ON THE FULL SPACE DOMAIN GREEN'S FUNCTIONS FOR MICROSTRIP GEOMETRIES

S. Marchetti
Universiy of Ancona,
78/a, via G. Galilei 60019 Senigallia (AN), ITALY

Abstract:

A new space domain closed form for the vector and scalar Green's functions pertaining to a microstrip geometry is presented. The novel formulation is the sum of an analytic function singular at the source point, an eventual analytic surface wave contribution and a corrective double series of Legendre polynomials versus the space variables. The coefficients in the series are only dependent on the frequency and dielectric properties and are evaluated by numerical integration of oscillating but fastly decaying integrands.

1. Introduction

The analysis and synthesis of printed circuits components and antennas for microwaves and millimeterwaves applications is becoming a space domain problem. This is due to the fact more and more complex printed conductors architectures must be realised when we want to increase component performances or realise multifunctionality of patch antennas. In this context classic accurate analysis methods like the Integral Equation Techniques solved by the Method of Moments require fast and accurate disposability of the Green's functions of the problem in the space domain. However the latter are easy to formulate only in a transformed space [1] and only recently [2], [3] they have been fastly and accurately recovered at the interface air dielectric. That result is all what we need in solving the internal problem, i.e. in evaluating charge and current densities distributions on the printed conductor. In the present study we formulate in closed form the Green's functions in any point of the space so as required in evaluating the diffracted fields in any point of the space around the scatterer.

2. Theoretical and computational results

By referring to the microstrip geometry illustrated in Fig.1, the external problem for printed circuits can be reduced to the problem of an x-directed horizontal dipole (HED) of current density $J_s\hat{x}$ and associated charge q_s posed at the origin of a cylindrical coordinate system and observed at a generic point $\vec{r} \equiv (\rho, \varphi, z)$ [1]. If the fields are descended from a vector $\vec{A}$ and scalar V potentials, the Green's functions required in the problem are indicated in the followings:

$$\begin{aligned} \vec{A}(\vec{r},\vec{0}) &= \left[\; G_A^{xx}(\vec{r},\vec{0})\,\hat{x} + G_A^{zx}(\vec{r},\vec{0})\,\hat{z}\; \right] J_s \\ V(\vec{r},\vec{0}) &= G_V(\vec{r},\vec{0})\, q_s \end{aligned} \qquad (1)$$

where in the air region it is:

$$G_A^{xx}(\vec{r},\vec{0}) = \frac{\mu_0}{4\pi}\int_0^{\infty} J_0(k_\rho\rho)\,\frac{2k_\rho}{D_{TE}}\,e^{-u_0 z}\,dk_\rho \tag{2}$$

$$G_A^{zx}(\vec{r},\vec{0}) = \frac{\mu_0}{4\pi}(1-\varepsilon_r)\cos\varphi\left[\int_0^{\infty} J_1(k_\rho\rho)\,\frac{2k_\rho^2}{D_{TE}D_{TM}}\,e^{-u_0 z}\,dk_\rho - j\pi R_A^{zx}\right] \tag{3}$$

$$G_v(\vec{r},\vec{0}) = \frac{1}{4\pi\varepsilon_0}\left[\int_0^{\infty} J_0(k_\rho\rho)\,2k_\rho\,\frac{u_0 + u\,\mathrm{th}(uh)}{D_{TE}D_{TM}}\,e^{-u_0 z}\,dk_\rho - j\pi R_v\right] \tag{4}$$

J_n is the Bessel function of order n whereas we indicate with:

$$D_{TE} = u_0 + u\,\mathrm{cth}(uh) \qquad D_{TM} = \varepsilon_r u_0 + u\,\mathrm{th}(uh)$$

$$u_0 = \sqrt{k_\rho^2 - k_0^2} \qquad u = \sqrt{k_\rho^2 - k_1^2}$$

$$k_0 = \omega\sqrt{\mu_0\varepsilon_0} \qquad k_1 = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_r}$$

$\omega = 2\pi f$ is the radian frequency and ε_0, μ_0 are the permittivity and permeability of the vacuum. For usual values of relative dielectric constants ε_r and dielectric thickness h, D_{TM} presents a 0 in $k_\rho = k_{\rho p}$ so that the integrand in (3), (4) present a pole of residuals R_A^{zx}, R_V respectively [1]. The inverse Bessel transform in equations (2), (3), (4) can be operated by expressing the functions $J_{0,1}(k_\rho\rho)$ and $e^{-u_0 z}$ in uniformly convergent series of Legendre polynomials versus ρ and z respectively (see [3], [4]) and integrating term by term. Further, a function singular at the source point ought be put in evidence if we want everywhere convergent the remainder double series allowing so to write:

