LOW FREQUENCY RESONANCES IN MAGNETODIELECTRIC CYLINDER COVERED BY HELICALY CONDUCTING SURFACE.

Shatrov Aleksander Dmitrievich and Pribytko Maksim Petrovich
Institute of Radio Engineering and Elecronics
1 Vvedensky Squ. , Fryazino , Moscow reg. 141120 , RUSSIA
E-*mail*: vss137@ire216.msk.su , *fax*: (7095)2038414

In scope of designing of artificial mediums with anisotropic properties with respect to electromagnetic wave polarization the advance has been achieved. The base element for construction of more complicated electromagnetic structures with anisotropic properties not only in wave polarization scope but in frequency one has been investigated. It is assumed that such element will find applications in filters and antennas design.

The problem of diffraction of electromagnetic wave normaly incident on dielectric cylinder covered by helical conducting surface has been investigated. The cylinder is made of uniform magnetodielectric characterized by magnetic permeability μ and complex dielectric permittivity $\varepsilon = \varepsilon' - j\varepsilon''$. The anisotropic surface conductivity provides the existence of surface current along helical curves with right-hand rotation and twirl angle ψ (Fig. 1).

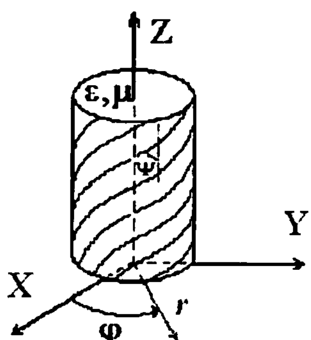

Fig. 1

On the cylider surface $(r = a)$ the boundary conditions are used :

$$(1) \qquad E_z^+ = E_z^-, E_\varphi^+ = E_\varphi^-, E_z \cos\psi + E_\varphi \sin\psi = 0,$$

$$(H_z^+ - H_z^-)\cos\psi + (H_\varphi^+ - H_\varphi^-)\sin\psi = 0,$$

where the signs "+" and "-" correspond to external $(r = a + 0)$ and internal $(r = a - 0)$ surface sides respectively.

It was shown [1] that in case of $\varepsilon = 1, \mu = 1, \psi << 1$ in low frequency region $ka << 1$ resonances occured. The aim of current investigation is to explore the influence of dielectric heat losses on resonator parameters.

The incident field is supposed to be independent of the coordinate z. In this case the problem of diffraction is two-dimentional and two-potential. It is convenient to take $E_z(r,\varphi)$ and $H_z(r,\varphi)$ as potential functions.

In the area $r \geq a$ the incident electromagnatic wave can be expanded into series of Bessel functions

$$(2) \quad E_z^0 = \sum_{n=-\infty}^{\infty} A_1^{(n)} J_n(kr) e^{jn\varphi} \quad , \quad H_z^0 = \sum_{n=-\infty}^{\infty} A_2^{(n)} J_n(kr) e^{jn\varphi} \quad ,$$

where coefficients $A_{1,2}^{(n)}$ are known.

The total internal ($r \leq a$) field can be represented as a series of Bessel functions with unknown coefficients $C_{1,2}^{(n)}$:

(3) $E_z = \sum_{n=-\infty}^{\infty} C_1^{(n)} J_n(k\sqrt{\varepsilon\mu}r)e^{jn\varphi}, H_z = \sum_{n=-\infty}^{\infty} C_2^{(n)} J_n(k\sqrt{\varepsilon\mu}r)e^{jn\varphi}$

The scattered field outside the cylinder $(r \geq a)$ can be represnted as a superposition of two fields:

(4) $E_z^s = E_{z0}^s + E_{z1}^s \quad , \quad H_z^s = H_{z0}^s + H_{z1}^s \quad ,$

where E_{z0}^s, H_{z0}^s is the field scattered by isotropic cylinder

(5) $$E_{zo}^s = -\sum_{n=-\infty}^{\infty} A_1^{(n)} \frac{J_n(ka)}{H_n^{(2)}(ka)} H_n^{(2)}(kr)e^{jn\varphi},$$

$$H_{zo}^s = -\sum_{n=-\infty}^{\infty} A_2^{(n)} \frac{J_n'(ka)}{H_n^{(2)'}(ka)} H_n^{(2)}(kr)e^{jn\varphi}.$$

The second summands in (4) can be represented as series with unknown coefficients $B_{1,2}^{(n)}$:

