DIFFRACTION OF A PLANE WAVE BY SLIT IN IMPEDANCE SCREEN

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1. INTRODUCTION

Rigorous solution is presented of the problem of diffraction of a plane electromagnetic wave by infinitely long slit in a plane impedance screen. Most of results of similar problems are restricted to perfectly conducting screens[1,2]. However, some interest is the consideration of more general class of problems, when impedance boundary conditions are given on the screen.

2. FORMULATION OF THE PROBLEM

Consider a slit of width $2a$ in plane screen with impedance ZZ_0 ($Z_0=\sqrt{\mu_0/\varepsilon_0}$ - free-space impedance) and choose a rectangular co-ordinate system x, y, z so that coincides with the plane z=0 and two edges of the slit are given by $|x|=a$.

A plane wave incidence from the upper half-space (z>0) with its plane of incidence xz - plane, angle of incidence θ_0. Permittivities of the half-space z>0 and z<0 are (ε_0, μ_0) and $(\varepsilon_0\varepsilon, \mu_0)$ respectively. Since we discuss a two-dimensional and steady-state problem, $\partial/\partial y = 0$ and the time dependence $e^{-i\omega t}$ were understood. The horizontal polarisation (E_y, H_x, H_z) is considered.

3. SOLUTION

The electromagnetic field in the region z>0 can be described superposition of incidence, reflected, and scattered waves and in the region z<0 as the scattered wave:

$$(1)\quad E_y(x,z) = \exp\{-ik_0(x\sin\theta_0+z\cos\theta_0)\} + R\exp\{-ik_0(x\sin\theta_0-z\cos\theta_0)\}+$$

$$+\ 1/2\pi \int_{-\infty}^{+\infty} A(\alpha)\exp\{i\alpha x + iW_0 z\}d\alpha \quad (z>0)$$

$$(2)\quad E_y(x,z) = 1/2\pi \int_{-\infty}^{+\infty} B(\alpha)\exp\{i\alpha x - iWz\}d\alpha \quad (z<0)$$

Here, $W_0=\sqrt{k_0^2-\alpha^2}$, $W=\sqrt{k^2-\alpha^2}$, *and* $k_0, k=k_0\sqrt{\varepsilon}$ - are the wave numbers, $R=\dfrac{Z\cos\theta_0-1}{Z\cos\theta_0+1}$ - is the reflection coefficient from the impedance plane. The magnetic components can be obtained from Maxwell equations.

The boundary conditions on the plane z=0 may be written as :

(3a) $E_y(x,\pm 0) \mp ZZ_0 H_x(x,\pm 0) = 0 \qquad |x|>a$

(3b) $\begin{cases} E_y(x,+0) = E_y(x,-0) \\ H_x(x,+0) = H_x(x,-0) \end{cases} \qquad |x|<a$

For the future treatment it is convenient to enter into the solution two auxiliary functions $F_{1,2}(x/a)$ as

(4) $F_{1,2}(x/a) = E_y(x,\pm 0) \mp ZZ_0 H_x(x,\mp 0), \quad |x| < \infty$

and the Fourier transforms of $F_{1,2}(x)$; i.e.

(5) $f_{1,2}(\alpha) = \int_{-a}^{a} F_{1,2}(x/a) e^{-i\alpha x} dx$

Substituting the field equations (1,2) into (4) we can express $A(\alpha)$, $B(\alpha)$ in terms of $f_{1,2}(\alpha)$, obtaining

(6) $A(\alpha) = \dfrac{k_0 f_1(\alpha)}{k_0 + ZW_0}, \quad B(\alpha) = \dfrac{k_0 f_2(\alpha)}{k_0 + ZW}$

Taking into account (6), the paired integral equations in $f_{1,2}$ can be defined on the basis of boundary conditions (3a,b) as following

(7)
$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{K}(\xi)\vec{f}(\xi)e^{i\xi u}d\xi = -\vec{p}_0 e^{-i\xi_0 u}, \quad |u| < 1$$
$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} \vec{f}(\xi)e^{i\xi u}d\xi = 0, \quad |u| > 1$$

