A NEW EXPRESSION FOR THE SCATTERING OF A GAUSSIAN BEAM BY A CONDUCTING RECTANGULAR CYLINDER - BEM -

Makoto Ohki[+], Kazuto Shimizu[+], Haruo Sakurai[++]
and Syogo Kozaki[+]
[+]Faculty of Engineering, Gunma University,
1-5-1 Tenjin-cho, Kiryu-shi, Gunma 376, Japan.
[++]Gunma College of Technology,
580 Toriba-machi, Maebashi-shi, Gunma 371, Japan

Abstract:
Resonant solution involved in the integral equation for the electromagnetic beam scattering by conducting rectangular cylinder is investigated. A simpler expression than in conventional analysis is represented. The surface current distribution is also presented.

1. Introduction

In indoor experiment it is very difficult to make the plane wave. In the microwave range, the horn antennas contain amplitude as well as phase distribution. Under the condition the electromagnetic beam wave scattering by conducting circular cylinder has been widely studied[1]. On the other hand the electromagnetic beam wave by conducting rectangular cylinder has also already investigated by different beam expression and B.E.M.[3].

Author reported a new expression for the scattering of a Gaussian beam by a conducting cylinder[1] and extended into the more complicated problemes, e.g., rectangular cylinder in this paper.

2. A new expression of the beam

We consider the following wave equation for the incident beam wave

$$\frac{\partial^2 E_z^i}{\partial x'^2} + \frac{\partial^2 E_z^i}{\partial y'^2} + k_0^2 E_z^i = 0 \qquad (1)$$

Let us assume the Gaussian beam (Fig. 1) at x'=0 as

$$E_z^i(x'=0, y') = E_0 e^{-\beta^2 y'^2} \qquad (2)$$

Applying Fourier transforms in Eq(1), we obtain

$$E_z^i(x',y') = \frac{E_0}{2\sqrt{\pi}\beta}\int_{-\infty}^{\infty}\exp\left(-\frac{\alpha^2}{4\beta^2} - jx'\sqrt{k_0^2-\alpha^2} - jy'\alpha\right)d\alpha \tag{3}$$

We put

$$x' = r_q\cos(\theta_q+\theta_i) + d_{q,o}\cos(\phi_{q,o}+\theta_i) \tag{4a}$$

$$y' = r_q\sin(\theta_q+\theta_i) + d_{q,o}\sin(\phi_{q,o}+\theta_i) \tag{4b}$$

$$\gamma = \sin^{-1}(\alpha/k_0) \tag{4c}$$

where θ_i is the incident angle and using the Bessel coefficients, we have

$$E_{z,q}^i(r_q,\theta_q) = E_0\sum_{n=-\infty}^{\infty} j^{-n}J_n(k_0r_q)e^{jn\theta_q}W_{n,q} \tag{5a}$$

where

$$W_{n,q} = \frac{e^{jn\theta_i}}{2\sqrt{\pi}\beta}\int_{-\infty}^{\infty} e^{-\frac{\alpha^2}{4\beta^2}-jk_0d_{q,o}\cos(\phi_{q,o}-\gamma)-jn\gamma}d\alpha \tag{5b}$$

The expression of the beam is represented in a much simpler form compared with the conventional one (e.g.[3]) and the product of a plane wave expression ($W_{n,q}=1$) and weghting function of $W_{n,q}$.

3. Boundary element method (B.E.M)

The tangential component of the total electric field E_z on C vanishes and the following integral equation is given by [2].

$$E_z^i(\bar{r}) = \frac{k_0\eta_0}{4}\int_c K_z(\bar{r})H_0^{(2)}(k_0(\bar{r}-\bar{r}'))d\bar{l}' \tag{6a}$$

where $K_z(\bar{r})$ is the electric current and assumed to be defined by

$$K_z(\bar{r}) = \sum_{q=1}^{N}\alpha_q f_q(\bar{r}_q) \tag{6b}$$

where

$$f_q(\bar{r}_q) = \begin{cases} 1 & \text{on } \Delta c_n \\ 0 & \text{on all other } \Delta c_n \end{cases}$$

The matrix equation is obtained as

$$\left[E_p^i\right] = \left[A_{pq}\right]\left[\alpha_q\right] \tag{7a}$$

from which the unknown coefficient $\left[\alpha_q\right]$ is determined.

