An Exact solution for wave equation in lossless TEM transmission lines with time dependent media and nonlinear time dependent load(using time domain analysis)

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Abstract

In this paper an exact soulotion for wave equation in lossles TEM transmission lines with time dependent Dielectric (e(t)) will be presented. There is no restriction about the nature of the source.

We suppose the capacitance and inductance per unit lenght of the line are aveilable as respectively, time - dependent function C(t), and constant L.

also the load may be nonlinear time - dependent load (Z(t)).

1 - Introduction

In this paper the transmission line charactristic impedance will be considered as time - dependent function Z(t), but this time dependency is raised from e(t) or C(t) (time - dependent dielectric) and m(or L) is constant.

To avoid from complicated computations (Transformation from frequency to time domin) we will use time domin approach directly.

Such more complicated problem was considered numerically for some simple cases such as pulse input and time - dependent load[3] but the present soloution is a complete exact soulotion and no approximation was used.

2 - Diffrential Equations

For a lossless transmission line as in fig.1 we have.

$$\frac{\partial v}{\partial x} = \frac{-\partial (LI)}{\partial t} \qquad (1)$$

$$\frac{\partial I}{\partial x} = \frac{-\partial q}{\partial t} \qquad (2)$$

$$q(x,t) = C(t)\ V(x,t) \qquad (3)$$

Applying (3) to (1) and (2) yields

$$\frac{\partial^2 I(x,t)}{\partial x^2} = LC(t)\,\frac{\partial^2 I(x,t)}{\partial t^2} \qquad (4)$$

$$\frac{\partial^2 q(x,t)}{\partial x^2} = LC(t)\,\frac{\partial^2 q(x,t)}{\partial t^2} \qquad (5)$$

Eqs (4) and (5) have soloutions as below

$$q(x,t) = q^+(t-\frac{x}{v}) + q^-(t+\frac{x}{v}) \qquad (6)$$

$$I(x,t) = I^+(t-\frac{x}{v}) + I^-(t+\frac{x}{v}) \qquad (7)$$

In which $V(t) = \frac{1}{\sqrt{LC(t)}}$ is the wave velocity and functions with and (-)superscripts indicates forward (in +x direction)and backward (in - x direction) traveling waves. forms as (6) and (7) was solved by the author using time- domain approach [1], the only diffrences in this case are time dependent nature of the wave velocity and charactristic impedance of the line ($Z(t)=\sqrt{L/C(t)}$, Generelization the result in [1] one can prove that the below function for voltage and current in the line satisfies (4) and (5) and also the boundary conditions in input and load terminals.

$$V(x,t) = \frac{1}{C(t)}\sum_{m=0}^{\infty} M(t-\frac{x}{v(t)}-\frac{2md}{v(t)})\,[V_gC(t-\frac{x}{v(t)}-\frac{2md}{v(t)})]$$

$$\prod_{k=1}^{m} k_g k_l(t-\frac{x}{v(t)}-\frac{2(k-1)d}{v(t)} + k_L(t+\frac{x}{v(t)}-\frac{d}{v(t)})\sum_{m=1}^{\infty} M(t+\frac{x}{v(t)}-\frac{2md}{v(t)})$$

$$[V_gC(t+\frac{x}{v(t)}-\frac{2md}{v(t)})]\prod_{k=2}^{m} k_g k_L\,(t+\frac{x}{v(t)}-\frac{2(k-1)d}{v(t)}) \qquad (8)$$

Because

$$I^+(t+\frac{x}{v(t)}) = \frac{1}{Z(t)C(t)}\,q^+(t-\frac{x}{v(t)}) \text{ and } I^-(t-\frac{x}{v(t)}) = \frac{-1}{Z_0(t)C(t)}\,q^-(t-\frac{x}{v(t)})$$

We can derive the below formula for current in the line.

$$I(x,t) = \frac{1}{\sqrt{LC(t)}}\{\sum_{m=0}^{\infty} M(t-\frac{x}{v(t)}-\frac{2md}{v(t)})\,[V_gC(t-\frac{x}{v(t)}-\frac{2md}{v(t)})]$$

$$\prod_{k=1}^{m} k_L k_g\left(t - \frac{x}{v(t)} - \frac{2(k-1)d}{v(t)}\right) - k_L\left(t + \frac{x}{v(t)} - \frac{d}{v(t)}\right) \sum_{m=1}^{\infty} M\left(t + \frac{x}{v(t)} - \frac{2md}{v(t)}\right)$$

$$[V_g C(t) + \frac{x}{v(t)} - \frac{2md}{v(t)}) \prod_{k=2}^{m} k_L k_g \left(t + \frac{x}{v(t)} - \frac{2(k-1)d}{v(t)}\right) \quad (9)$$

