ELECTROMAGNETIC SCATTERING BY THE LUNEBERG LENS REFLECTOR

Haruo SAKURAI+, Takeshi HASHIDATE++, Makoto OHKI++,
Kuniyuki MOTOJIMA++ and Syogo KOZAKI++
+Gunma College of Tehnology
580, Toriba-machi, Maebashi-shi, Gunma 371, Japan.
++Faculty of Engineering, Gunma University
1-5-1, Tenjin-cho, Kiryu-shi, Gunma 376, Japan.

1. Introduction

The Luneberg lens is a spherical lens that focuses collimated waves incoming from a point on one side of the lens to a point at the surface on the other side. Although the Luneberg lens was originally proposed as a optical lens, it is commercialized as the radar reflector that has a large radar cross section over a wide conical viewing angle.

The Luneberg lens has a refractive index n which varies with normalized radial coordinates ρ according to the relation $n = \sqrt{2-\rho^2}$, however, it is fabricated by using several different homogeneous dielectric layered media practically. And it can be made into a reflector by adding a reflecting surface (metallic cap) to the point where the incident plane wave is focused.

The electromagnetic scattering by the Luneberg lens reflector is not able to analyze as a classical boundary value problem any longer. Sanford [1] utilizes the modal expansion technique and Physical Optics whose method is effective to analyze the far field.

In this paper we analyze the near field of the lens by using the modal expansion technique and Point Matching Method (PMM) [2]. In order to check the theory, a microwave model experiment is performed by using Tokipec Company six different homogeneous layered lens. The experimental values are a good agreement with theoretical ones.

2. Formulation of the problem

The geometry of the problem is shown in Fig. 1. We consider the Luneberg lens reflector which has six layered structure and a metallic cap in a portion of the sphere. The plane wave traveling in the z-direction is assumed to be given by

$$\mathbf{E}^i = E_0 e^{-jkz}\mathbf{i}_x = E_0 \sum_{n=1}^{\infty}(-j)^n \frac{2n+1}{n(n+1)}\left\{\mathbf{M}^{(1)}_{o1n}(k_0) + j\mathbf{N}^{(1)}_{e1n}(k_0)\right\} \tag{1}$$

The scattered and transmitted waves in the each shell p are represented as

$$\mathbf{E}_p^s = E_0 \sum_{n=1}^{\infty} (-j)^n \frac{2n+1}{n(n+1)} \left\{ \alpha_n^p \mathbf{M}_{oln}^{(3)}(k_p) + j\beta_n^p \mathbf{N}_{eln}^{(3)}(k_p) \right\} \tag{2a}$$

$$\mathbf{E}_p^t = E_0 \sum_{n=1}^{\infty} (-j)^n \frac{2n+1}{n(n+1)} \left\{ a_n^p \mathbf{M}_{oln}^{(1)}(k_p) + j b_n^p \mathbf{N}_{eln}^{(1)}(k_p) \right\} \tag{2b}$$

where $\mathbf{M}_{oln}$, $\mathbf{N}_{eln}$ are the vector mode functions [3] and defined by

$$\begin{aligned} \mathbf{M}_{oln}(k_p) &= z_n(k_p r) \frac{P_n^1(\cos\theta)}{\sin\theta} \cos\varphi \mathbf{i}_\theta \\ &\quad - z_n(k_p r) \frac{\partial P_n^1(\cos\theta)}{\partial\theta} \sin\varphi \mathbf{i}_\varphi \end{aligned} \tag{3a}$$

$$\begin{aligned} \mathbf{N}_{eln}(k_p) &= n(n+1) \frac{z_n(k_p r)}{kr} P_n^1(\cos\theta) \cos\varphi \mathbf{i}_r \\ &\quad + \frac{\left[k_p r z_n(k_p r)\right]'}{kr} \frac{\partial P_n^1(\cos\theta)}{\partial\theta} \cos\varphi \mathbf{i}_\theta \\ &\quad + \frac{\left[k_p r z_n(k_p r)\right]'}{kr} \frac{P_n^1(\cos\theta)}{\sin\theta} \sin\varphi \mathbf{i}_\varphi \end{aligned} \tag{3b}$$

The scattered wave exterior to the lens corresponds with E_p^s in the case of p=0. The unknown coefficients a_n^p, b_n^p, α_n^p, β_n^p are determined by using PMM. The approach is summerized below.

