

# End-to-End Path Loss Inference Algorithm with Network Tomography

Xiangyu Cao ,Ying Wang, Xuesong Qiu, Luoming Meng

State Key Laboratory of Networking and Switching Technology

Beijing University of Posts and Telecommunication

Beijing, China

caoxiangyu@bupt.edu.cn, wangy@bupt.edu.cn, xsqiu@bupt.edu.cn, lmmeng@bupt.edu.cn,

**Abstract**—Network path loss rate is an important indicator of the network performance. Given a network with  $n$  end hosts, existing systems require  $O(n \log n)$  measurements, and thus consume more management time and costs. An efficient network path loss rates inference method is proposed in this paper, which only needs to measure less part of paths to infer the loss rate of all remaining paths with higher accuracy, and ensure the quality of monitoring and reduce the management cost. The simulation results show that our method saves 5% ~ 14% probing paths than the existing method. Moreover our algorithm reduces the management cost of the network monitoring to some extent.

**Keywords**-Network measurement and monitoring; Path selection; End-to-End path loss rate; Route matrix

## I. INTRODUCTION

As the rapid development of the Internet technique, people increasingly dependent on network service, and they also put forward higher requirements on the performance of the network. Thus, how to measure the performance of the network accurately and efficiently is an important task.

Congestion is difficult to avoid. Network congestion may lead network unable to provide normal service, and resulting in economic losses and security risks. It is an important indicator of network performance, so how to quickly and accurately get path loss rate is becoming a hot topic of many researchers.

In order to save network overhead, existing active network measurement method mainly select fewer probing paths as far as possible for data acquisition [1] [2] [3]. However, the algorithms need to be carried out for all the paths one by one to analyze whether the path meet the independent conditions, which has a higher complexity. And the paths that these methods selecting always are more concentrated, that leads overloading to some links, and affecting the inference accuracy.

In this paper, we propose an end-to-end path loss inference algorithm with network tomography. We can infer the loss rate of the remaining paths accurately by measuring  $r$  independent paths that with uniform distribution. The new method adaptively selects the next path by using the results of a probe in the previous detection returned. The deletion of the non-congested paths and links makes the size of routing matrix smaller, which reduces the number of independent paths obviously. And our method can obtain the loss rates of all

remaining paths without solving the value of the link loss rate, which greatly reducing the complexity of the algorithm. The simulation results show that our method not only guarantee the accuracy of inference, but also decrease the number of independent paths to 5% ~ 14% compared to the existing classical method [2].

## II. MODELING AND PROBLEM FORMULATION

### A. Network Topology

We use a directed graph  $M = (V, E)$  to model the network.  $V$  represents the routers and hosts.  $E = \{e_1, e_2, \dots, e_{n_e}\}$  represents the links,  $n_e$  is the number of links,  $n$  is the number of end hosts. We define  $P = \{p_1, p_2, \dots, p_{n_p}\}$  as the set of all paths, here path  $p_i$  as a sequence of links that starts at a source host and ends at a destination host,  $n_p = |P|$  is the number of paths. Given a network  $M = (V, E)$  and a set of path  $P$ , we can obtain the routing matrix  $G$  of dimension  $n_p \times n_e$  as follows: each row of the matrix represents a path in the network, and the column represents links.  $G_{ij} = 1$  when path  $p_i$  traverses link  $e_j$ , and  $G_{ij} = 0$  otherwise. For example, the routing matrix of Fig.1 can be written as below.

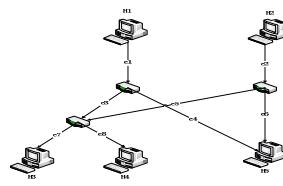

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Figure 1. An example network

### B. Algebraic Model

The transmission rate of path and link is more intuitive than the loss rate of them, so the remaining part of this article will describe the inference process of transmission rate only. We define the threshold of congestion as  $\varphi_r$ , and define the path is

“good” if the path transmission rate  $\varphi_i \geq \varphi_T$ . If one path is good, all the links that the path traverses are good.

Let  $\varphi_i$  represents the transmission rate of path  $p_i$ , and  $l_j$  represents the transmission rate of link  $e_j$ . As the path transmission rate is equal to the product of link transmission rate that the path traverses, then we can get the equation as (1).

$$\varphi_i = \prod_{j=1}^{n_e} l_j^{\nu_j} \quad (1)$$

Taking the logarithms on both sides of (1), we define a column vector  $x = [x_1 \cdots x_{n_e}]^T$  with element  $x_j = \log l_j$ , then we can rewrite (1) as below.

$$\log \varphi_i = \sum_{j=1}^{n_e} \nu_j \log l_j = \sum_{j=1}^{n_e} \nu_j x_j = x \nu^T \quad (2)$$