$$G_A^{xx}(\rho,\varphi,z,f) = \frac{\mu_0}{4\pi}\left[\frac{e^{-jk_0 r}}{r} + \sum_{n=0}^{n_M}\sum_{m=0}^{m_M} A_{nm}^{xx}(f)\,P_n(1-2\bar{\rho}^2)\,P_m(1-2\bar{z})\right]$$

$$G_A^{zx}(\rho,\varphi,z,f) = \frac{\mu_0\eta}{4\pi}\cos\varphi\left[\frac{z}{\rho}\left(\frac{e^{-jk_0 r}}{r} - \frac{e^{-jk_0 z}}{z}\right) - \rho\sum_{n=0}^{n_M}\sum_{m=0}^{m_M} A_{nm}^{zx}(f)\,P_n(1-2\bar{\rho}^2)\,P_m(1-2\bar{z}) - j2\pi\varepsilon_M R_A^{zx}\right]$$

$$G_v(\rho,\varphi,z,f) = \frac{1}{4\pi\varepsilon_0\varepsilon_M}\left[\frac{e^{-jk_0 r}}{r} + \sum_{n=0}^{n_M}\sum_{m=0}^{m_M} A_{nm}^{v}(f)\,P_n(1-2\bar{\rho}^2)\,P_m(1-2\bar{z}) - j\pi\,\varepsilon_M\,R_V\right]$$

with:

$$A_{nm}^{xx}(f) = 2(2n+1)(2m+1)\int_0^{\infty}\frac{J_{2n+1}(k_\rho\rho_M)}{k_\rho\rho_M}\,\frac{i_m(u_0 z_M/2)}{e^{u_0 z_M/2}}\left[\frac{2k_\rho}{D_{TE}} - \frac{k_\rho}{u_0}\right]dk_\rho$$

$$A_{nm}^{zx}(f) = 2(2n+1)(2m+1)\int_0^{\infty}\frac{J_n(k_\rho\rho_M)}{k_\rho\rho_M}\,\frac{i_m(u_0 z_M/2)}{e^{u_0 z_M/2}}\left[\frac{4\varepsilon_M k_\rho^2}{D_{TE}D_{TM}} - 1\right]dk_\rho$$

$$A_{nm}^{v}(f) = 2(2n+1)(2m+1)\int_0^{\infty}\frac{J_{2n+1}(k_\rho\rho_M)}{k_\rho\rho_M}\,\frac{i_m(u_0 z_M/2)}{e^{u_0 z_M/2}}\left[2\varepsilon_M\frac{k_\rho(u_0 + u\,\mathrm{th}(uh))}{D_{TE}D_{TM}} - \frac{k_\rho}{u_0}\right]dk_\rho$$

where:

$$\varepsilon_M = \frac{\varepsilon_r+1}{2}, \qquad \eta = \frac{\varepsilon_r-1}{\varepsilon_r+1}, \qquad i_m(v) = \sqrt{\frac{\pi}{2v}}\, I_{m+1/2}(v) \quad \text{(I modified Bessel function)}$$

$$J_n = \left(1 - J_0(k_\rho \rho_M) - 2\sum_{i=1}^{n} J_{2i}(k_\rho \rho_M)\right) / \rho_M, \qquad \bar{\rho} = \rho / \rho_M, \qquad \bar{z} = z / z_M$$

ρ_M, z_M are the maximum ρ and z values where Green's functions are required whereas n_M, m_M are proper truncation indices of the series. Complete demonstrations of these formulas will be given in further works. Here we only point out that the singular function is the one of the free space for G_A^{xx} and G_v whereas for G_A^{zx} it is a novel function combination of a spherical and cylindrical term. The double series present the useful property of separability versus space and frequency and converge to an everywhere bounded function with maximum modules at the source point as depicted in Fig.2 for G_A^{xx} at f=10 GHz. The computing time for recovering the Green's functions is determined by the numerical evaluation of the coefficients $A_{nm}(f)$ (see Fig.3). Fortunately the integrands in the expressions of the coefficients are fastly decaying because of the properties of the Bessel functions whereas $A_{nm}(f)$ are smooth functions of frequency that consents considerable saving of computer time using interpolation when a wideband analysis is performed. Further, the truncation indices n_M, m_M in the series can be reduced by putting in evidence all analytic direct term functions as reported in [5]. Notice that the present inverse transform technique is applicable to situation where the source dipole is anyhow oriented and located into a multilayered medium. Within the layers without source no analytic direct terms are present and the Green's functions are smoother, hence, their double series representation converges much faster. Derivation of the EM fields from the presented Green' functions expressions will be given in further works.

