Applicability of Michaelis–Menten model for signal strength estimation in wireless networks

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Abstract – **In modern times the appliance of wireless technologies is enhanced with the rapid and widespread usage of Internet of Things and Smart technologies. The methods for accurate estimation of the Received Signal Strength Indicator (RSSI) become increasingly important. This paper presents the research on the applicability of the Michaelis–Menten enzyme kinetics model for RSSI estimation.**

Keywords – **Bluetooth Low Energy (BLE), RSSI estimation, indoor propagation loss, Michaelis–Menten model**

I. Introduction

In modern times a variety of wireless technologies have floated the market. The increasing deployment of wireless communication technologies is enhanced with the rapid and widespread appliance of Internet of Things (IoT) and smart technologies systems and environments (smart city, smart agriculture, smart grid, etc.). Considering the extensive usage of those technologies, the methods for accurate estimation of Received Signal Strength Indicator (RSSI) become increasingly important both in indoor and outdoor environments. The accurate signal strength estimation can be very helpful e.g. for planning wireless sensor nodes deployment in indoor wireless sensor networks (WSN) or IoT systems in the urban and suburban areas.

Although, there is a variety of existing indoor propagation models such as the ITU Indoor Path Loss or Log-distance Path Loss Model, the applicability of other models should be analyzed as well. This paper is focused on the applicability of the Michaelis–Menten (MM) equation commonly used as a model for enzymatic reactions. The Michaelis–Menten model is evaluated in this research as a model for RSSI estimation of indoor based Bluetooth Low Energy (BLE) devices.

This paper discusses the applicability of the MM model with the comparison of its accuracy with the two most popular indoor propagation loss models such as ITU and Log-Distance. This paper is structured as follows. After the Introduction, the short description of Michaelis–Menten is given, followed by the short description of ITU and Log-Distance Propagation Loss models. In the next section, a brief explanation of the experiment and data sets is given. The Results and the concluding remarks are given in the last two sections of the paper.

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II. Michaelis–Menten (MM) equation

Michaelis–Menten (MM) equation is a commonly used model for enzymatic reactions [1]. This model plays an important role in pharmacokinetics, from a theoretical as well as a computational point of view [2]. The application of the model is various, e.g. Michaelis–Menten kinetics is one of the best-known models of enzyme kinetics in vitro drug elimination or drug-drug interaction experiments [3] and it is shown in Eq. (1):

$$V = \frac{V_{max} \times [S]}{K_m + [S]} \qquad (1)$$

The Michaelis–Menten equation (MM equation) consists of two parameters, the maximum reaction rate (V_{max}) and the Michaelis constant (K_m) describing the rate of enzymatic reactions by relating reaction rate (V) to the concentration of a substrate ($[S]$) [3]. However, this model requires at least a couple (e.g., eight or more) of measurements at different substrate concentrations to determine kinetic parameters [4].

The two most commonly used methods for determining the parameters of the MM equation are the Lineweaver-Burk plot and the Eadie-Hofstee plot. Those methods are linearization methods that transform the original nonlinear MM equation into a linear one, and the data is then fit by a linear regression, which can be displayed as a straight line in a 2-dimensional graph [3]. In this paper, five methods for parameter calculations are used: Lineweaver-Burk [5], Hanes or Woolf, Eadie-Hofstee, Hyperbolic, and. Eisenthal-C.Bowden [6, 7].

III. Indoor propagation loss models

The indoor propagation loss models can be divided into two main groups: site-specific and site-general models [8]. Site-specific models are very accurate but are closely connected with the accuracy of the model of an indoor environment. They can give accurate results only with the accurate modeling of indoor spaces with parameters such as wall/separation materials and thickness, furniture, and other obstacles deployment. So, accurate site-specific models can be very difficult and expensive to make.

The characteristics of site-general models are their wider applicability with significantly less accuracy. The low accuracy of the model can be the problem. The two most popular site-general models will be described.

A. ITU Propagation Loss model

ITU Propagation Loss model [8, 9] is shown in Eq. (2):

$$L = 20 \cdot log_{10}f + N \cdot log_{10}d + P_f(n) - 28 \qquad (2)$$

The Eq. (2) parameters are as follows: L is the total path loss in decibels (dB), f is the frequency of transmission in megahertz (MHz), d is the distance in meters (m), N is the distance power loss coefficient, n is the number of floors between the transmitter and receiver and $P_f(n)$ is the floor loss penetration factor. Since the experiment used in this research is performed on a single floor the equation may be reduced to:

$$L = 20 \cdot log_{10}f + N \cdot log_{10}d - 28 \qquad (3)$$

Since Bluetooth Low Energy (BLE) is used as a wireless technology the value for frequency f is 2,400MHz, and the value for distance depends on the location of the wireless node and it is given in Table I. Parameter N is recommended with [8] and other sources and can have values 22 for commercial, 28 for residential and 30 for office environments at the frequencies 1.8–2GHz. However, in this research value for N is calculated to fit the dataset presented in Table I because of greater accuracy. The calculation is made with the usage of GNU Octave and *lsqnonlin* function for solving nonlinear least-squares (nonlinear data-fitting) problems. The calculated fitted value of N is 46.

