THE SPECTRUM OF GRATING AS OPEN PERIODIC RESONATOR AND WAVEGUIDE IN THE VICINITY OF COMPLEX HYPERSURFACE MORSE CRITICAL POINT

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1. Introduction

Spectral characteristics of gratings in the vicinity of singular points of the complex hypersurfaces belonging both to "physical" and "non-physical" complex Riemannian manifold are analyzed in anomalous conditions in the report. Numerically-analytical description of a complex hypersurface of spectral characteristics of a grating in the vicinity of the Morse critical point (MCP) is carried out using the theory of singularities of smooth mapping [1]. The grating is considered both as open periodic resonator (spectral parameter, frequency) and as open periodic waveguide (spectral parameter, quasiperiodicity constant) [2]. The analytical description of spectral characteristics of gratings is carried out in the vicinity of the MCP of the complex hypersurface which is the function of three (two spectral and one non-spectral parameters) complex variables. The approach of this kind allows to analyze simultaneously the scattering and guiding properties of a grating in the vicinity of a single MCP plus to describe and analyze of the total transformation phenomenon of plane waves (wave packets) by periodic structures.

2. Spectral method of analysis

Spectrum points $\kappa=\omega(\varepsilon_0\mu_0)^{1/2}\in\Omega_\kappa$ (ω is the circular frequency, ε_0 and μ_0 are the electrodynamics medium parameters) if the grating viewed as open periodic resonator or spectrum points $\phi\in\Omega_\phi$ is quasiperiodicity constant and correspondent free field modes $U(y,z)$ of one-dimensional-periodic grating (see Fig. 1) are considered as non-trivial solutions of the boundary value problem

$$\left[\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+\kappa^2\varepsilon(y,z)\right]U(y,z)=0,\ 0\le y\le 2\pi,\ \{y,z\}\notin \operatorname{int} S; \qquad (1)$$

$$U(y,z)=\sum_{n=-\infty}^{\infty}\begin{Bmatrix}a_n\\ b_n\end{Bmatrix}e^{i[\phi_n y\pm\Gamma_n(z\mp 2\pi\delta)]},\ z\gtrless\pm 2\pi\delta \qquad (2)$$

$$U\begin{Bmatrix}\dfrac{\partial U}{\partial y}\end{Bmatrix}(2\pi,z)=e^{i2\pi\phi}U\begin{Bmatrix}\dfrac{\partial U}{\partial y}\end{Bmatrix}(0,z) \qquad (3)$$

satisfying the boundary conditions ordinary for an electromagnetic field at the surface S of ideally conducting metal generalizes of the grating and surfaces of discontinuity $\varepsilon(y,z)$ i.e. the relative permittivity of the layer in which it is placed. Here $U=E_x$ in the case of the E-polarization of the field and $U=H_x$ in the H-case; E_x, H_x are the components of vectors of the field intensity; $\phi_n=n+\phi$, $\Gamma_n=(\kappa^2-\phi_n^2)^{1/2}$; the time dependence is defined by the factor $\exp(-i\omega t)$. A complex-valued, segment-smooth function $\varepsilon(y,z)$ (in the H-case it is segment constant one) with $\mathrm{Re}\,\varepsilon>0$ and $\mathrm{Re}\,\kappa\,\mathrm{Im}\,\varepsilon\ge 0$ is periodic by y and equal identically to unit in zones of the grating radiation $|z|>2\pi\delta$.

A region of the change of the spectral parameter $\kappa\in\Omega_\kappa$ (or $\phi\in\Omega_\phi$) (an infinite-sheeted complex Riemann surface H (or Φ) with branch points of the second order $\kappa_n^\pm:\Gamma_n(\kappa_n^\pm)=0$ (or $\phi_n^\pm:\Gamma_n(\phi_n^\pm)=0$)) is fully determined by boundaries of the possible analytical continuation of the Green's function of the inhomogeneous problem (1)-(3) while the absence of material scatters in the region of complex values of κ (or ϕ) [2]. The first "physical" sheet of the surface H (pairs $\kappa, \{\Gamma_n(\kappa)\}_{n=-\infty}^{\infty}$) (or Φ (pairs $\phi, \{\Gamma_n(\phi)\}_{n=-\infty}^{\infty}$)) is determined by the condition of radiation (2) on the real axis $\operatorname{Im}\kappa = 0$ (or $\operatorname{Im}\phi = 0$): $\operatorname{Im}\Gamma_n \geq 0$, $\operatorname{Re}\kappa \operatorname{Re}\Gamma_n \geq 0$, $n = 0, \pm 1, \ldots$, correlated with a requirement for absence of waves coming from the infinity. Following "non-physical" sheets of the surface H (or Φ) are differ from the first one by the fact that for the finite number of quantities of indices n sings Γ_n are substituted by the opposite ones.

