DIFFRACTION OF A PLANE WAVE BY AN IMPEDANCE STRIP

Serge A. KOMAROV, Pavel M. ZATCEPIN, Eugene A. BARANCHUGOV
Altai State University, Radiophysics Department
66 Dimitrov str., Barnaul, 656099, RUSSIA

1. INTRODUCTION

In this paper, the quasi-three-dimensional problem of the diffraction of a plane wave by an impedance strip was solved. The problem was transformed to the systems of integral vector equations which were solved by method of moments (Galerkin's method). Algorithm was worked out and numerical results were received for some characteristics of a scattered field.

Studying of the diffraction's characteristics of fields associated with the strip structure under the plane wave illumination are usually considered to be absolutely conducting [1-3]. More than that the discussion of more general class of problems of incidence waves which are not perpendicular to strip and the impedance boundary conditions are satisfied on a strip, present definite interest.

2. FORMULATION OF THE PROBLEM

We consider the diffraction of a plane wave in free space by an impedance strip defined by $S:\{x \in (-a,a); y \in (-\infty,\infty); z = 0\}$, which has an impedance Z on each face of a strip. The initial plane wave makes incidence from the upper half-space under angle Θ_0 to z-axis and Φ_0 to x-axis and we let a plane wave to have the vertical polarization. If we take dependent of time in the form of $e^{-i\omega t}$, then we can write the initial field through a magnetic component, those absolute value is equal to one, in the form of:

(1) $$\vec{H}_t^{(i)} = \vec{H}_0 e^{ik_0\vec{n}\vec{r}}; \qquad \vec{E}_t^{(i)} = -Z_0\vec{n} \times \vec{H}_t^{(i)}$$

where k_0, Z_0 are the wavenumber and impedance in the free space. $\vec{H}_0=\{-\sin\Phi_0, \cos\Phi_0, 0\}$,

$\vec{n}=\{-\sin\Theta_0\cos\Phi_0, -\sin\Theta_0\sin\Phi_0, -\cos\Theta_0\}$, $\vec{r}=\{x, y, z\}$.

There is an incidence wave which is scattered by a two-dimensional plane structure in the quasi-three-dimensional problem and by this reason, any fields can be presented by the following expression:

(2) $$\vec{U}(x,y,z) = \vec{U}(x,z)e^{-ip_0y}, \text{ were } p_0 = k_0\sin\Theta_0\cos\Phi_0$$

2. SOLUTION

The total field is the superposition of initial field and the field scattered by a strip. If we introduce magnetic vector potential $\vec{A}^m$, then we can use known expressions, and the field can be presented in the form of:

(3) $$\vec{E} = -rot\vec{A}^m; \quad \vec{H} = i\varpi\varepsilon_0\vec{A}^m - \frac{1}{i\varpi\mu_0} graddiv\vec{A}^m$$

In the case of the vertical polarization the vector potential will be defined as:

(4) $$\vec{A}^m = \left\{A_x^m, A_y^m, 0\right\} = \vec{A}$$

If the vector potentials represented through Fourier integral on coordinate x, then we can rewrite expressions for the electric and the magnetic components of the total field in the form of:

(5)
$$\vec{E}_t(x,z) = \mp\frac{1}{2\pi}\int_{-\infty}^{\infty} iw(\vec{v}_0 \times \vec{a}_\pm(\alpha))e^{\pm iwz+i\alpha x}d\alpha$$
$$\vec{H}_t(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{u}\vec{a}_\pm(\alpha)e^{\pm iwz+i\alpha x}d\alpha$$

Here, $\vec{v}_0$ is the norm vector for axis z, $w = \sqrt{k_0{}^2 - \alpha^2 - p_0^2} = \sqrt{k_\perp^2 - \alpha^2}$, $\vec{a}_\pm$ is the spectral function of the vector potential for upper and lower half-space respectively, $\tilde{u}$ is a matrix operator:

(6)
$$\tilde{u} = \begin{pmatrix} ik_0 + \frac{1}{ik_0}\alpha^2 & \frac{1}{ik_0}\alpha p_0 \\ \frac{1}{ik_0}\alpha p_0 & ik_0 + \frac{1}{ik_0}p_0^2 \end{pmatrix}$$

For the complete resolve it is useful to introduce the densities of the electric and the magnetic superficial currents, which we determine in the following way:

(7) $\vec{j}^e(x/a) = \vec{v}_0 \times (\vec{H}_t(x,+0) - \vec{H}_t(x,-0))$, $\vec{j}^m(x/a) = Z_0^{-1}\vec{v}_0 \times (\vec{E}_t(x,+0) - \vec{E}_t(x,-0))$;

were $\left|\vec{j}^{e,m}(x/a)\right| \equiv 0$, for $|x| > a$.

