Ring Grounding Electrode and Coaxial Vertical Cylindrical Ground Non-homogeneity

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Abstract — **Satisfactorily accurate approximation of soil inhomogeneity providing simple determination of the soil structure influence on grounding system characteristics is important part of the procedure for grounding system analysis. Very often, ground non-homogeneity can be approximated as vertical cylinder tube (wells, various tubes, pillar foundation etc.). The method based on using of Method on Moments and Green's function, applicable for analysis of earthing electrodes positioned in the surrounding soil of this type is presented. The procedure application is illustrated at the problem of ring ground electrode coaxially placed related to cylindrical non-homogeneity.**

Keywords — **Green's function method; Grounding; Non-homogeneous media**

I. INTRODUCTION

Modeling ground non-homogeneity as cylinder or semi-cylinder placed in surrounding ground can be applied for analysis of various grounding problems. For example, it can be used for approximation of pillar foundation [1] or road [2-4] and characterize their influence on the features of the earthing electrode placed nearby them. This paper deals with influence of the ground non-homogeneity, assumed as homogeneous infinitely long vertical cylinder, on grounding system placed in its vicinity, will be considered in this paper. Such approximation can be used for characterization of the earthing systems realized in the vicinity of wells, various tubes etc. The applied procedure is based on using the Green function [5] and standard application of Method of Moment (MoM) [6] and it provides determining the resistance of the grounding system and electric scalar potential distribution.

In this paper, the described procedure application will be illustrated on the problem of characterization of ring grounding electrode coaxially placed related to the vertical cylindrical non-homogeneity.

II. THE THEORETICAL BASIS

A. Problem Descriptions

The procedure for characterization of the of ring grounding electrode coaxially placed related to the vertical cylindrical non-homogeneity will be presented, Fig. 1. The electrode is supplied with stationary current I_g via insulated conductor. The specific conductivity of the soil is σ_2, while the non-homogeneity specific conductivity value is σ_1. The described geometry includes Descartes' (x, y, z) and cylindrical (r, θ, z) coordinate systems ($r=\sqrt{x^2+y^2}$, $\theta=\arctan(y/x)$).

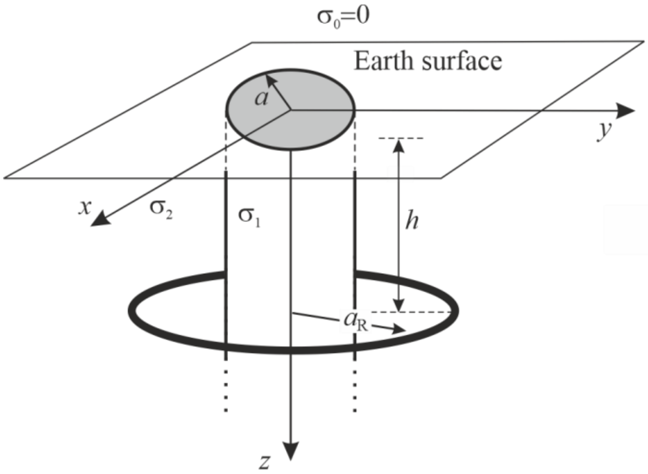

Fig. 1 Ring grounding electrode coaxially placed related to the vertical cylindrical non-homogeneity

The ring electrode of radii a_R buried at depth h from the earth surface is observed. The electrode is supplied by total current I_g, while label I_L denotes longitudinal current along the ring wire which is assumed as linear function.

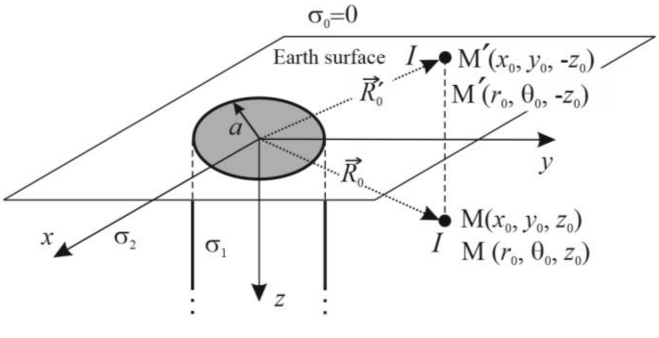

Fig. 2. The current point source and corresponding image.

B. Green's function

In [7] the Green function for the electric scalar potential of the stationary current point source positioned within the domain of infinitely long cylinder having circularly-shaped cross-section has been derived as solution of Laplace, i.e. Poisson's equation. The analogue procedure has been applied

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in [5] for forming Green function expressions for the current point source placed outside the cylinder. Those expressions, combined with the image theory are used for deriving Green`s function of the current point source positioned nearby the infinitely long semi-cylinder, Fig. 2.