In this equations

$$\vec{f}(\xi) = \begin{bmatrix} f_1(\xi) \\ f_2(\xi) \end{bmatrix}, \qquad \hat{K}(\xi) = \begin{bmatrix} \dfrac{\kappa_0}{\kappa_0 + Zw_0} & -\dfrac{\kappa_0}{\kappa_0 + Zw} \\ \dfrac{w_0}{\kappa_0 + Zw_0} & \dfrac{w}{\kappa_0 + Zw} \end{bmatrix}, \quad \vec{p}_0 = \begin{bmatrix} \dfrac{2Z\cos\theta_0}{1 + Z\cos\theta_0} \\ \dfrac{2\cos\theta_0}{1 + Z\cos\theta_0} \end{bmatrix}$$

and

$$u = x/a, \quad \xi = \alpha a, \quad w_0 = \sqrt{\kappa_0^2 - \xi^2}, \quad w = \sqrt{\kappa^2 - \xi^2}, \quad \kappa_0 = k_0 a, \quad \kappa = ka,$$

$$\xi_0 = \kappa_0 \cos\theta_0, \quad f_{1,2}(\xi) = a f_{1,2}(\alpha)$$

The integral equations (7) can be resolved using the moment's method. For this purpose we expand $F_{1,2}(x/a)$ as a series in orthogonal set of functions. Taking into account conditions on the edges of structure Chebyshev polynomials were chosen. There after the expansion of the functions $f_{1,2}(\xi)$ can be obtained from (5) as the following series in Bessel functions:

(8)

$$\begin{bmatrix} f_1(\xi) \\ f_2(\xi) \end{bmatrix} = \pi \sum_{m=0}^{\infty} \left[\begin{bmatrix} C_{2m} \\ D_{2m} \end{bmatrix} \frac{J_{2m+s}(\xi)}{\xi^2} - i \begin{bmatrix} C_{2m+1} \\ D_{2m+1} \end{bmatrix} \frac{J_{2m+1+s}(\xi)}{\xi^2} \right] \quad \text{where} \quad s = \begin{cases} 0 & Z \neq 0 \\ 1 & Z = 0 \end{cases}.$$

Substituting there results into (7) and taking into account the expressions of

$e^{i\xi_0 u}$, $e^{-i\xi_0 u}$ in term of Chebyshev polynomials we have the following infinite sets of equations :

$$\sum_{m=0}^{\infty}[\,Q_{11}(2m+s,2n+1)\,C_{2m} + Q_{12}(2m+s,2n+1)\,D_{2m}\,] = -\frac{2Z\cos\theta_0}{1+Z\cos\theta_0}\frac{J_{2n+1}(\xi_0)}{\xi_0}$$

$$\sum_{m=0}^{\infty}[\,Q_{21}(2m+s,2n+1)\,C_{2m} + Q_{22}(2m+s,2n+1)\,D_{2m}\,] = -\frac{2\cos\theta_0}{1+Z\cos\theta_0}\frac{J_{2n+1}(\xi_0)}{\xi_0}$$

$$(9)\qquad \sum_{m=0}^{\infty}[Q_{11}(2m+1+s,2n+2)C_{2m+1} + Q_{12}(2m+1+s,2n+2)D_{2m+1}\,] = \frac{2Z\cos\theta_0}{1+Z\cos\theta_0}\frac{J_{2n+2}(\xi_0)}{\xi_0}$$

$$\sum_{m=0}^{\infty}[Q_{21}(2m+1+s,2n+2)C_{2m+1}+Q_{22}(2m+1+s,2n+2)\,D_{2m+1}\,]= \frac{2\cos\theta_0}{1+Z\cos\theta_0}\frac{J_{2n+2}(\xi_0)}{\xi_0}$$

$n=0,1,2...$

Here the matrix elements Q_{ij} are defined as :

$$(10)\qquad \tilde{Q}(\mu,\nu) = \int_0^{\infty}\frac{J_{\mu}(\xi)J_{\nu}(\xi)}{\xi^{s+1}}\tilde{K}(\xi)d\xi$$

The infinite sets of equations were solved by reduction method with computer. Some numerical results were obtained for the scattered field in the far zone of radiation.

References

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