By putting  $n_p$  equations in form (2) together, the vector  $\nu^T$  form a rectangular matrix  $G \in \{0,1\}^{n_p \times n_e}$ . Let  $b = [b_1 \cdots b_{n_p}]^T$  as a column vector with element  $b_i = \log \varphi_i$ , then we rewrite the  $n_p$  equations in form (2) as below.

$$Gx = b \quad (3)$$

we obtain a reduced system from (3) as below.

$$\bar{G}\bar{x} = \bar{b} \quad (4)$$

Here  $\bar{G} \in R^{r \times n_s}$  is the maximum linear independent paths group selected from  $G$ ,  $n_s$  is the column number of  $\bar{G}$ ,  $\bar{x} \in R^{n_s}$  is the logarithm of link transmission rate being selected, and  $\bar{b} \in R^r$  consists of corresponding  $r$  rows of  $b$ .

### III. ACCURATE PATH LOSS INFERENCE ALGORITHM

#### A. Select the measurement paths

In order to choose  $r$  linear independent paths, we group the paths, and then select and detect the paths according to certain rules. In the process of selection, we simplify the matrix by removing good links and paths, and finally obtained a simplified linear system as the formula (4).

We assume that the original matrix is  $G$ , the matrix to be processed is  $G_{new}$ ,  $G_{new} = G$ . The linear independent paths are stored in the matrix  $G_{inde}$ . we group the matrix  $G$  according to the starting node. We select one path that is linear independent with matrix  $G_{inde}$  from each group by turn. If the selected path is linear independent, we judge whether the path is congested or not. If the path is congested, put this path into  $G_{inde}$ , and then continue to select from the next group. Else if the path is good, delete the links that this path traverses from matrix  $G_{new}$  and  $G_{inde}$ , and then select the next path from the same group.

We group the matrix according to the starting node, the row numbers of the paths of each group are scored in set  $S_i$ , and the row number of the paths that does not meet the condition is temporarily stored in set  $T_i$ . We get the first data of the set  $S_i$  to obtain the row number and the corresponding row vector  $v$  in the matrix  $G$ . Then we analyze whether  $v$  is linear independent with the vectors of matrix  $G_{inde}$ . If they are linear independent, we get the corresponding path transmission rate  $\varphi_{S_i[1]}$ . If  $\varphi_{S_i[1]} \geq \varphi_T$ , we delete this row number from  $S_i$ , and delete the column that the value is 1 of this row vector in matrix  $G_{new}$  and  $G_{inde}$ , then continue to get the first data of this set to select path. Else if  $\varphi_{S_i[1]} < \varphi_T$ , we draw conclusion that this path meet the condition, then we store this row vector in matrix  $G_{inde}$  and delete the row number from set  $S_i$ . We deal the remaining sets under this rule, after we have finish the last set  $S_n$ , then goto the first set  $S_1$ . If the set  $S_i$  is empty during the processing, then we goto the next set  $S_{i+1}$ . If  $G_{inde}$  is not the maximum linear independence path set when all set  $S$  are empty, then we assign the value of set  $T$  to set  $S$  and continue the process.

$G_{inde}$  may have nonlinear independent paths after the link deletion of this matrix. We can use path selection algorithm which is a variant of the QR decomposition with column pivoting (the Reselect Path algorithm) [4].

Our algorithm has the complexity of  $O(rn_p^2)$ . Although the complexity of our algorithm is similar to classical algorithms, but the probing paths that our algorithm get are obvious less than the classical algorithm.

#### B. Path Loss Rate Calculations

We propose a method which does not require solving the link transmission rates to obtain all path transmission rates.

We obtain the value of  $\bar{b}$  by the transmission rates of the maximum linear independent paths that we detected. Then we restructure  $G_{new}$ , make  $G_{final} = \begin{bmatrix} \bar{G} \\ G_{rem} \end{bmatrix}$ , here  $G_{rem}$  is the matrix that composed by the remaining paths after remove  $\bar{G}$  from  $G_{new}$ . We can use the vector of  $\bar{G}$  to represent all the vectors of  $G_{rem}$  as the linear combination. Similarly, the transmission rates of remaining paths can be represented as the linear combination of  $\bar{b}$ .

Elementary column transformation is carried out on  $G_{final}$  and simplest form matrix  $G_1$  are obtained, here the linear correlation of row vector in  $G_1$  is unchanged and the first  $r$  rows of the matrix is transformed to the identity matrix. For example, by carrying out the elementary column

transformation on matrix that after the restructuring of the routing matrix  $G$  in Figure 1, we can get the simplest form matrix  $G_1$  shown as below:

$$G_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The first  $r$  column vectors are assigned to the matrix  $M$ , and we can get that  $b = M\bar{b}$ , here  $b$  is the transmission rate of all the paths.