3. Conclusions

This study presents the first basic step in the solution of the problem of space domain closed form for the Green's functions pertaining to microstrip geometries. Essentially they are discovered to be easily expressible like the sum of an analytic function singular at the source point and a remainder double series that is bounded at the source point and with coefficients numerically determined versus frequency and dielectric properties. This formulation is addressed to solve external problems in such a way far and near fields are given by a unique formulation and practically with the same accuracy.

References:

[1] J.R. Mosig, "Integral Equation Techniques", from "Techniques for Microwave and Millimeter-Wave Passive Structures", by Itoh, p. 133-213, 1989.
[2] S. Marchetti and J.M. Laheurte, "On the Space Domain Green's Functions for Microstrip Geometries", Proc. of the IEEE A&P International Symp., Newport Beach, C, June 1995.
[3] S. Marchetti and al., " Series Form of Space Domain Green's Functions on the Conductor Plane of Microstrip Antennas", Proc. of IEEE A&P Int. Symp., Univ. of Michigan, Ann Harbor, MI, June 1993.
[4] S. Marchetti and J. Citerne,"Series Form of Space Domain Green's Functions on the Free Space of Microstrip Antennas", Proc. of IEEE A&P Int. Symp., University of Washington, Seattle Washington, June 1994.
[5] S. Marchetti ," Green's Functions and EM Fields Direct Terms for Printed Circuits", Proc. of the IEEE A&P Int.-Symp., Baltimore (MD), July 1996.

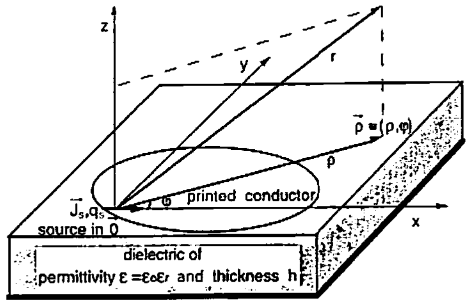

Fig.1 : Microstrip Geometry

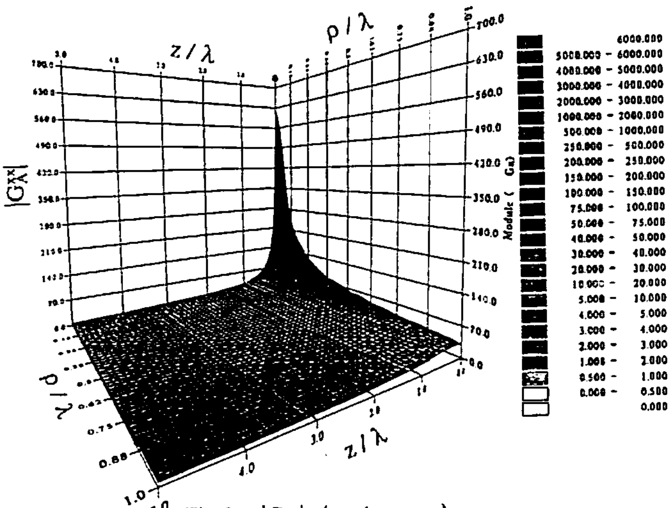

Fig.2 : $|G_A^{xx}|$ (series part)

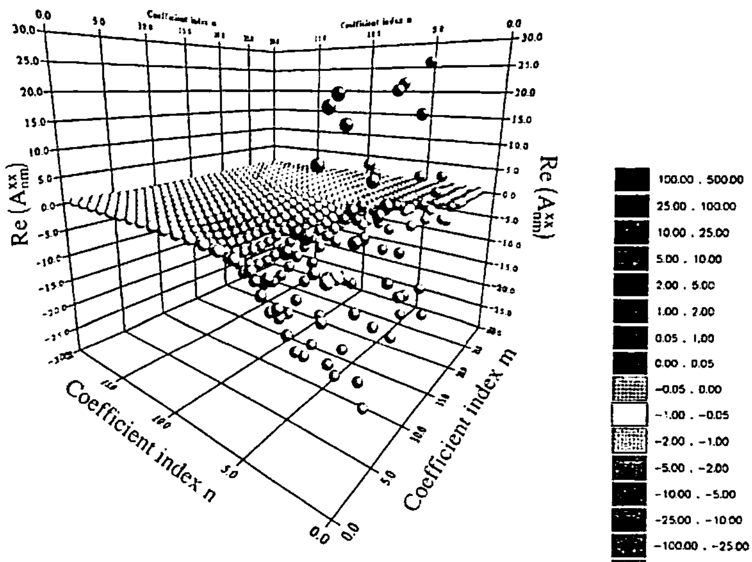

Fig.3 : Re (A_{nm}^{xx})