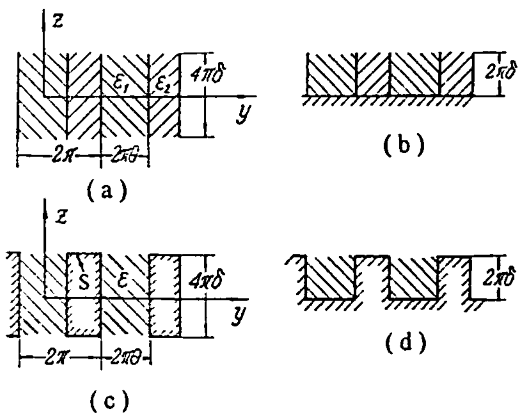

Fig. 1 Geometry of open periodic structures.

The problem (1)-(3) reduces the partial inversion method to the operator equation

$$(E + A(\kappa, \phi, \tau))\bar{a} = \bar{0}, \begin{cases} (\kappa \in \Omega_\kappa, \ \phi = const, \ \tau = const) \in \mathrm{H} \times R \times T \\ (\kappa = const, \ \phi \in \Omega_\phi, \ \tau = const) \in R \times \Phi \times T \end{cases} \tag{4}$$

Due to the properties of the operator function $A(\kappa, \phi, \tau)$ investigated in detail. Shown that the spectrum of gratings as open periodic resonator $\Omega_\kappa(\phi, \tau)$and waveguides $\Omega_\phi(\kappa, \tau)$ is discrete, finite-to-one, and is determined by the roots of the equations

$$f(\kappa, \phi, \tau) = \det(E + A(\kappa, \phi, \tau) = 0 \begin{cases} \kappa(\phi, \tau) \in \Omega_\kappa(\phi, \tau) \subset \mathrm{H} \\ \phi(\kappa, \tau) \in \Omega_\phi(\kappa, \tau) \subset \Phi \end{cases} \tag{5}$$

where $\tau \in T \subset R$ (or C) is a non-spectral parameter, R and C is the fields of real and complex numbers.

3. Morse critical points and the spectrum of gratings

It woos established that an analysis of solution of problems on natural oscillation and waves of one-dimensional-periodic structures is essentially simplified when using the results of the theory of features of smooth mappings [1].

The unity of analysis for scattering and guiding properties of a grating proceeds to execute the analytical prolongation of the dependence of dispersion to the range of complex values of parameters, i.e.

$$f(\kappa, \phi, \tau) = \det(E + A(\kappa, \phi, \tau)) : \mathrm{H} \times \Phi \times C \to C \tag{6}$$

and to consider the complex hypersurface $\Omega(\kappa, \phi, \tau) = \{(\kappa, \phi, \tau) \in \mathrm{H} \times \Phi \times C : f(\kappa, \phi, \tau) = o\}$.
Where $\mathrm{H} \times \Phi$ is the infinite-sheeted complex Riemannian manifold with branch points is subject to the conditions $\Gamma_n(\kappa_n^\pm, \phi_n^\pm) = 0$ (i.e. $\kappa_n^\pm = \pm\phi_n^\pm$). Here

$$\Omega(\kappa, \phi, \tau) = \begin{cases} \Omega_\kappa(\phi, \tau), \text{if } \operatorname{Im}\phi = 0 \text{ and } \operatorname{Im}\tau = 0 \ (\text{or } \tau \in T) \\ \Omega_\phi(\kappa, \tau), \text{if } \operatorname{Im}\kappa = 0 \text{ and } \operatorname{Im}\tau = 0 \ (\text{or } \tau \in T). \end{cases}$$

Let us conduct an analysis of spectral characteristics of the grating is considered both as open periodic resonator and as open periodic waveguide by means of Morse critical points (MCP). The MCP $(\kappa_0, \phi_0, \tau_0)$ is determined form the solution of the functional equations

$$\frac{\partial f}{\partial \kappa}=0, \quad \frac{\partial f}{\partial \phi}=0, \quad \frac{\partial f}{\partial \tau}=0; \quad (\kappa,\phi,\tau)=(\kappa_0,\phi_0,\tau_0) \tag{7}$$

$$\det J(\kappa,\phi,\tau)\neq 0; \quad (\kappa,\phi,\tau)=(\kappa_0,\phi_0,\tau_0) \tag{8}$$