(6) $$E_{z1}^s = \sum_{n=-\infty}^{\infty} B_1^{(n)} H_n^{(2)}(kr)e^{jn\varphi}, \quad H_{z1}^s = \sum_{n=-\infty}^{\infty} B_2^{(n)} H_n^{(2)}(kr)e^{jn\varphi}.$$

The quantities $B_{1,2}^{(n)}, C_{1,2}^{(n)}$ can be obtaned from the boundary conditions (1). For the sake of implicity it is convenient to introduce the following vectors and values:

$$\vec{A}^{(n)} = \left\{A_1^{(n)}, A_2^{(n)}\right\} \quad , \quad \vec{B}^{(n)} = \left\{B_1^{(n)}, B_2^{(n)}\right\} \quad , \quad \vec{C}^{(n)} = \left\{C_1^{(n)}, C_2^{(n)}\right\}.$$

$$\vec{L}^{(n)} = \left\{\frac{\sin\psi}{H_n^{(2)}(ka)}, \frac{j\cos\psi}{H_n^{(2)'}(ka)}\right\}, \vec{N}^{(n)} = \left\{\frac{\sin\psi}{H_n^{(2)}(ka)}, \frac{-j\cos\psi}{H_n^{(2)'}(ka)}\right\},$$

$$\vec{M}^{(n)} = \left\{\frac{\sin\psi}{J_n(ka\sqrt{\varepsilon\mu})}, \frac{j\sqrt{\varepsilon}\cos\psi}{\sqrt{\mu}J_n'(ka\sqrt{\varepsilon\mu})}\right\},$$

$$D_n = \frac{kaH_n^{(2)}(ka)}{H_n^{(2)'}(ka)}\cos^2\psi - \frac{kaH_n^{(2)'}(ka)}{H_n^{(2)}(ka)}\sin^2\psi -$$

$$-\frac{1}{\mu}\frac{ka\sqrt{\varepsilon\mu}J_n(ka\sqrt{\varepsilon\mu})}{J_n'(ka\sqrt{\varepsilon\mu})}\cos^2\psi + \frac{1}{\mu}\frac{ka\sqrt{\varepsilon\mu}J_n'(ka\sqrt{\varepsilon\mu})}{J_n(ka\sqrt{\varepsilon\mu})}\sin^2\psi.$$

If introduced parameters are used, the vectors $\vec{B}^{(n)}, \vec{C}^{(n)}$ can be represented in a brief form:

(7) $$\vec{B}^{(n)} = \frac{2j(\vec{A}^{(n)}, \vec{N}^{(n)})}{\pi D_n}\vec{L}^{(n)} \; , \; \vec{C}^{(n)} = \frac{2j(\vec{A}^{(n)}, \vec{N}^{(n)})}{\pi D_n}\vec{M}^{(n)}.$$

where parentheses are determined as $(\vec{A}^{(n)}, \vec{N}^{(n)}) = A_1^{(n)} N_1^{(n)} + A_2^{(n)} N_2^{(n)}$.

Suppose that

(8) $ka\sqrt{\varepsilon'\mu} << 1 \quad , \quad \varepsilon''/\varepsilon' << 1.$

The asymptotic expansion of Bessel functions of small argument (8) can be used. The values D_n are real in higher order:

$$\operatorname{Re} D_0 = \frac{2}{\mu}\cos^2\psi - \frac{\ln\left(\gamma \frac{ka}{2}\right)}{\left|\ln\left(j\gamma \frac{ka}{2}\right)\right|^2}\sin^2\psi$$

$$\operatorname{Re} D_n = \left(1 + \frac{1}{\mu}\right)|n|\sin^2\psi - \frac{(ka)^2}{|n|}(1+\varepsilon')\cos^2\psi \ , \ n \neq 0$$

where $\gamma = 1.78...$

The value $\operatorname{Re} D_0$ is positive in any case (8) but $\operatorname{Re} D_n, n \neq 0$ became zero when the following equation is satisfied:

(9) $ka = |n|\sqrt{\frac{1}{\mu}}\sqrt{\frac{1+\mu}{1+\varepsilon'}}\tan\psi$

The low frequency resonances occure only when $\psi << 1$.

The Q-factor of the base $(|n| = 1)$ resonance defined by the most frequency dependable parameter D_n and given by the equation:

$$Q = \left|\frac{k\frac{d}{dk}\operatorname{Re} D_{\pm 1}}{2\operatorname{Im} D_{\pm 1}}\right| = \frac{2(\varepsilon'+1)^2}{\pi\psi^2\left(2 + \frac{1}{\mu} + \varepsilon'\right) + 2\varepsilon''(1+\varepsilon')}$$

calculated at the base resonance frequency (9).

