Evaluation of the refractive index structure constant profile

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***Abstract*— The refractive index structure parameter C_n^2, is of high importance to estimate and predict the channel behavior characterizing satellite to ground optical links. Several models have been studied to describe C_n^2 profiles. Trinquet-Vernin, Dewan and Masciadri are three different Parametric models for obtaining vertical turbulence profiles of C_n^2 present in this paper. Moreover, we show a three-day Seeing measurements in Changchun, China and evaluate Masciadri model based on ECMWF model in Paranal, Chile. The study presents in this paper aims to identify the atmospheric turbulence modelling and validate the Masciadri model.**

Keywords—Astronomical seeing, Turbulence profile, C_n^2

I. Introduction

Atmospheric turbulences is the major limitation of free-space laser communications' performances. Free-space optical channels are described by different parameters such as the Isoplanatic Angle, the Rytov variance, the Fried parameter, Seeing etc. These parameters derive from the particular atmospheric turbulences model chosen.
In order to design reliable ground-to-satellite optical communications links, vertical profiles modellings of the strength of refractive turbulences, so-called C_n^2 (in $m^{-2/3}$) structure parameter are needed.

The turbulence altitude profile is characterized by, C_n^2 which is a measure of the amount of refraction present in the air. Generally, observed values range from $10^{-12}\,m^{-2/3}$ to $10^{-16}\,m^{-2/3}$ [1]. High values (10^{-12}) are the sign of a turbulent atmosphere, resulting in visual blurring or image distortion and a negligible effect is considered by low values (10^{-16}).

Several models have been developed to address optical turbulence in the atmosphere such as, Dewan, Trinquet-Vernin, etc. These numerical models use a various models with some inputs based on the location for which a profile is best suited. Boundary layer height estimation is not considered in this paper.

II. Refractive Index Structure Parameter, C_n^2 Models

There are two different parametric models and a modification model for deriving vertical turbulence profiles of C_n^2 present in this paper.

A. Dewan model

This model converts radiosonde data into profiles of C_n^2, using the following formula [2]:

$$C_n^2 = 2.8\,\Big(\frac{(-79x10^{-6}P)}{T\,\theta}\Big)^2 L_0^{\,4/3}\Big(\frac{d\theta}{dh}\Big)^2 \qquad (1)$$

where θ is potential temperature gradient at altitude h (above mean sea level), P is pressure in mbar and T in Kelvin.

The calculation of C_n^2 necessitates the knowledge of the outer scale L_0. Two relations are proposed for troposphere and stratosphere layers as follows:

$$L_0^{4/3} = 0.1^{\,4/3}\,10^{\,1.64 + 42.0\,S} \qquad \text{Troposphere} \qquad (2)$$

$$L_0^{4/3} = 0.1^{\,4/3}\,10^{\,0.506 + 50.0\,S} \qquad \text{Stratosphere} \qquad (3)$$

$$S = \Big[\Big(\frac{dV_x}{dh}\Big)^2 + \Big(\frac{dV_y}{dh}\Big)^2\Big]^{1/2} \qquad (4)$$

where *S*, is the wind shear, V_x and V_y are the north and east horizontal wind components at altitude *h*.

It is pointed out that the Tatarski's formulation (Eq. 1) is not expected to be applicable in the lower atmosphere during unstable condition and does not relate to the convective boundary layer.

Three cases of static stability (Fig. 1) for better understanding the optical turbulence formation and the potential temperature mechanism are as follows [3]:

- When the potential temperature decreases with height but slower than the adiabatic lapse rate, the atmosphere is stable, $\frac{d\theta}{dh} > 0$
- When potential temperature decreases upward, the atmosphere is top heavy and unstable, $\frac{d\theta}{dh} < 0$.
- And when the potential temperature is vertically uniform, the atmosphere is neutral, $\frac{d\theta}{dh} = 0$