The impedance boundary conditions for z=0

(8) $\vec{E}_t(x,\pm0) \mp ZZ_0\vec{v}_0 \times \vec{H}_t(x,\pm0) = 0$

If the expression (5) is taken into boundary conditions (8) then the system of the integral vector equations for spectral functions of the densities superficial currents is received:

(9a)
$$\frac{1}{2\pi}\int_{-\infty}^{\infty} iw(\vec{v}_0 \times \tilde{u}^{-1}(\vec{v}_0 \times \vec{J}^e(\alpha)))e^{i\alpha x}d\alpha + \frac{Z}{2\pi}\int_{-\infty}^{\infty}\vec{J}^e(\alpha)e^{i\alpha x}d\alpha = 2\vec{E}_t^{(i)}(x) = 2(\vec{n}\times\vec{H}_t^{(i)}(x)); |x| < a$$
$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\vec{J}^e(\alpha)e^{i\alpha x}dx = 0; \quad |x| > a$$

(9b)
$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\vec{J}^m(\alpha)e^{i\alpha x}d\alpha - \frac{Z}{2\pi}\int_{-\infty}^{\infty}\frac{1}{iw}\tilde{u}\vec{J}^m(\alpha)e^{i\alpha p}d\alpha = 2Z\vec{H}_t^{(i)}(x); \quad |x| < a$$
$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\vec{J}^m(\alpha)e^{i\alpha x}dx = 0; \quad |x| > a$$

Here, $\vec{J}^{e,m}(\alpha) = \int_{-a}^{a}\vec{j}^{e,m}(x/a)e^{-i\alpha x}dx$

3. METHOD OF MOMENTS

This system of the integral vector equations can be resolved by the method of moments (Galerkin's method). For this purpose the components of unknown functions of the densities superficial currents are represented in the form of the decomposition on the complete system of orthogonal functions. It is taken for granted that the behavior of the field on the edge of the strip in the form of the power law with the degree of the singularity for the transvertial to edges component of the field equal to -1/2 [4]. Pursuant to it a number of functions is limited by satisfaction to this conditions in each components of the decomposition. For this power low of the behavior of the fields and currents in the edge's vicinity of a strip it is useful to take the Chebyshev's polynomials. In this case spectral functions of superficial currents can be received in the form of:

$$
(11)\quad
\begin{aligned}
J^e_{xc}(\xi) &= \sum_{m=0}^{\infty} C_{2m}\frac{J_{2m+1}(\xi)}{\xi}, & j^e_{yc}(\xi) &= \sum_{m=0}^{\infty} D_{2m}J_{2m}(\xi)\\
J^e_{xs}(\xi) &= \sum_{m=0}^{\infty} C_{2m+1}\frac{J_{2m+2}(\xi)}{\xi}, & j^e_{ys}(\xi) &= \sum_{m=0}^{\infty} D_{2m+1}J_{2m+1}(\xi)\\
J^m_{xc}(\xi) &= \sum_{m=0}^{\infty} F_{2m}\frac{J_{2m+1}(\xi)}{\xi}, & j^m_{yc}(\xi) &= \sum_{m=0}^{\infty} G_{2m}J_{2m}(\xi)\\
J^m_{xs}(\xi) &= \sum_{m=0}^{\infty} F_{2m+1}\frac{J_{2m+2}(\xi)}{\xi}, & j^e_{ys}(\xi) &= \sum_{m=0}^{\infty} G_{2m+1}J_{2m+1}(\xi)
\end{aligned}
$$

Here, $J^{e,m}_{x,ys}$ are odd and $J^{e,m}_{x,yc}$ are even parts of spectral functions of superficial currents.

From this place we'll use the undimensional variable $\xi=a\alpha$.

The substitution expressions (11) in the system of integral equations (10) and the usage of Chebyshev's polynomials like "trial" function comes to infinite system of linear algebraic equations for coefficients C_{2m}, D_{2m}, E_{2m}, F_{2m} and C_{2m+1}, D_{2m+1}, E_{2m+1}, F_{2m+1}, which relate to both even and odd parts of the problem solution in relation to coordinate x:

$$
(12a)\;\begin{cases}
\sum_m Q_{11}(2m+1,2n+1)C_{2m} + \sum_m Q_{12}(2m+1,2n+1)D_{2m+1} = \cos\Theta_0\cos\Phi_0\dfrac{J_{2n+1}(k_0\cos\Theta_0\cos\Phi_0)}{k_0\cos\Theta_0\cos\Phi_0}\\
\sum_m Q_{21}(2m+1,2n+2)C_{2m} + \sum_m Q_{22}(2m+1,2n+2)D_{2m+1} = -\cos\Theta_0\sin\Phi_0\dfrac{J_{2n+2}(k_0\cos\Theta_0\cos\Phi_0)}{k_0\cos\Theta_0\cos\Phi_0}
\end{cases}
$$