In Eqs(8) and (9) d , V(t) and $V_g(t)$ are respectively the line lenght, velocity of wave propagation and source voltage. Also $k_g(t)$, $k_L(t)$ are the reflection coeffietionts in input and load terminals.

$$k_g(t) = \frac{R_g - Z_0(t)}{R_g + Z_0(t)} \quad (10)$$

$$k_L(t) = \frac{Z_L(t) - Z_0(t)}{Z_L(t) + Z_0(t)} \quad (11)$$

$$M(t) = \frac{1}{2}(1 - k_g(t)) \quad (12)$$

and for simplicity we use the notation $V_g C(x,t)$ and $k_g k_L(x,t)$ for

$V_g(x,t)C(x,t)$ and $k_g(x,t)\ k_L(x,t)$ respectively.

3 - Examples

In below two examples we have d = 400m and $R_g = 50\Omega$

a) C(t) is as in Fig(2) and input is the sine pulse as in Fig(3). Voltag in the load and input terminals are plotted in (4) and (5).

b) C(t) is an Fig(2) and the load is $R_L(t) = A \cos\omega_0 t + B$ which A, B and ω_0 are arbitrary constant. The voltage in the load and input terminal are plotted in (6) and (7) for unit step input.

4 - Results and future aspects

The given formulas (8) and (9) are indeed the exact soulotion for the wave equation in time - dependent media and were used for TEM transmission line case. Formulas (8) and (9) are still valid even when the media is nonlinear time - dependent media. This method can be improved for other transmission lines and other wave guides which are the future work of the aouther.

5 - Refrences

[1] A.cheldavi, H.oraizi and M.Kamarei "Time Domain Analysis of Transmission lines" Proc. 25th EuMC, september 4.8. 1995, Bologna. Italy.

[2] C.H.Durney, T.G stockham, Jr.K.Moten "Electromagnetic pulse propagation" Elect.Eng.Dept. univ. utah, tech. Rep, Feb.1987.

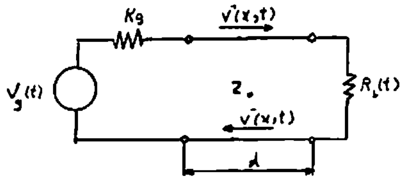

Fig.1

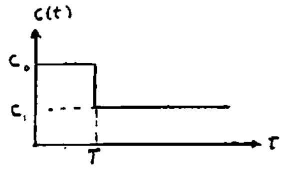

Fig.2

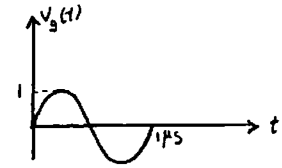

Fig. 3

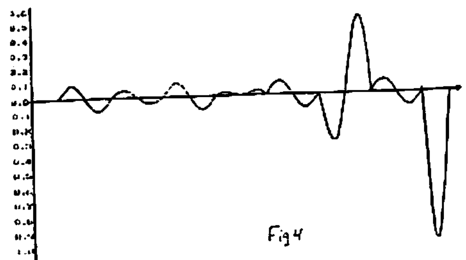

Fig4

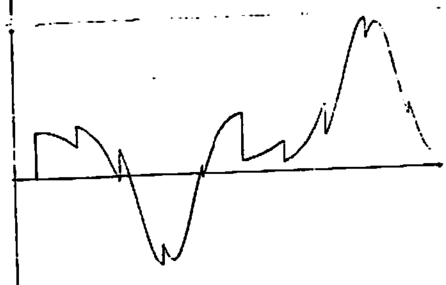

Fig 5

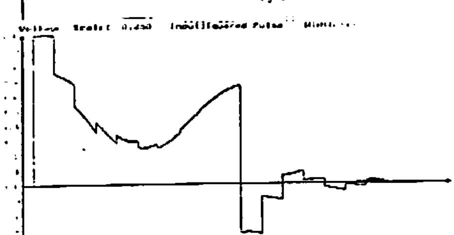

Fig 6

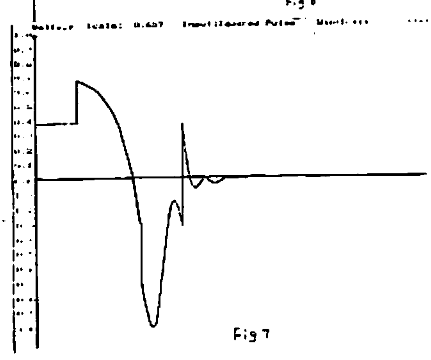

Fig 7