$$

where c is the velocity of light. In this paper, we set $\Delta u = 0.01$[m] and $\Delta t = 0.02$[nsec].

In the PML media surrounding the analysis region, according to Ref.[6], we divide the electric component E_z^s into two subcomponents E_{zx}^s and E_{zy}^s, i.e., we define as $E_z^s = E_{zx}^s + E_{zy}^s$. Then the field components of the scattered wave are calculated by the following equations

$$H_x^s(i,j+\frac{1}{2})^{n-\frac{1}{2}} = \exp\{-\sigma_y^*(j+\frac{1}{2})\Delta t/\mu_0\} H_x^s(i,j+\frac{1}{2})^{(n-1)-\frac{1}{2}} - \frac{1-\exp\{-\sigma_y^*(j+\frac{1}{2})\Delta t/\mu_0\}}{\sigma_y^*(j+\frac{1}{2})\Delta u}\left\{E_{zx}^s(i,j+1)^{n-1} + E_{zy}^s(i,j+1)^{n-1} - E_{zx}^s(i,j)^{n-1} - E_{zy}^s(i,j)^{n-1}\right\}, \quad (5)$$

$$H_y^s(i+\frac{1}{2},j)^{n-\frac{1}{2}} = \exp\{-\sigma_x^*(i+\frac{1}{2})\Delta t/\mu_0\} H_y^s(i+\frac{1}{2},j)^{(n-1)-\frac{1}{2}} + \frac{1-\exp\{-\sigma_x^*(i+\frac{1}{2})\Delta t/\mu_0\}}{\sigma_x^*(i+\frac{1}{2})\Delta u}\left\{E_{zx}^s(i+1,j)^{n-1} + E_{zy}^s(i+1,j)^{n-1} - E_{zx}^s(i,j)^{n-1} - E_{zy}^s(i,j)^{n-1}\right\}, \quad (6)$$

$$E_{zx}^s(i,j)^n = \exp\{-\sigma_x(i)\Delta t/\varepsilon_0\} E_{zx}^s(i,j)^{n-1} + \frac{1-\exp\{-\sigma_x(i)\Delta t/\varepsilon_0\}}{\sigma_x(i)\Delta u}\left\{H_y^s(i+\frac{1}{2},j)^{n-\frac{1}{2}} - H_y^s(i-\frac{1}{2},j)^{n-\frac{1}{2}}\right\}, \quad (7)$$

$$E^s_{zy}(i,j)^n = \exp\{-\sigma_y(i)\Delta t/\varepsilon_0\}E^s_{zy}(i,j)^{n-1} - \frac{1-\exp\{-\sigma_y(i)\Delta t/\varepsilon_0\}}{\sigma_y(i)\Delta u}\left\{H^s_x(i,j+\frac{1}{2})^{n-\frac{1}{2}} - H^s_x(i,j-\frac{1}{2})^{n-\frac{1}{2}}\right\}, \quad (8)$$

where (σ_x, σ_y) and (σ^*_x, σ^*_y) are, respectively, electric and magnetic conductivities of the point specified by the i or j index. These parameters are defined as follows:
— in the PML medium of the left and right sides of the analysis domain,

$$\sigma_y = \sigma^*_y = 0, \qquad \sigma^*_x = \frac{\mu_0}{\varepsilon_0}\sigma_x, \qquad \sigma_x(\rho) = \sigma_m\left(\frac{\rho}{d}\right)^2 \quad (0 \le \rho \le d), \quad (9)$$

— in the PML medium of the upper and lower sides of the analysis domain,

$$\sigma_x = \sigma^*_x = 0, \qquad \sigma^*_y = \frac{\mu_0}{\varepsilon_0}\sigma_y, \qquad \sigma_y(\rho) = \sigma_m\left(\frac{\rho}{d}\right)^2 \quad (0 \le \rho \le d), \quad (10)$$

where σ_m and ρ are, respectively, the maximum value of electric conductivities and the distance from each boundary between the analysis domain and the PML medium. In this paper, we set $\sigma_m = 0.7$ and $d = 0.08$[m] empirically.

3. Numerical Results

As the incident wave, we have used the Gaussian pulse defined as follows:

$$E^{inc}_z(i,j)^n = \begin{cases} e^{-\alpha(\tau-\beta\Delta t)^2} & (0 \le \tau \le 2\beta\Delta t), \\ 0 & (\text{otherwise}), \end{cases} \quad (11)$$

where

$$\tau = t - \frac{\Delta u}{c}(i-1) = n\Delta t - \frac{\Delta u}{c}(i-1), \quad (12)$$

and we have set $\alpha = \{4/(\beta\Delta t)\}^2$ and $\beta = 125$ in this paper. The shape of the pulse is shown in Fig.3. In this case, $\dot{E}^{inc}_z$ in Eq.(3) is given as follows:

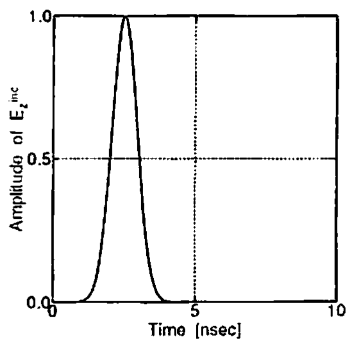


Fig.3. Incident Gaussian pulse.