Here $J(\kappa,\phi,\tau)$ is the matrix Hess for mapping (6), i.e.

$$J=\begin{pmatrix} j_{11} & j_{12} & j_{13} \\ j_{21} & j_{22} & j_{23} \\ j_{31} & j_{32} & j_{33} \end{pmatrix}=\begin{pmatrix} \partial^2 f/\partial\kappa^2 & \partial^2 f/\partial\kappa\partial\phi & \partial^2 f/\partial\kappa\partial\tau \\ \partial^2 f/\partial\phi\partial\kappa & \partial^2 f/\partial\phi^2 & \partial^2 f/\partial\phi\partial\tau \\ \partial^2 f/\partial\tau\partial\kappa & \partial^2 f/\partial\tau\partial\phi & \partial^2 f/\partial\tau^2 \end{pmatrix}$$

The complex hypersurface $\Omega(\kappa,\phi,\tau)$ to within are cubically small terms can be written as follows in the vicinity of MCP $(\kappa_0, \phi_0, \tau_0)$:

$$\begin{aligned} &j_{11}(\kappa-\kappa_0)^2+j_{22}(\phi-\phi_0)^2+j_{33}(\tau-\tau_0)^2+ \\ &2\{j_{12}(\kappa-\kappa_0)(\phi-\phi_0)+j_{31}(\kappa-\kappa_0)(\tau-\tau_0)+j_{23}(\phi-\phi_0)(\tau-\tau_0)\}+ \\ &2f(\kappa_0,\phi_0,\tau_0)=0. \end{aligned} \tag{9}$$

The approach of this kind allows to analyze simultaneously the scattering (spectral parameter, frequency) and guiding (spectral parameter, quasi-periodicity constant) properties of a grating is the vicinity of a single MCP [3].

4. Numerical analysis

Consider a specific example of using the method. Fig. 2 illustrate the results of the application of the (7)-(9) approach for investigation of the effects of intertype coupling of electromagnetic fields of gratings and of total transformation phenomenon of wave packets by periodic structures in the case of the E-polarization of the field. Spectral characteristics are placed on the "non-physical" Remann manifold sheet that differ from the "physical" one by the fact that for the finite number of quantities of indices $n=0$ and 1 sings Γ_0 and Γ_1 are substituted by the opposite ones. Parameter δ used as an alternative to a non-spectral parameter τ. The description of gratings spectral characteristics is carried out in vicinity of the MCP (κ_0 = 2,479249 $\cdot10^0$ + i 5,863054$\cdot10^{-5}$; ϕ_0 = 4,960388$\cdot10^{-1}$ $-$ i 4,114606 $\cdot10^{-2}$; δ_0 = 2,136720 $\cdot10^0$ $-$ i 6,267661 $\cdot10^{-5}$) of complex hypersurface $\Omega(\kappa,\phi,\delta)$, form (9) with coefficients: j_{11} = $-$5,43465 $\cdot10^7$ +i 7,73659 $\cdot10^5$; j_{22} = +1,02887 $\cdot10^1$ +i 2,04123 $\cdot10^2$; j_{33} = $-$2,97915 $\cdot10^6$ +i 9,66990 $\cdot10^4$; j_{12} = $-$4,10402$\cdot10^4$ +i 7,11342 $\cdot10^4$; j_{31} = $-$3,18777 $\cdot10^7$ + i 4,68671 $\cdot10^5$; j_{23} = $-$4,67307 $\cdot10^4$ +i 7,67111 $\cdot10^3$; $f(\kappa_0,\phi_0,\delta_0)$ = $-$7,50376 $\cdot10^{-2}$ $-$ i 1,18058 $\cdot10^{-1}$, shown in Fig. 2. A fragment of the spectral surface $\Omega_\kappa(\phi,\delta)$ (Fig. 2, a) and spectral lines $\kappa(\delta)$ draw the thick curves (Fig. 2, b) whose point determine the existence conditions of electromagnetic oscillations $H_{0,2,22}$ and $H_{0,11,12}$ types under the intratypal connection. They restored as a result of application of (7)-(9) approach. The spectral lines $\kappa(\delta)$draw the thin curves (Fig. 2, b) corresponding to solution (1)-(3) problem. In Fig. 2, b you can see that the solution (1)-(3) and (7)-(9) problems are consistent in the vicinity of complex hypersurface MCP.