$$
(12b)\;\begin{cases}
\sum_m Q_{11}(2m+2,2n+1)C_{2m+1} + \sum_m Q_{12}(2m,2n+1)D_{2m} = -\cos\Theta_0\cos\Phi_0\dfrac{J_{2n+2}(k_0\cos\Theta_0\cos\Phi_0)}{k_0\cos\Theta_0\cos\Phi_0}\\
\sum_m Q_{21}(2m+2,2n+1)C_{2m+1} + \sum_m Q_{22}(2m,2n+1)D_{2m} = \cos\Theta_0\sin\Phi_0\dfrac{J_{2n+1}(k_0\cos\Theta_0\cos\Phi_0)}{k_0\cos\Theta_0\cos\Phi_0}
\end{cases}
$$

Here:

$$
(13)\quad \tilde{Q}(\mu,\nu)=\frac{1}{2\pi}\int_0^{\infty}\tilde{q}J_{\mu}(\xi)J_{\nu}(\xi)d\xi; \qquad
\tilde{q}=\begin{pmatrix}
\dfrac{1}{\xi^2}\left(Z-\dfrac{(ak_0)^2-\xi^2}{ik_0w}\right) & \dfrac{ap_0}{ik_0w}\\
\dfrac{ap_0}{ik_0w\xi} & \dfrac{1}{\xi}\left(Z-\dfrac{(ak_{\perp})^2}{ik_0w}\right)
\end{pmatrix}
$$

Where, $J_{\mu}(\xi)$, $J_{\nu}(\xi)$ are Bessel's functions of the orders m and n.

$$
(14a)\begin{cases}\sum_m R_{11}(2m+1,2n+1)F_{2m}+\sum_m R_{12}(2m+1,2n+1)G_{2m+1} = -Z\sin\Phi_0\dfrac{J_{2n+1}(k_0\cos\Theta_0\cos\Phi_0)}{k_0\cos\Theta_0\cos\Phi_0}\\ \sum_m R_{21}(2m+1,2n+2)F_{2m}+\sum_m R_{22}(2m+1,2n+2)G_{2m+1} = -Z\cos\Phi_0\dfrac{J_{2n+2}(k_0\cos\Theta_0\cos\Phi_0)}{k_0\cos\Theta_0\cos\Phi_0}\end{cases}
$$

$$
(14b)\begin{cases}\sum_m R_{11}(2m+2,2n+2)F_{2m-1}+\sum_m R_{12}(2m,2n+1)G_{2m} = Z\sin\Phi_0\dfrac{J_{2n+2}(k_0\cos\Theta_0\cos\Phi_0)}{k_0\cos\Theta_0\cos\Phi_0}\\ \sum_m R_{21}(2m+2,2n+1)G_{2m-1}+\sum_m R_{22}(2m,2n+1)G_{2m} = Z\cos\Phi_0\dfrac{J_{2n+1}(k_0\cos\Theta_0\cos\Phi_0)}{k_0\cos\Theta_0\cos\Phi_0}\end{cases}
$$

Here:

$$
(15)\qquad \tilde{R}(\mu,\nu)=\frac{1}{2\pi}\int_0^\infty \tilde{r}J_\mu(\xi)J_\nu(\xi)d\xi;\qquad \tilde{r}=\begin{pmatrix}\dfrac{1}{\xi^2}\left(1+Z\dfrac{(ak_0)^2-\xi^2}{ik_0w}\right) & -Z\dfrac{ap_0}{ik_0w}\\ -Z\dfrac{ap_0}{ik_0w\xi} & \dfrac{1}{\xi}\left(1+Z\dfrac{(ak_\perp)^2}{ik_0w}\right)\end{pmatrix}
$$

Infinite system of linear algebraic equations (12a,b), (14a,b) can be solved by reduction method numerically. The analysis of the solution was made for the section of energy of a wave scattered to the upper half-space in the double-dimensional case, $\Phi_0=0$.

REFERENCES

1. Nomura Y., Katsura S. Diffraction of electromagnetic waves by ribbon and slit. J. Phys. Soc. Japan, 12, №2, 1957 pp.190-201.
2. Otsuki T., Reexamination of diffraction problem of a slit by method of Fourier-Orthogonal functions transformation. J. Phys. Soc. Japan, 41, №6, 1957 pp.2046-2051.
3. Veliev E.I. Diffraction of waves on a plane ribbon of a finite size. Doklady AN USSR. 1988. V.301. №3. p.598.
4. Mittra R., Lee S. Analiticheskie metody teorii volnovodov. M.: Mir, 1974.