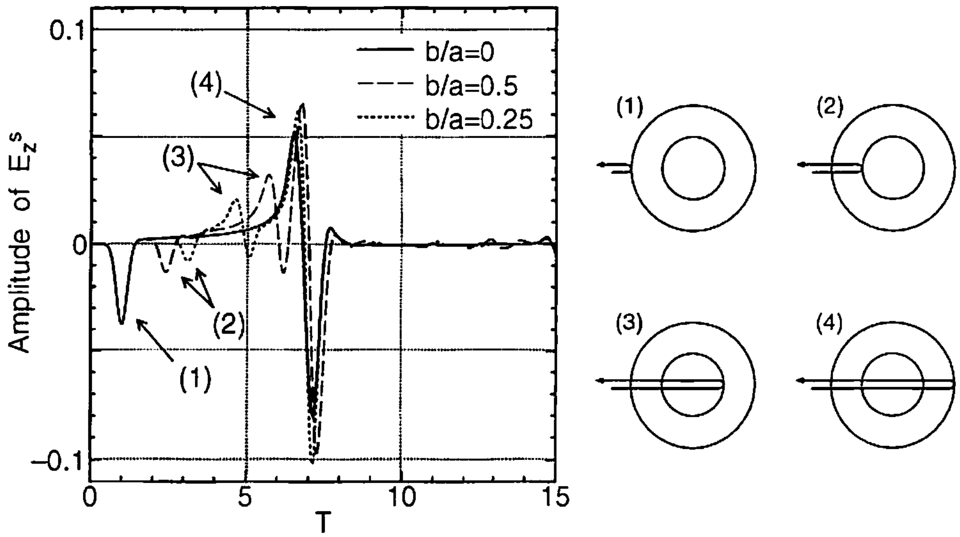
$$\dot{E}^{inc}_z(i,j)^n = -2\alpha(\tau-\beta\Delta t)e^{-\alpha(\tau-\beta\Delta t)^2}. \quad (13)$$

We have set the analysis domain as $L = 4.5$[m] and $W = 11.25$[m], and have performed computer simulations for the cylinder with $\varepsilon_1/\varepsilon_0 = 2.0$, $\varepsilon_2/\varepsilon_0 = 3.0$, $a = 0.75$[m] and $(x_c, y_c) = (9.09\text{[m]}, 2.34\text{[m]})$. The observation point has been set at $(x_o, y_o) = (0.84\text{[m]}, 2.34\text{[m]})$. Figure 4 shows back-scattering responses of the Gaussian pulse by the two-layered dielectric cylinder when the radius b of the inner layer of the cylinder is changed. In this figure, the time parameter T is defined as $T = ct/a$, and we have set $T = 0$ at the time when the front of the scattered wave reaches the observation point. We can see from Fig.4 that

- Though reflections at the surface of the cylinder (responses (1) and (4)) are only observed in the case of $b/a = 0$, reflections at the surface of the inner layer (responses (2) and (3)) are observed, in addition to reflections at the surface of the cylinder, in cases of $b/a = 0.5$ and 0.25.

- The difference in radii of the inner layer appears clearly as that in response timings and hence we can estimate the inner structure of the cylinder using the characteristic.

In the above cases, no definite response is observed concerning creeping waves. But in the case that the cylinder is a strong scatterer, precise analyses should be given to the responses.



(a) Response waveforms (b) Paths of the responses with corresponding numbers

Fig.4. Back-scattering responses of a Gaussian pulse by a two-layered cylinder. Parameters are $\varepsilon_1/\varepsilon_0 = 2.0$, $\varepsilon_2/\varepsilon_0 = 3.0$, $a = 0.75$[m], $(x_c, y_c) = (9.09$[m]$, 2.34$[m]$)$ and $(x_o, y_o) = (0.84$[m]$, 2.34$[m]$)$.

4. Conclusion

We have analyzed back-scattering responses of a pulsed wave by a two-layered dielectric cylinder using the FD-TD method. We have shown that the difference in three types of the inner structure of the cylinder can be seen clearly as that in response waveforms. We will investigate scattering characteristics of pulsed waves by lossy and complicated objects precisely, and consider the application of the FD-TD method to inverse scattering problems.

Acknowledgement

The authors wish to thank Prof. Mitsuru Tanaka, Oita University, for his encouragement and helpful suggestions.

References

[1] Ikuno H., Nishimoto M. and Matsumoto K.: "Extended ray solution for scattering from a dielectric cylinder", Proc. 1989 URSI Int. Symp. EM theory, pp.178-180, Stockholm (1989).

[2] Itoh A., Hosono T. and Hirao Y.: "Transient electromagnetic fields on a conducting sphere excited by a pulsed plane wave", IEICE Trans. Electron., E75-C, 1, pp.107-112 (Jan. 1992).

[3] Shirai H., Nakano E. and Yano M.: "Analysis of time transient EM field response from a dielectric spherical cavity", IEICE Trans. Electron., E75-C, 5, pp.627-634 (May 1992).

[4] Yee K. S.: "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media", IEEE Trans. Ant. Prop., 14, 3, pp.302-307 (May 1966).

[5] Kunz K. S. and Luebbers R. J.: The finite difference time domain method for electromagnetics, CRC Press (1993).

[6] Berenger J.-P.: "A perfectly matched layer for the absorption of electromagnetic waves", J. Comput. Phys., 114, pp.185-200 (1994).