Evaluation of the refractive index structure constant profile

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Abstract—The refractive index structure parameter $C_n^2$, is of high importance to estimate and predict the channel behavior characterizing satellite to ground optical links. Several models have been studied to describe $C_n^2$ profiles. Trinquet-Vernin, Dewan and Masciadri are three different Parametric models for obtaining vertical turbulence profiles of $C_n^2$ present in this paper. Moreover, we show a three-day Seeing measurements in Changchun, China and evaluate Masciadri model based on ECMWF model in Paranal, Chile. The study presents in this paper aims to identify the atmospheric turbulence modelling and validate the Masciadri model.

Keywords—Astronomical seeing, Turbulence profile, $C_n^2$

I. INTRODUCTION

Atmospheric turbulences is the major limitation of free-space laser communications’ performances. Free-space optical channels are described by different parameters such as the Isoplanatic Angle, the Rytov variance, the Fried parameter, Seeing etc. These parameters derive from the particular atmospheric turbulences model chosen.

In order to design reliable ground-to-satellite optical communications links, vertical profiles modellings of the strength of refractive turbulences, so-called $C_n^2$ (in m$^{-2/3}$) structure parameter are needed.

The turbulence altitude profile is characterized by $C_n^2$ which is a measure of the amount of refraction present in the air. Generally, observed values range from $10^{-12}$ m$^{-2/3}$ to $10^{-16}$ m$^{-2/3}$ [1]. High values ($10^{-12}$) are the sign of a turbulent atmosphere, resulting in visual blurring or image distortion and a negligible effect is considered by low values ($10^{-16}$).

Several models have been developed to address optical turbulence in the atmosphere such as, Dewan, Trinquet-Vernin, etc. These numerical models use a various models with some inputs based on the location for which a profile is best suited. Boundary layer height estimation is not considered in this paper.

II. REFRACTIVE INDEX STRUCTURE PARAMETER, $C_n^2$ MODELS

There are two different parametric models and a modification model for deriving vertical turbulence profiles of $C_n^2$ present in this paper.

A. Dewan model

This model converts radiosonde data into profiles of $C_n^2$ using the following formula [2]:

$$C_n^2 = 2.8 \left( \frac{-1.79 \times 10^{-6} p}{T_\theta} \right)^2 L_0^{4/3} \left( \frac{d\theta}{dh} \right)^2$$

(1)

where $\theta$ is potential temperature gradient at altitude $h$ (above mean sea level), $P$ is pressure in mbar and $T$ in Kelvin.

The calculation of $C_n^2$ necessitates the knowledge of the outer scale $L_0$. Two relations are proposed for troposphere and stratosphere layers as follows:

$$L_0^{4/3} = 0.1 \times 10^{4/3} \left( \frac{1.64 + 42.0 S}{T_\theta} \right)$$

(2)

and

$$L_0^{4/3} = 0.1 \times 10^{4/3} \left( \frac{0.506 + 50.0 S}{T_\theta} \right)$$

(3)

where $S$ is the wind shear, $V_x$ and $V_y$ are the north and east horizontal wind components at altitude $h$.

It is pointed out that the Tatarski’s formulation (Eq. 1) is not expected to be applicable in the lower atmosphere during unstable condition and does not relate to the convective boundary layer.

Three cases of static stability (Fig. 1) for better understanding the optical turbulence formation and the potential temperature mechanism are as follows [3]:

- When the potential temperature decreases with height but slower than the adiabatic lapse rate, the atmosphere is stable, \( \frac{dT}{dh} > 0 \)
- When potential temperature decreases upward, the atmosphere is top heavy and unstable, \( \frac{dT}{dh} < 0 \).
- And when the potential temperature is vertically uniform, the atmosphere is neutral, \( \frac{dT}{dh} = 0 \).
B. Trinquet–Vernin model