The boundary conditions on the metallic cap which exists in a portion of the lens surface are as follows:

$$E^i(k_0 r_1) + E_0^s(k_0 r_1) = E_1^t(k_1 r_1) + E_1^s(k_1 r_1) = 0 \tag{4}$$

where $E_0^s(k_0 r_1)$ is the scattered wave exterior to the lens. The boundary conditions on the dielectric surface are as follows:

$$\begin{aligned} E^i(k_0 r_1) + E_0^s(k_0 r_1) &= E_1^t(k_1 r_1) + E_1^s(k_1 r_1) \\ H^i(k_0 r_1) + H_0^s(k_0 r_1) &= H_1^t(k_1 r_1) + H_1^s(k_1 r_1) \end{aligned} \tag{5}$$

On the other hand, the boundary conditions on the inner shells of the sphere are as follows:

$$\begin{aligned} E_{p-1}^t(k_{p-1} r_p) + E_{p-1}^s(k_{p-1} r_p) &= E_p^t(k_p r_p) + E_p^s(k_p r_p) \\ H_{p-1}^t(k_{p-1} r_p) + H_{p-1}^s(k_{p-1} r_p) &= H_p^t(k_p r_p) + H_p^s(k_p r_p) \end{aligned} \tag{6}$$

Substituting Eq(2a), (2b) and the expressions for the H field which are omitted here into Eq(6), we can find the following relation

$$\begin{bmatrix} a_n^1 \\ \alpha_n^1 \end{bmatrix} = U_n^2 U_n^3 \cdots U_n^6 \begin{bmatrix} a_n^6 \\ \alpha_n^6 \end{bmatrix}, \quad \begin{bmatrix} b_n^1 \\ \beta_n^1 \end{bmatrix} = V_n^2 V_n^3 \cdots V_n^6 \begin{bmatrix} b_n^6 \\ \beta_n^6 \end{bmatrix} \tag{7}$$

where U_n and V_n are complex 2×2 matrixes whose entries are expressions involving *n*th order sperical Bessel functions and their derivatives [1]. The α_n^6 and β_n^6 equal zero by the conditions of finiteness for the fields at the origin. Hence, we can obtain a_n^1, α_n^1, b_n^1 and β_n^1 from (7). In other word, the undefined coefficient is only α_n^0 that is the coefficient of the scattered wave exterior to the lens.

To determine the coefficient α_n^0 , PMM is now adopted for numerical calculations. We apply the boundary conditions given by Eq(4) and (5) , which depend on θ angle only. The sampling points on the surface of the lens are

$$\theta_k = \frac{\pi}{2N}(2k-1) \qquad k = 1,2,\cdots,N \tag{8}$$

The scattered and transmitted coefficients a_n^p , b_n^p , α_n^p , β_n^p in the each shell are easily obtained by using the relations Eq(7). Thus the all coefficients are determined and the entire fields can be found.

3. Results

A model experiment was performed at f=10 GHz. A block diagram is shown in Fig.2. Tokipec company lens (ka=15.7) is used as the model of the Luneberg lens, which has six layered structure whose dielectric constants (ε_1, ε_2,$\cdots$, ε_6) are given in Table 1. It is covered with a reflecting surface 'metallic cap' in a half of the sphere. The wave from a large horn antenna is normally incident to the metallic cap. The monopole antenna (1/4 λ length) are used for the receiver.

The amplitude of the near field distribution along the z axis is shown in Fig. 3. It is normalyzed by the the amplitude of the incident wave at the center of the sphere in the absense of the sphere. It shows the comparison between theoretical and experimental value, which is good agreement with ones.

The reflectivity of the spherical lens reflector will be described in details in the future.

Acknowledgment

We thank Mrs. Jun Hoshino and Fumio Yamano (under graduated students) who engaged in the experiment.

References

[1] Sanford, J. R., "Analysis of sperical radar cross-section enhancers," IEEE Trans. Microwave Theory Tech., MTT-43, pp.1400-1403, 1995.

[2] Mullin, C. R., Sanbury R. and Velline C. D., "A numerical technique for the determination of scattering cross sections of infinite cylinders of arbitrary geometrical cross section," IEEE Trans. Antennas Propagat., AP-13, pp.141-149, 1965.