4. Extended boundary condition

In order to avoid the resonant solution involved in Eq.(7a) the following extended boundary condition must be adopted[2]

$$-\frac{k_0\eta_0}{4}\int_c K_z(\vec{r})H_0^{(2)}(k_0R)d\vec{l}\,' = -E_z^i(\vec{r}) \tag{8a}$$

Applying the additional theorem of the Hankel function $H_0^{(2)}(k_0R)$, we have

$$-\frac{k_0\eta_0}{4}\int_c K_z(\vec{r})H_m^{(2)}(k_0r')e^{-jm\theta'}d\vec{l}\,' = (-j)^m e^{-jm\theta_i}W_{m,q} \qquad (m = 0,\pm1,\pm2,....) \tag{8b}$$

5. Numerical result

The surface current distribution along the corner is illustrated in Fig. 2(a) in which dotted line shows the result without resonant condition and solid line denotes the with resonant solution. Thus the elimination of the erroneous solution was made for the induced current of the beam.

Numerical calculations of the near scattered field along the conducting surface has also been performed in the paper, however, these results are omitted here to be described.

This method the above can also be applied to the various problems, e.g., dielectric square cylinder and conducting stripes etc.

References:

[1] S.Kozaki: "A new expression for the scattering of a Gaussian beam by a conducting cylinder", IEEE Trans., Antennas and propagat., vol. 30, no. 5, Sep. 1982.

[2] N.Morita: "Resonant solution involved in the integral equation approach to scattering from conducting and dielectric cylinder, IEEE Trans on Antennas and propagat., vol.27, no. 6, Nov. 1979.

[3] Y.Miyazaki, W.Asano and K.Tanaka: "Microwave and laser scattering studies for electromagnetic compatibility problem in mobile communication and broadcasting system", EMC, ISP408, pp. 386-389, 1994.

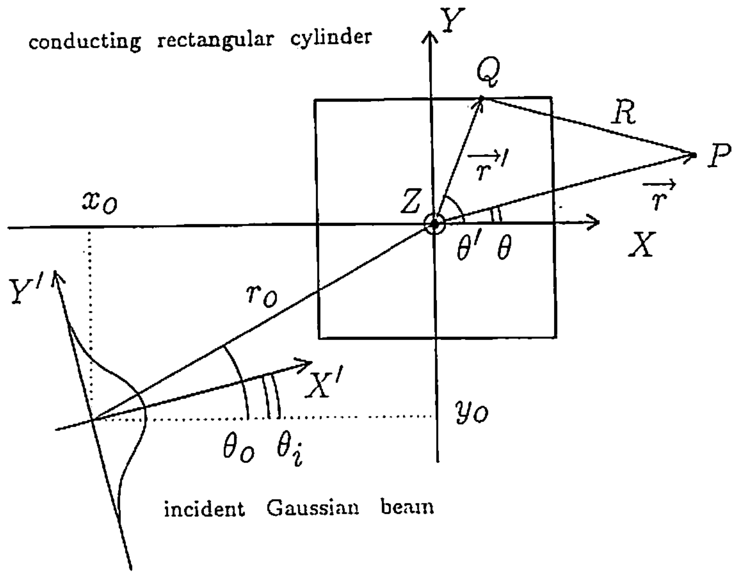

Fig 1. Geometry of the problem

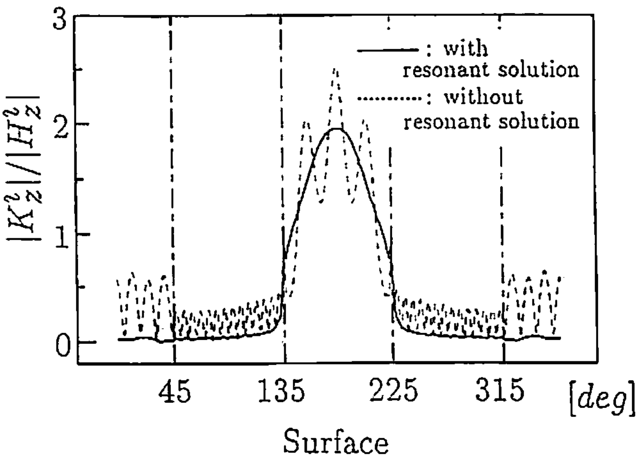

Fig 2. Surface current distribution

$(k_o a = k_o w_o = 24.602, k_o r_o = 24.602\pi,$
$\theta_i = \theta_o = 0^o, m = 5)$