This model is linked with Weather Research and Forecasting software (WRF) [4] for the simulation of pressure, Temperature and humidity profiles and gives an alternative approach to Tatarski: the calculation of $C_n^2$ from the temperature structure constant $C_T^2$. Both quantities are related by the Gladstone equation:

$$C_n^2 = \left( \frac{\theta_0}{T} \right)^2 C_T^2(h)$$  \hspace{1cm}(5)$$

where $P$ is the pressure in hPa and $T$ is the absolute temperature. Since $C_T^2$ is a positive value, only positive values of $\frac{\theta}{dh}$ and $S(h)$ are valid. It is clear from Eq. 6 that $C_n^2$ is proportional to the vertical wind shear $S(h)$ and the gradient of the potential temperature ($\frac{\theta}{dh}$) as follows:

$$C_n^2(h) = \Phi(h) \left( \frac{\theta}{dh} \right) S(h) \frac{1}{2}$$  \hspace{1cm}(6)$$

This vertical profile, $\Phi(h)$, has been evaluated by Trinquet-Vernin, using about 160 radiosoundings [4] and denotes the median values of temperature structure constant, potential temperature gradient and wind shear. The profiles are given in table 1.

<table>
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<th>$\phi$</th>
<th>Free atmosphere</th>
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C. Masciadri model

The Gladstone relation applied in Trinquet-Vernin can be used to estimate the value of $C_n^2$. Here, we use the modification version of the Dewan to calculate the $C_n^2$ [6].

$$C_n^2 = \left( \frac{\theta_0}{T(0)} \right)^2 C_T^2$$  \hspace{1cm}(7)$$

where $\theta$ is the potential temperature and $C_T^2$ can be estimated as follows:

$$C_T^2 = KL \frac{1}{4} \left( \frac{\theta}{dh} \right)^2$$  \hspace{1cm}(8)$$

where $h$ is the altitude, $K$ is equal to 6, $\frac{\theta}{dh}$ is the gradient of the potential temperature and $L$ is the scale of the largest energy input into the turbulent flow and can be defined as:

$$L(h) = \frac{E}{\theta(h) (\frac{\theta}{dh})^2}$$  \hspace{1cm}(9)$$

where $E$ is the turbulent kinetic energy. Here, $E = S^2$, where $S$ is the vertical wind shear (Eq. (4)).

Therefore, to estimate $C_n^2$ we have the following.

$$C_n^2 = K \left( \frac{\theta_0}{T(h)} \right)^2 \frac{\theta(h)}{S(h)} \frac{1}{2}$$  \hspace{1cm}(10)$$

D. Seeing

Seeing parameter is an essential physical parameter that needs to be measured in modern astronomical site and Differential Image Motion Monitor (DIMM) is currently a commonly used astronomical instrument for calculating seeing parameter. DIMM (Fig. 2) is a small instrument with differential technique to precisely measure the seeing conditions.

Figure 2: DIMM system at Lijiang Astronomical Observatory in China

In General, The ratio between wavelength, $\lambda$ and coherence length (Fried parameter) $r_0$, is indicating the telescope angular resolution or is called astronomical seeing, $v$ in arcseconds. Smaller $v$ means a better seeing.
Seeing is calculated from the three $C_n^2$ models (Dewan, Trinquet-Vernin, Masciadri) using any meteorological profiles (such as Weather Research and Forecasting (WRF) or European Centre for Medium-Range Weather Forecasts (ECMWF) data as input. The Seeing is calculated by [7]:

$$\varepsilon = 0.98 \lambda r_0^{-1}$$  \hspace{1cm} (11)

$$\varepsilon = 5.25 \lambda^{-1.5} (\sec z \int_0^h C_n(h) dh)^{3/5}$$  \hspace{1cm} (12)

where $r_0$ is the Fried parameter and $z$ is the zenith angle.

Following figures show an example which is carried out at Lijiang base of Changchun Astronomical Observatory in China on July 5th, 6th and 7th, 2016 to measure Astronomical Seeing parameter based on DIMM instrument to obtain a general understanding about the range of the seeing values in arcsec.

