



## Asymptotic behaviour of blinking (stochastically switched) dynamical systems

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**Abstract**—We discuss the behavior of continuous-time dynamical systems that have external input signals that are constant in small time intervals and that can only take the values 0 and 1. They can be interpreted as switching between  $2m$  different dynamical systems if there are  $m$  such input signals. Switching is supposed to be fast with respect to the time constants of the (non-switched) different systems. Therefore, one expects that the switched (blinking) system behaves like the time-averaged system. More precisely, we suppose the switching to be stochastic such that the value of external signal at a certain time interval is a random variable and that all these random variables are independent.

In general, the solutions of the blinking and the averaged system starting from the same initial state, stay close together if the switching is fast, but this property holds only for finite time. However, if the solution of the averaged system converges to an attractor, this is also true for the corresponding rapidly switched blinking system under some weak hypotheses. In general, the solutions do not stay close together forever, but they converge to the same attractor.

Strictly speaking, this is only possible if the attractor of the averaged system is an invariant set to the blinking system. However, if this is not the case, the solution of the blinking system will still come close to attractor of the averaged system and stay close in a probabilistic sense. Furthermore, if the averaged system has more than one attractor (multi-stability), there is a small, but positive probability that the solution of the blinking system converges to another attractor than the solution of the averaged system.

Hence, there are 4 cases to distinguish, according to whether or not the averaged system has more than one attractor and whether or not the attractor(s) of the averaged system is (are) invariant under the blinking system. We give for each case an example and we prove a theorem that characterizes the relation between the asymptotic behavior of the averaged and the blinking system. Depending on the case, convergence of the blinking system to the attractor of the averaged system is either in the strong sense (for almost all switching sequences) or in the weak sense (with probability converging to one) if switching is fast.