[3] Stratton, J. A., Electromagnetic Theory, McGraw-Hill, New York, pp.416-419, 1941.

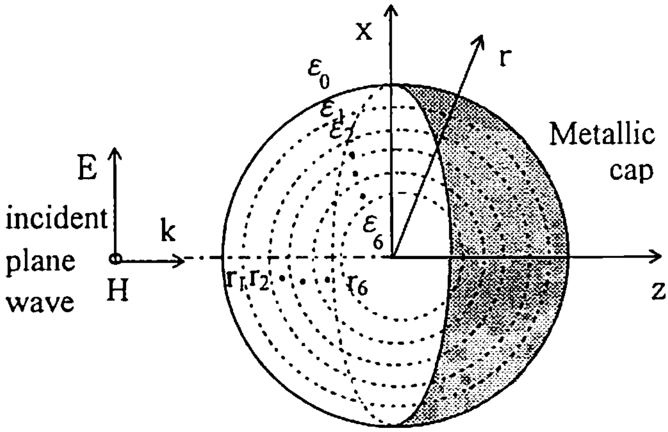

Fig. 1 Geometry of the problem.

Table 1 Dielectric constant of the Luneberg lens.

Radius (m)	ε_r
.04	1.903
.048	1.801
.056	1.701
.064	1.505
.072	1.353
.075	1.148

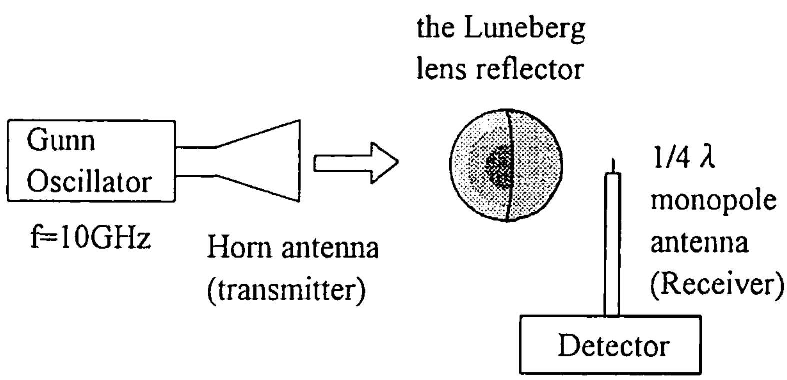

Fig. 2 . Block diagram for the experiment.

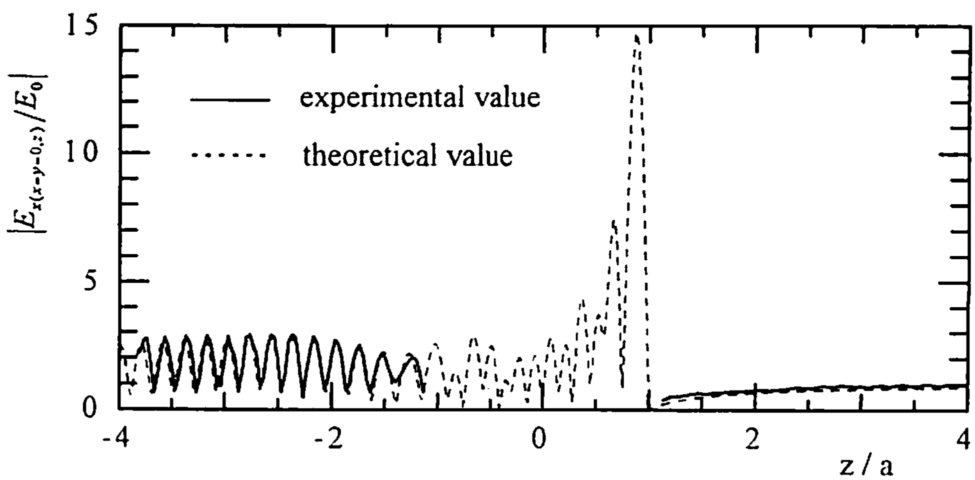

Fig. 3 The amplitude of the near field distribution along the z axis. (The comparison between theoretical and experimental value)