Projection of the MCP denote by sign cross (+) in Fig. 2, b. The eigen real frequency κ: $\mathrm{Im}\,\kappa(\phi,\delta)=0$, ϕ and $\delta\in R$ of the "non-physical" sheet $\mathrm{H}\times\Phi$ denote by symbol circle ($\circ$). At this point, it is total transformation phenomenon of plane waves by periodic structures [4]. Parameters of the regime: $\kappa = 2{,}47919$; $\delta = 2{,}136704$. Here the equation (2) reduces to equation $U(y,z) = U_1(y,z) + U_2(y,z)$. Packets $U_1(y,z)$ and $U_2(y,z)$ non-crossing the set $n = 0, \pm 1, \ldots$ from the partial field components of a free field mode with a frequency $\kappa{:}\mathrm{Im}\,\kappa = 0$. The situation when $U_1(y,z)$ consists of two homogeneous waves (amplitude of the incident waves: $a_0 = 2$; $a_1 = -0{,}208 \cdot 10^1 - i\ 0{,}122 \cdot 10^1$), and $U_2(y,z)$ is an infinite spectrum of inhomogeneous waves and three homogeneous waves (amplitude of the outcoming waves: $a_{-2} = -0{,}566 \cdot 10^{-1} + i\ 0{,}227 \cdot 10^1$; $a_{-1} = +0{,}112 \cdot 10^1 - i\ 0{,}183 \cdot 10^1$; $a_2 = -0{,}852 \cdot 10^{-1} + i\ 0{,}415 \cdot 10^{-1}$). A field of a free oscillation $U(y,z)$ coincides with a full diffraction field.

It is shown that the modes of the full transformation of the plane wave packets can be realized as a result of the "interaction" of time-increasing $(\mathrm{Im}\,\kappa(\phi,\delta) > 0)$ and time-damping $(\mathrm{Im}\,\kappa(\phi,\delta) < 0)$ field oscillations under the conditions close to the longitudinal resonance for both waves, i.e. in the convergence of the corresponding natural frequencies of the "non-physical" Riemann manifold sheet.

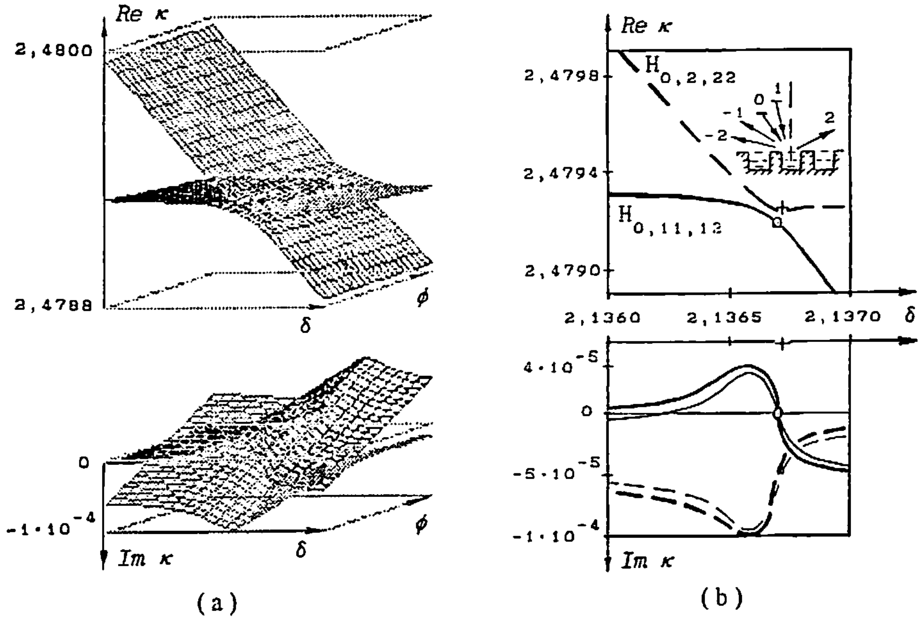

Fig. 2 Spectral characteristics of grating (see Fig. 1, d for $\varepsilon = 8$; $\theta = 0{,}8$) with the "non-physical" Riemann manifold sheet: (a) $\Omega_\kappa(\phi,\delta)$, $\delta \times \phi = [2{,}136; 2{,}137] \times [0{,}48; 0{,}52]$; (b) $\kappa(\delta)$ for $\phi = 0{,}48$.

5. Conclusion

It is shown that the Morse critical points are applied to effectively restore spectral characteristics connected with manifestation of the effects of intertype coupling and to study degenerate and anomalous states of free electromagnetic fields of gratings.

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