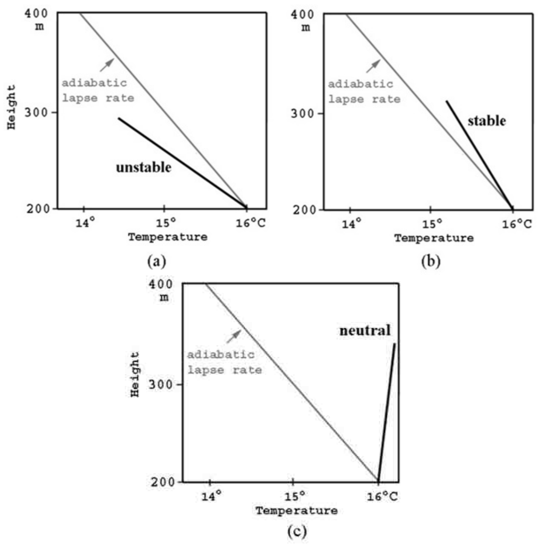

Figure 1: (a) $\frac{d\theta}{dh} < 0$ unstable, (b) $\frac{d\theta}{dh} > 0$ stable, (c) $\frac{d\theta}{dh} = 0$ neutral. The grey line refers to the adiabatic case.

B. *Trinquet–Vernin model*

This model is linked with Weather Research and Forecasting software (WRF) [4] for the simulation of pressure, Temperature and humidity profiles and gives an alternative approach to Tatarski: the calculation of C_n^2 from the temperature structure constant C_T^2. Both quantities are related by the Gladstone equation:

$$C_n^2 = \left(\frac{80x10^{-6}P}{T^2}\right)^2 C_T^2(h) \tag{5}$$

where P is the pressure in *hPa* and T is the absolute temperature.

Since C_T^2 is a positive value, only positive values of $\frac{d\theta}{dh}$ and *S(h)* are valid. It is clear from Eq. 6 that C_T^2 is proportional to the vertical wind shear *S(h)* and the gradient of the potential temperature ($\frac{d\theta}{dh}$) as follows :

$$C_T^2(h) = \Phi(h) \left(\frac{d\theta}{dh}\right) S(h)^{1/2} \tag{6}$$

This vertical profile, *Φ (h)* has been evaluated by Trinquet-Vernin, using about 160 radiosoundings [4] and denotes the median values of temperature structure constant, potential temperature gradient and wind shear. The profiles are given in table 1.

TABLE 1. Vertical profile of *Φ* (*h*) deduced from statistical analyses for different altitudes (Taken from [5])

Boundary layer		Free atmosphere	
Altitude (m)	ϕ	Altitude (m)	ϕ
5	2.834 992 0	1500	0.220 223 9
55	0.782 577 3	2500	0.123 299 4
105	0.285 124 6	3500	0.122 084 7
155	0.224 789 3	4500	0.111 699 2
205	0.233 936 9	5500	$7.956\,506\,3 \times 10^{-2}$
255	0.236 869 7	6500	$7.661\,102\,0 \times 10^{-2}$
305	0.139 371 8	7500	$9.468\,948\,1 \times 10^{-2}$
355	0.169 790 4	8500	$8.243\,700\,1 \times 10^{-2}$
405	0.135 091 6	9500	$8.556\,377\,9 \times 10^{-2}$
455	0.115 170 5	10 500	$7.964\,827\,9 \times 10^{-2}$
505	0.120 165 6	11 500	$5.956\,235\,9 \times 10^{-2}$
555	0.124 200 0	12 500	$4.449\,683\,1 \times 10^{-2}$
605	0.152 836 5	13 500	$4.532\,294\,3 \times 10^{-2}$
655	0.125 810 8	14 500	$3.857\,794\,8 \times 10^{-2}$
705	0.103 847 3	15 500	$4.923\,798\,9 \times 10^{-2}$
755	$9.600\,337\,6 \times 10^{-2}$	16 500	$4.553\,578\,8 \times 10^{-2}$
805	$8.320\,550\,6 \times 10^{-2}$	17 500	$4.589\,249\,6 \times 10^{-2}$
855	0.106 195 8	18 500	$3.965\,354\,7 \times 10^{-2}$
905	$9.471\,563\,2 \times 10^{-2}$	19 500	$4.126\,950\,0 \times 10^{-2}$
955	0.102 255 2		

C. *Masciadri model*

The Gladstone relation applied in Trinquet-Vernin can be used to estimate the value of C_n^2. Here, we use the modification version of the Dewan to calculate the C_n^2 [6].