#### IV. SIMULATION

In this section, we verify the performance of our algorithm under the simulation network environment, and compare our algorithm with a former representative algorithm of [2] on different setting of networks.

##### A. Experiment Setup

In this paper we compare the two algorithms under two types of network topologies INET and Brite-WAXMAN[5]. We define non-leaf-nodes as routing nodes and leaf-nodes in the network topologies as the end hosts. The network sizes of this experiment are range from 100 to 1000 nodes. We use the LLRD1 model to assign the value of link loss rate. We define the congested probability as  $P_{cr}$ . In each detection, end host sends probe packets to each other, and the number of probe packet of each path is  $s = 1000$ . And we set the threshold of congestion as  $\varphi_r = 0.998$  and congested probability as  $P_{cr} = 0.1$ .

##### B. Simulation Results

We use the number of end-to-end probing paths, to compare detection overhead of these two algorithms. We also use the ERMS to measure the accuracies in estimating the path loss rate. Error Root Mean Square (ERMS) is the mean square of the error of path transmission rates. The ERMS is defined as follows.

$$\text{ERMS} = \sqrt{\frac{\sum (l_k - \hat{l}_k)^2}{n_e}} \quad (5)$$

Fig.2 and Fig.3 compares the results of the two algorithms on different scales in INET and BRITE-Waxman network topologies. Fig.2 (a) and Fig.3 (a) shows the number of probing paths of the two algorithms. As we can see that our algorithm saves 5% ~ 14% measurement paths under different types of topologies. Moreover, with the increase of nodes number in the topology, the optimization effect is more obvious. Fig.2 (b) and Fig.3 (b) shows the Error Root Mean Square (ERMS) of the two algorithms under the two topologies. As we can see that the ERMS of our algorithms is not greater than 0.02 and 0.027, that means the results of our algorithm are closer to the ground truth.

#### V. CONCLUSION

Path loss rate is a necessary part of locating congestion. How to use less probes and load as far as possible to obtain the congestion status that is closest to the real situation is the focus of our study. We improve the performance of end-to-end path measurement, and put forward a novel path loss rate algorithm which saves more detection cost. The algorithm only needs to measure less paths of the existing algorithm can infer the loss rates of all paths. Moreover, we optimize the distribution of selected paths, which reduces the load of links. The deletion of the routing matrix reduces the complexity of the calculation process, which makes our algorithm better.

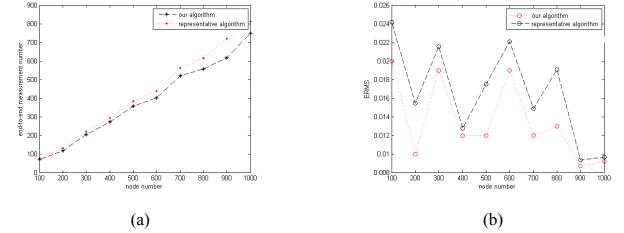


Figure 2. the results of the two algorithm under INET topologies

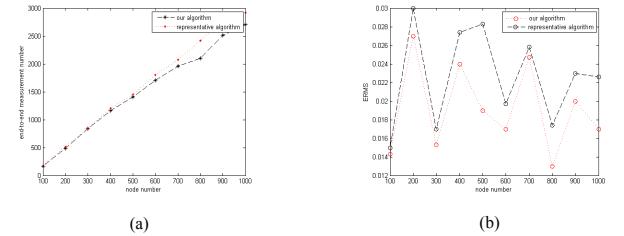


Figure 3. the results under BRITE-Waxman topologies

#### ACKNOWLEDGMENT

This work was partly supported by the National High Technology Research and Development Program of China (2013AA013502), the Fundamental Research Funds for the Central Universities BUPT 2013RC1103, Funds for Creative Research Groups of China (61121061).

#### REFERENCES

- [1] Y. Chen, D. Bindel, H. Song, and R. H. Katz, An algebraic approach to practical and scalable overlay network monitoring[C], Proceedings of ACM SIGCOMM, Portland, Oregon, USA, Aug 2004: 55-66.
- [2] Yanjie Ren, Yan Qiao, Xue-song Qiu and Shun-an Wu , Scalable Deterministic End-to-End Probing and Analytical Method for Overlay Network Monitoring, Proceedings of the 7th International Conference on Network and Services Management, Austria ,2011: 460-464.
- [3] Shun-an Wu, Qiao Yan, Xue-song Qiu, Yanjie Ren, A Probe Prediction Approach to Overlay Network Monitoring, Proceedings of the 7th International Conference on Network and Services Management, Austria ,2011: 465-469.
- [4] G.H. Golub and C.F. Van Loan, Matrix Computations, The Johns Hopkins University Press, 1989.
- [5] Clegg R G, Cairano-Gilfedder C D, Zhou S . A critical look at power modeling of the Internet [J].Computer Communications, 2010, 33(3): 259-268.