![Figure 3: Astronomical Seeing parameter measured by DIMM at Changchun on July 5th, 2016.](image)

![Figure 4: Astronomical Seeing parameter measured by DIMM at Changchun on July 6th, 2016.](image)

![Figure 5: Astronomical Seeing parameter measured by DIMM at Changchun on July 7th, 2016.](image)

III. ECMWF AND SIMULATION

ECMWF is a non-hydrostatic model which is refreshed every 6 hours and provides a forecast with the time resolution of 1 hour. Pressure level and model level are available to be used in ECMWF. The resolution of the altitude near the ground is generally a few tens of meters and above the tropopause is a few kilometres.

A. ECMWF

The aim of this section is to validate Masciadri model with the ECMWF data at Cerro Paranal in Chile based on ERA5 catalogue in 0.3-degree grid.

Figure 6, presents median turbulence profile for the night beginning 2nd July 2016 based on Author data [6]. The Altitude shown is from observatory level.

![Figure 6. The red curve is the median turbulence profile forecast from ECMWF data. The altitude is 2.6 km above sea level (Modified from [6])](image)

B. Matlab Simulation

In this section, $C_n^2$ is calculated using ECMWF model based on Masciadri model. In this simulation the time resolution is 1 hour with varying vertical resolution starting around 200m.

0.25-degree grid has been chosen to interpolate between the four nearest data point. Product type has been selected reanalysis as an input whereas author [6] has selected forecast.

We use available data from ECMWF-ERA5 catalogue for the whole July 2016 to simulate hourly profiles of pressure, temperature and humidity on the Paranal site to produce hourly profiles of the $C_n^2$ for the whole time period on July 2nd (in Fig. 8), along with its median as follows:

![Figure 7. The blue curve is the median turbulence profile forecast from ECMWF data for the night beginning 2nd July 2016. The altitude is 1.07 km above sea level.](image)
A comparison between ECMWF forecast profile by [6] in Fig. 6 follows a fairly similar pattern over ECMWF reanalysis product-type simulated in Fig. 7.

IV. DISCUSSION

Tatarski relation is used in both Dewan and Masciadri models whereas Trinquet-Vernin parameterization uses Gladstone formulation for $C_n^2$.

Additionally, based on Tatarski relation, which is applied in Dewan and Masciadri models, $C_n^2$ is proportional to Potential temperature gradient by power by two $(\frac{\partial T}{\partial z})^2$ instead of power by one $(\frac{\partial T}{\partial z})$ in Trinquet–Vernin model. This Nature may be a case for the Trinquet-Vernin to have a negative value in Potential temperature gradient; therefore, the model becomes invalid for this meteorological condition.

Trinquet-Vernin model can be employed to the whole atmosphere whereas both Dewan and Masciadri models are applied to the free atmosphere only.

Since the value of $\alpha C_n^2$ calculated based on Masciadri and Dewan is not valid in planetary boundary layer, there should be a need to describe the $C_n^2$ parameter model in the boundary layer only. Bulk Richardson number proposed by the Integrated Forecasting System (IFS) documentation [8] reference to [9] and Vogelezang and Holtslag [10] are two methods to estimate the height of the Atmospheric Boundary Layer.

In order to have a better validation for the astronomical seeing or other important optical turbulence parameters based on each $C_n^2$ profiles, the real measurement are needed to enable a better comparison with the ECMWF data or other General circulation models (GCM).

V. CONCLUSION

In this paper, we summarized and investigated three different methods to calculate Refractive index structure constant, $C_n^2$ and consequently the Seeing parameter. Three models have been studied and compared. Masciadri and Dewan model are intended to the free atmosphere only. In other words, they are applicable above the boundary layer therefore there is a need to estimate the atmospheric boundary layer height to calculate the total seeing (boundary layer + free atmosphere).

Consequently, we illustrated a three-day campaign to measure the astronomical seeing with the DIMM in Changchun from July 5th July to 7th July 2016. Furthermore, to validate the Masciadri model, we have shown a comparison of ECMWF forecast performed by [6] and ECMWF reanalysis simulated by Matlab which shows both trends follows a fairly similar pattern. The differences in magnitude in both graphs (Fig. 6, Fig. 7) can be related to the chosen degree grid which is 0.25 in our simulation and 0.3 in [6] resulting in different Altitude above see level or the product type which has been selected reanalysis as a input whereas [6] has selected forecast.

REFERENCES