$$C_n^2 = \left(\frac{80x10^{-6}P}{T\,\theta}\right)^2 C_T^2 \tag{7}$$

where θ is the potential temperature and C_T^2 can be estimated as follows:

$$C_T^2 = K\,L^{4/3}\left(\frac{\overline{d\theta}}{dh}\right)^2 \tag{8}$$

where h is the altitude, K is equal to 6, $\frac{\overline{d\theta}}{dh}$ *is* the gradient of the potential temperature and L is the scale of the largest energy input into the turbulent flow and can be defined as:

$$\mathrm{L}(h) = \sqrt{\frac{2\,\mathrm{E}}{\frac{g}{\theta}\left(\frac{\overline{d\theta}}{dh}\right)}} \tag{9}$$

where E is the turbulent kinetic energy. Here, E = S^2, where S is the vertical wind Shear (Eq. (4)).

Therefore, to estimate C_n^2 we have the following,

$$C_n^2 = \mathrm{K}\left(\frac{80x10^{-6}P}{T(h)\,\theta(h)}\right)^2 L(h)^{4/3}\left(\frac{\overline{d\theta}}{dh}\right)^2 \tag{10}$$

D. *Seeing*

Seeing parameter is an essential physical parameter that needs to be measured in modern astronomical site and Differential Image Motion Monitor (DIMM) is currently a commonly used astronomical instrument for calculating seeing parameter. DIMM (Fig. 2) is a small instrument with differential technique to precisely measure the seeing conditions.

Figure2: DIMM system at Lijiang Astronomical Observatory in China

In General, The ratio between wavelength, λ and coherence length (Fried parameter) r_0, is indicating the telescope angular resolution or is called astronomical seeing, ε *in* arcseconds. Smaller ε means a better seeing.

Seeing is calculated from the three C_n^2 models (Dewan, Trinquet-Vernin, Masciadri) using any meteorological profiles (such as Weather Research and Forecasting (WRF) or European Centre for Medium-Range Weather Forecasts (ECMWF) data as input. The Seeing is calculated by [7]:

$$\varepsilon = 0.98\, \lambda\, r_0^{-1} \tag{11}$$

$$\varepsilon = 5.25\, \lambda^{-1/5} \left(\sec z \int_{h_0}^{\infty} C_n^2(h)\, dh\right)^{3/5} \tag{12}$$

where r_0 is the Fried parameter and z is the zenith angle.

Following figures show an example which is carried out at Lijiang base of Changchun Astronomical Observatory in China on July 5th, 6th and 7th, 2016 to measure Astronomical Seeing parameter based on DIMM instrument to obtain a general understanding about the range of the seeing values in arcsec.

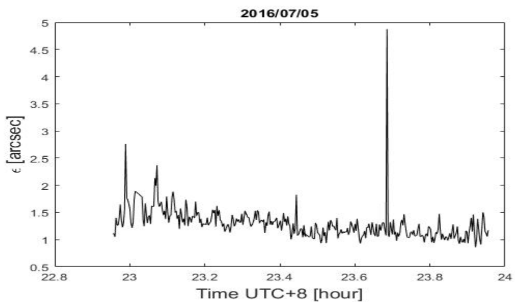

Figure 3: Astronomical Seeing parameter measured by DIMM at Changchun on July 5th, 2016.

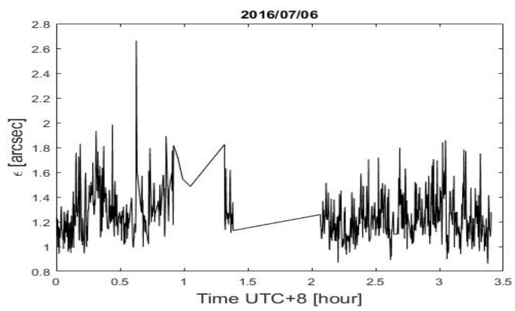

Figure 4: Astronomical Seeing parameter measured by DIMM at Changchun on July 6th, 2016.

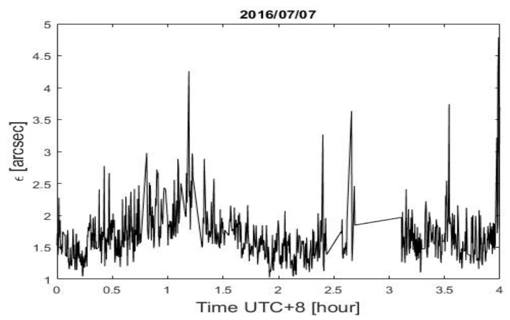

Figure 5: Astronomical Seeing parameter measured by DIMM at Changchun on July 7th, 2016.