Consider the arbitraryly polarized incident wave $E_z^0 = \alpha_1 e^{-jkx}$, $H_z^0 = \alpha_2 e^{-jkx}$, $A_{1,2}^{(n)} = (-j)^n \alpha_{1,2}$ where the coefficients α_1, α_2 are complex and satisfy the power norming $(|\alpha_1|^2 + |\alpha_2|^2 = 1)$. The power losses P_- achieve the maximum value when the complex dielectric permittivity satisfies the equation :

$$\varepsilon'' = \frac{\pi(1 + 2\mu + \varepsilon'\mu)}{2\mu(1+\varepsilon')}\psi^2$$

The dependence of scattered and absorpted power on incident electromagnetic wave polarization is characterized by the factor $\left|\sqrt{\mu(\varepsilon'+1)}\alpha_1 + j\sqrt{\mu+1}\alpha_2\right|^2$.

This factor achieves its minimum (zero) and maximum values when: $\frac{\alpha_1}{\alpha_2} = -j\sqrt{\frac{1+\mu}{\mu(1+\varepsilon')}}$, $\frac{\alpha_1}{\alpha_2} = j\sqrt{\frac{\mu(1+\varepsilon')}{\mu+1}}$ respectively.

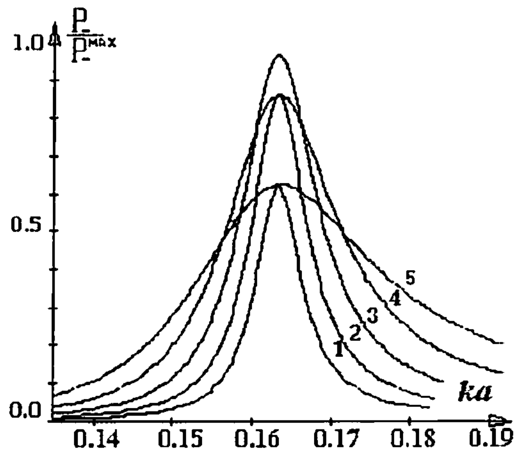

Fig. 2

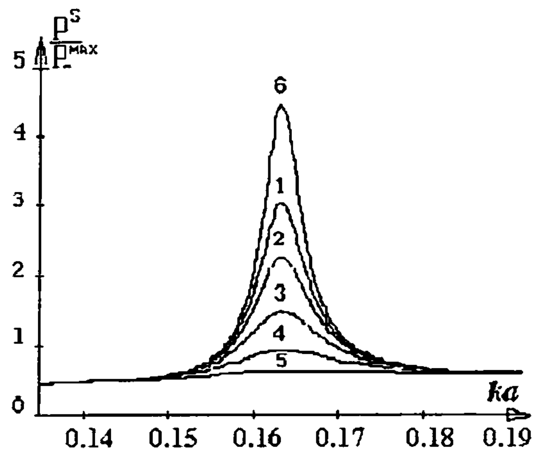

Fig. 3

Maximum cross-section equaled $8/k$ is achieved in resonance .

A set of curves showing the dependence of heat losses on frequency for different ε'' ($1{:}\,\varepsilon'' = 0.026;\ 2{:}\,\varepsilon'' = 0.052;\ 3{:}\,\varepsilon'' = 0.104;\ 4{:}\,\varepsilon'' = 0.208;\ 5{:}\,\varepsilon'' = 0.416;$) is presented in Fig.2. The appropriate dependences of scattered power on frequency are presented in Fig.3 ($6{:}\,\varepsilon'' = 0$). The twirl angle $\psi = 0.2$, $\varepsilon' = 2$ and $\mu = 1$ are the same for all curves. The maximum absorption is achieved when $\tan\delta = \varepsilon''/\varepsilon' \doteq 0.052$.

References

[1] **A. D. Chuprin, A. D. Shatrov, A. N. Sivov** " A NOVEL TYPE OF LOW FREQUENCY RESONANCE RELATED TO A HOLLOW CIRCULAR CYLINDER FORMED BY AN ANISOTROPIC SURFACE WITH HELICAL CONDUCTIVITY " /Proc. 24th European Microwave Conference, 1994 , Cannes, pp 596-601