Image theory application, includes necessity of taking the image source into account. For air, specific conductivity is $\sigma_0 \approx 0$ and consequently the reflection coefficient R_{20} is

$$R_{20} = (\sigma_2 - \sigma_0)/(\sigma_2 + \sigma_0) \approx 1, \tag{1}$$

The image source current is $R_{20} I = I$. The source and image positions (points M and M') are defined by the field vectors

$$\vec{R}_0 = x_0\hat{x} + y_0\hat{y} + z_0\hat{z} \text{ and } \vec{R}'_0 = x_0\hat{x} + y_0\hat{y} - z_0\hat{z} = r_0\tilde{r} - z_0\hat{z} \tag{2}$$

respectively. The field vector of corresponding observation point is

$$\vec{R} = x\hat{x} + y\hat{y} + z\hat{z} = r\hat{r} + z\hat{z} \tag{3}$$

Now, taking into consideration potentials of the original and image sources and using expressions for Green`s function of the point source positioned outside the infinitely long cylinder [5], total potential of the system from Fig. 2 inside cylinder ($r < a$) can be expressed as

$$\begin{aligned}\varphi_1 = \frac{I}{2\pi^2\sigma_2}\sum_{m=0}^{\infty}\Big\{\xi_m \cos[m(\theta-\theta_0)]\times \\ \times\Bigg[\int_0^\infty A_m(\lambda) I_m(\lambda r) I_m(\lambda r_0)\cos[\lambda(z-z_0)]d\lambda + \quad \text{and} \\ + \int_0^\infty A_m(\lambda) I_m(\lambda r) I_m(\lambda r_0)\cos[\lambda(z+z_0)]d\lambda\Bigg]\Big\}, r < a,\end{aligned} \tag{4}$$

$$\begin{aligned}\varphi_2 = \frac{I}{2\pi^2\sigma_2}\sum_{m=0}^{\infty}\Big\{\xi_m \cos[m(\theta-\theta_0)]\times \\ \times\Bigg[\int_0^\infty [B_m(\lambda) I_m(\lambda r_0) K_m(\lambda r) + I_m(\lambda r) K_m(\lambda r_0)]\times \\ \times\big[\cos[\lambda(z-z_0)] + \cos[\lambda(z+z_0)]\big]d\lambda\Bigg]\Big\}, \\ \xi_0 = 1, \xi_m = 2,\ m = 1,2,\ldots, r > a.\end{aligned} \tag{5}$$

outside the cylinder ($r > a$). The coefficients $A_m(\lambda)$ and $B_m(\lambda)$, $m = 0,1,2,\ldots$ in (4) and (5) can be determined using expressions

$$\begin{gathered}A_m = \Delta_A / \Delta,\ B_m = \Delta_B / \Delta,\ m = 0,1,2,\ldots \\ \Delta_A = K_m(\lambda r_0) I_m(\lambda r_0)/(\lambda a) \\ \Delta_B = I_m(\lambda a) I_m(\lambda r_0) I'_m(\lambda a) K_m(\lambda r_0)\left(1 - \frac{\sigma_1}{\sigma_2}\right) \\ \Delta = \left[I_m(\lambda r_0)\right]^2\left[\frac{\sigma_1}{\sigma_2} I'_m(\lambda a) K_m(\lambda a) - I_m(\lambda a) K'_m(\lambda a)\right].\end{gathered} \tag{6}$$

In (4)-(6), the labels I_m and K_m correspond to the modified Bessel functions of the first, i.e. and second kind.

Finally, the stationary current point source Green`s functions shown in Fig. 1 are obtained as

$$G_1(\vec{R}, \vec{R}_0) = \varphi_1 / I \text{ and } G_2(\vec{R}, \vec{R}_0) = \varphi_2 / I. \tag{7}$$

C. *Potential and resistance determining*

The potential value he at an arbitrary point of the earthing system's surroundings can be determined as

$$\varphi_{1/2}(\vec{R}) = \int_{l_R} G_{1/2}(\vec{R}, \vec{s}'_k) I_{\text{leak}}(s')\,\mathrm{d}s' \tag{8}$$

where index "1" corresponds to the potential within the cylinder ($r < a$), while "2" labels the potential outside the cylindrical domain ($r > a$). In (8), $\vec{s}'$ correspond to the elementary vector of corresponding part of ring electrode, while the total leakage current from the wire element $\mathrm{d}s'$, is denoted as $\mathrm{d}I = I_{\text{leak}}\,\mathrm{d}s'_k$, where

$$I_{\text{leak}} = -\partial I_L(s')/\partial s' = -I'_L(s'). \tag{9}$$

is the leakage current density per unit length. As it has been already emphasized above, the longitudinal current is assumed as linear function and consequently the leakage current density per unit length is constant i.e.

$$I_{\text{leak}} = I_g / l_R \tag{10}$$

where l_R denotes ring electrode length. This approach is justified for quasi-stationary regime.