III. ECMWF and Simulation

ECMWF is a non hydrostatic model which is refreshed every 6 hours and provides a forecast with the time resolution of 1 hour. Pressure level and model level are available to be used in ECMWF. The resolution of the altitude near the ground is generally a few tens of meters and above the tropopause is a few kilometres.

A. *ECMWF*

The aim of this section is to validate Masciadri model with the ECMWF data at Cerro Paranal in Chile based on ERA5 catalogue in 0.3-degree grid.

Figure 6, presents median turbulence profile for the night beginning 2nd July 2016 based on Author data [6]. The Altitude shown is from observatory level.

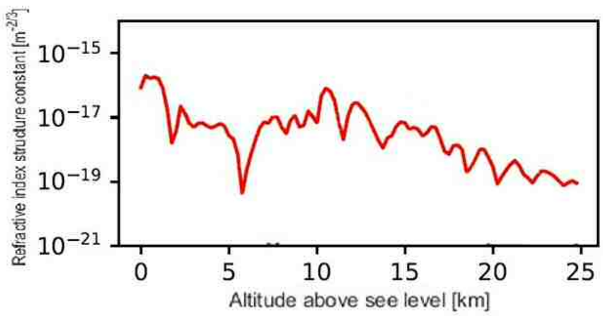

Figure 6. The red curve is the median turbulence profile forecast from ECMWF data. The altitude is 2.6 km above sea level (Modified from [6])

B. *Matlab Simulation*

In this section, C_n^2 is calculated using ECMWF model based on Masciadri model. In this simulation the time resolution is 1 hour with varying vertical resolution starting around 200m.

0.25-degree grid has been chosen to interpolate between the four nearest data point. Product type has been selected reanalysis as an input whereas author [6] has selected forecast.

We use available data from ECMWF-ERA5 catalogue for the whole July 2016 to simulate hourly profiles of pressure, temperature and humidity on the Paranal site to produce hourly profiles of the C_n^2 for the whole time period on July 2nd (in Fig. 8), along with its median as follows:

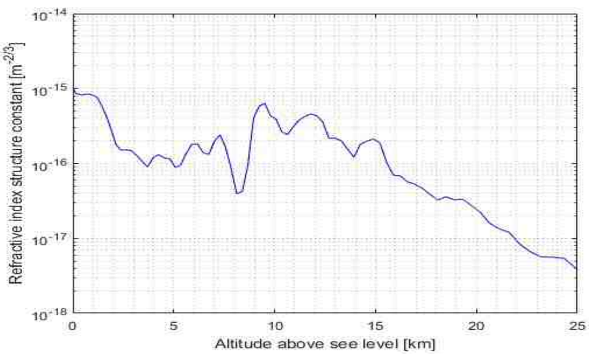

Figure 7. The blue curve is the median turbulence profile forecast from ECMWF data for the night beginning 2nd July 2016. The altitude is 1.07 km above sea level.

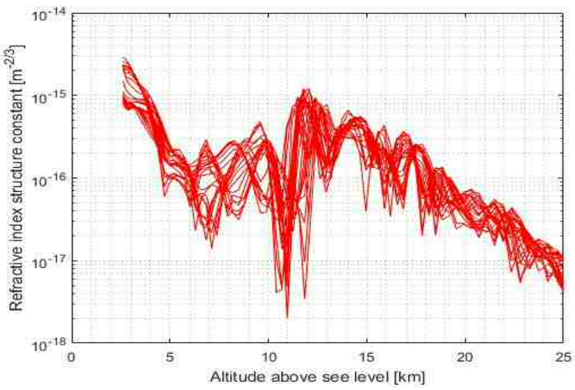

Figure 8. The red curve is the turbulence profile forecast from ECMWF data for the night beginning 2nd July 2016.

A comparison between ECMWF forecast profile by [6] in Fig. 6 follows a fairly similar pattern over ECMWF reanalysis product-type simulated in Fig. 7.

IV. Discussion

Tatarski relation is used in both **Dewan** and **Masciadri** models whereas **Trinquet-Vernin** parameterization uses Gladstone formulation for C_n^2.

Additionally, based on Tatarski relation, which is applied in **Dewan** and **Masciadri** models, C_n^2 is proportional to Potential temperature gradient by power by two ($\frac{d\theta}{dh}$) ^{2} instead of power by one ($\frac{d\theta}{dh}$) in **Trinquet–Vernin** model. This Nature may be a case for the Trinquet-Vernin to have a negative value in Potential temperature gradient; therefore, the model becomes invalid for this meteorological condition.

Trinquet-Vernin model can be employed to the whole atmosphere whereas both **Dewan** and **Masciadri** models are applied to the free atmosphere only.

Since the value oC_n^2 calculated based on **Masciadri** and **Dewan** is not valid in planetary boundary layer, there should be a need to describe the C_n^2 parameter model in the boundary layer only. Bulk Richardson number proposed by the Integrated Forecasting System (IFS) documentation [8] reference to [9] and Vogelezang and Holtslag [10] are two methods to estimate the height of the Atmospheric Boundary Layer.

In order to have a better validation for the astronomical seeing or other important optical turbulence parameters based on each C_n^2 profiles, the real measurement are needed to enable a better comparison with the ECMWF data or other General circulation models (GCM).

V. Conclusion

In this paper, we summarized and investigated three different methods to calculate Refractive index structure constant, C_n^2 and consequently the Seeing parameter. Three models have been studied and compared. Masciadri and Dewan model are intended to the free atmosphere only. In other words, they are applicable above the boundary layer therefore there is a need to estimate the atmospheric boundary layer height to calculate the total seeing (boundary layer + free atmosphere).

Consequently, we illustrated a three-day campaign to measure the astronomical seeing with the DIMM in Changchun from July 5th July to 7th July 2016. Furthermore, to validate the Masciadri model, we have shown a comparison of ECMWF forecast performed by [6] and ECMWF reanalysis simulated by Matlab which shows both trends follows a fairly similar pattern. The differences in magnitude in both graphs (Fig. 6, Fig. 7) can be related to the chosen degree grid which is 0.25 in our simulation and 0.3 in [6] resulting in different Altitude above see level or the product type which has been selected reanalysis as a input whereas [6] has selected forecast.

References

[1] A. D. Tunick, “The Refractive Index Structure Parameter/Atmospheric Optical Turbulence Model: CN2“ Army Research Laboratory. MD 20783-1197,1998.

[2] T. Cherubini and S. Businger, “Another Look at the Refractive Index Structure Function,” Journal of applied meteorology and climatology, 2012

[3] J. Vernin, Z. Benkhaldoun, C. M. Tunon, “Mechanisom of formation of optical turbulence” ASP Conference Series, Vol. 266,2002.

[4] H. Trinquet, J. Vernin, “A statistical model to forecast the profile of the index structure constant,“ Environmental Fluid Mechanics. pp. 397-407, October 2007.

[5] C. Giordano,J. Vernin, H. Trinquet and C. Munoz, “Weather Research and Forecasting prevision model as a tool to search for the best sites for astronomy: application to La Palma, Canary Islands,” Monthly Notices of the Royal Astronomical Society, April 2014.

[6] J. Osborn and M. Sarazin, “Atmospheric turbulence forecasting with a General Circulation Model for Cerro Paranal “, Monthly Notices of the Royal Astronomical Society. 480. 10.1093/mnras/sty1898, 2018

[7] L.-Y. Liu, C. Giordano, Y.-Q. Yao, J. Vernin, M. Chadid, H.-S. Wang, J. Yin, Y.-P. Wang; ”Optical turbulence characterization at LAMOST site: observations and models, Monthly Notices of the Royal Astronomical Society,” Vol. 451, August 2015, pp. 3299–3308.

[8] IFS Documentation – Cy45r1 Operational implementation-Part IV : Physical Processes, European Centre for Medium-Range Weather Forecasts, England, June 2018.

[9] D.J. Seidel,Y. Zhang, A. Beljaars, J.-C. Golaz, A. R. Jacobson, and B. Medeiros, ”Climatology of the planetary boundary layer over the continental United States and Europe,” J. Geophys. Vol. 117, 2012.

[10] D. H. P. Vogelezang, and A. A. M. Holtslag , ”Evaluation and model impacts of alternative boundary-layer height formulation, Boundary Layer Meteorology,” Vol. 81, 1996, pp. 245–269.