Further, the previously presented analysis combined with MoM is applied for analysis of the grounding electrode shown in Fig. 2. Assumption that the electrode surface is approximately equipotential, allows matching potential value $\varphi = U$ at the surface point of the ring wire which position corresponds to the field vector $\vec{R}_R$. The presented procedure results in a linear equation,

$$\varphi(\vec{R} = \vec{R}_R) \cong U = \frac{I_g}{l_R}\int_{l_R} G_2(\vec{R}_R, \vec{s}')\,\mathrm{d}s'. \tag{11}$$

The solution of the equation (11) is supplying current I_g. Now, earthing electrode resistance is

$$R_g = U/I_g. \tag{12}$$

III. NUMERICAL RESULTS

In order to illustrate proposed procedure, wire electrode having dimension corresponding to the conductors used in practice [8]. The electrode radii is a_R=1 m, and cross-section radii is 5 mm. It is placed at the *h*=0.7 m depth, parallel to the *x* axis and soil surface. The cylinder radii is a=0.5 m and surrounding soil specific conductivity value is σ_2=0.01 S/m.

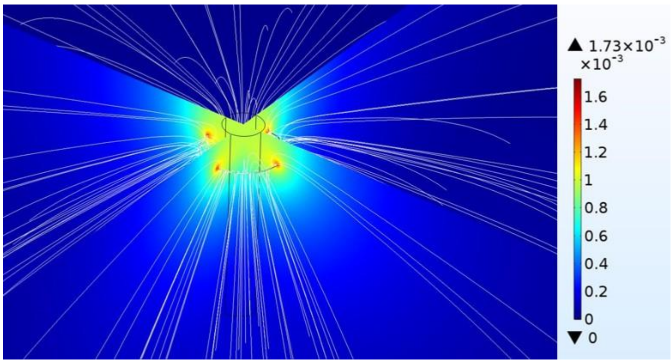

Fig. 3. Potential and current density distribution in the vicinity of the grounding electrode for σ_2/σ_1=10 and I_g=1 A.

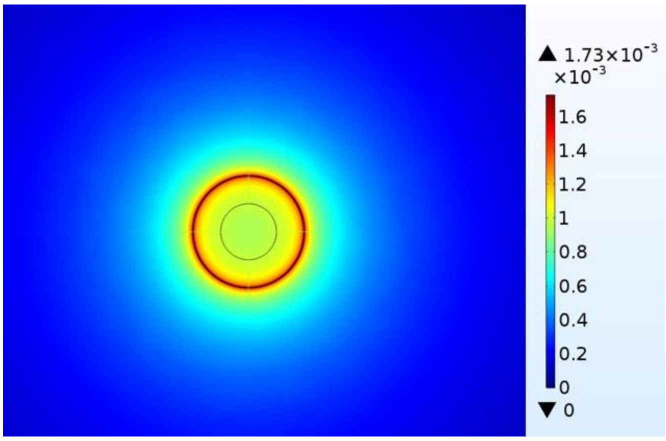

Fig. 4. Potential distribution at the grounding surface for σ_2/σ_1=10 and I_g=1 A.

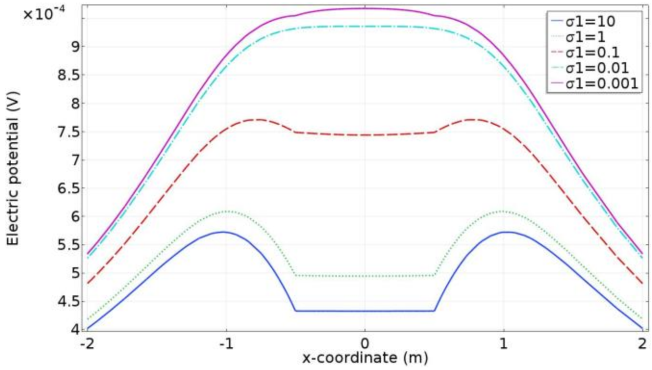

Fig. 5. Potential distribution at the grounding surface along x-axis for for different values of specific conductivity σ_1 and I_g=1 A

The potential distribution as well distribution of the current density in the vicinity of the grounding electrode for σ_2/σ_1=10 are presented in Fig. 3, where I_g=1 A. The potential at the ground surface for the same parameters is shown in Fig. 4 (plane z=0). In Fig. 5, the potential distribution along x axis for different values of specific conductivity σ_1 is presented.

The value of single wire electrode resistance versus ratio σ_2/σ_1 is shown in Table I.

TABLE I
ELECTRODE RESISTANCE VERSUS SPECIFIC CONDUCTIVITIES RATIO

σ_2/σ_1	$R_g[\Omega]$
0.001	18.400
0.01	18.826
0.1	20.508
1	21.785
10	22.033

IV. CONCLUSIONS

One method based on Green function [5] and Method of Moment for characterizing influence of cylindrical non-homogeneity on grounding system placed in surrounding ground is presented in the paper.

There are such non-homogeneities which can be justifiably approximated with infinitely long cylinder.

This approach can be used in the framework of semi-analytical approaches, providing more accuracy with less computing time. The modern computing facilities provide possibility for determining Green function which are complex for computing, related to previous times when such approach was not too reasonable.

The procedure is illustrated on the problem of single ring wire electrode coaxially placed in the ground, related to cylindrical non-homogeneity. It is not limited only on coaxial ring electrode (ring electrode can be arbitrary placed). The simple extension of described approach can be used for even for more complex earthing electrodes used in practice.

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