The expected RSSI value at the receiving side (R_x) can be calculated with the following formula:

$$P_L = G_{tx} + G_{rx} - L_{tx} - L_{rx} - L \qquad (4)$$

The parameters represent a gain of the receiver (G_{rx}) and transmitter (G_{tx}) antenna, and losses in cables and connectors at the receiver (L_{rx}) and transmitter (L_{tx}) side all in decibels (dB). The parameter L is path loss calculated with the ITU propagation loss model presented in Eq. (3). Since in this experiment the equipment with low values for gains and losses on both sides is used, the formula is reduced to:

$$P_L = -L \qquad (5)$$

B. Log-Distance Propagation Model

Log-Distance Propagation Loss model [8, 10, 11] is shown in Eq. (6):

$$L = PL_0 + 10 \cdot \gamma \cdot log_{10}\left(\frac{d}{d_0}\right) + X_s \qquad (6)$$

The Eq. (6) parameters are as follows: PL_0 is the path loss at the reference distance d_0 in decibel (dB), d is the length of the path, d_0 is the reference distance, usually 1 km (or 1 mile) for large cells or outdoor environments and 1 m to 10 m for microcells or indoor environments, γ is the path loss exponent and X_s is a normal (or Gaussian) random variable with zero mean, reflecting the attenuation (in decibels) caused by flat fading. The calculation of X_s parameter is shown in Eq. (7):

$$X_s = \delta \cdot z \qquad (7)$$

The value of the PL_0 can be calculated using Friis free-space propagation loss formula presented in Eq. (8) with a reference distance of 1 m or by measuring the RSSI values with the devices used in the experiment at reference distance (1m). For the experiment in this research, the free-space loss propagation formula is used.

$$PL_o = 20 \cdot log_{10}(d) + 20 \cdot log_{10}(f) - 27.55 \qquad (8)$$

If parameters d and f are given in meters and megahertz, respectively, the constant used in the formula has a value of −27.55. In other cases, the constant has different values. With the usage of Eq. (8) the calculated value of PL_0 parameter is 40.054 dB.

The values of γ and δ are proposed in numerous literature sources [8, 10. 11] and they are based on empirical results. The application of any of the proposed values will not give accurate results. The reason is that majority of proposed values are made in the 914 MHz band, mainly in various industrial environments. To achieve the maximum accuracy of Eq. (7), the model is fitted with experimental data set, for a combination of most suitable γ and δ values. To calculate the value of δ with Eq. (8) the value of 1.645 was assigned to parameter z, according to [8]. The fitting resulted in the following calculated values of δ=16 and γ=2. The formula of the model can be reduced as in Eq. (5).

IV. Experiment and Data Set

The experiment is explained in more detail in [12, 13, 14]. The 5 fixed location BLE nodes are used in the experiment. BLE devices used for the experiment are built on Arduino UNO Rev 3, well known open-source hardware platform. For the BLE connectivity, the low-cost BLE communication modules (AT-09) are used. The RSSI is measured with the smartphone Android application on multiple locations. The multiple locations and the signal path between BLE and measuring devices are also explained in detail in [12]. The results of measurements for a total of 40 locations are given. In Table I the measurement results are presented with a location number, the distance from the measuring device, and RSSI in dBm.

TABLE I
PAGE LAYOUT DESCRIPTION

Location	Distance [m]	RSSI [dBm]	Location	Distance [m]	RSSI [dBm]
1	14.93	-93.12	21	16.09	-88.78
2	19.02	-92.69	22	11.8	-88.33
3	16.15	-92.26	23	13.71	-88.11
4	15.79	-92.05	24	8.88	-87.37
5	13.43	-91.32	25	13.97	-86.36
6	16.42	-91.05	26	12.49	-86.34
7	14.28	-90.66	27	8.27	-85.10
8	22.57	-90.45	28	8.8	-85.04
9	9.08	-90.36	29	6.36	-84.21
10	19.21	-90.13	30	6.45	-82.91
11	11.43	-89.95	31	8.17	-81.74
12	15.22	-89.94	32	3.59	-81.61
13	12.01	-89.92	33	4	-81.16
14	7.43	-89.65	34	4.75	-80.86
15	12.53	-89.35	35	4.3	-79.19
16	13.64	-89.11	36	6.63	-79.11
17	15.65	-89.06	37	6.43	-78.53
18	17.86	-89.04	38	8.23	-77.62
19	11.77	-88.95	39	2.93	-73.68
20	7.2	-88.80	40	4.65	-71.13

A. Michaelis–Menten parameter determination

As it is written in the previous section, there are several methods for determining the MM equation parameters. In this research five methods are used (Lineweaver-Burk, Hanes or Woolf, Eadie-Hofstee, Hyperbolic and. Eisenthal-C.Bowden). The third-party script written in Python is used for calculations [15]. This script is written in Python 3 programming language [16, 17] with following numeric libraries: Numpy [18] (designed for numerical computing with powerful numerical arrays objects), Scipy [19, 20] (designed for optimization, regression, interpolation, etc) and Matplotlib (designed for 2-D visualization and plotting).

The results of parameter determination are shown in Table II. V_{max} represents the limiting rate, K_m is Michaelis constant, *SE_V* is the standard error of the limiting rate and SE_K_m is the standard error of the Michaelis constant.

TABLE II
PAGE LAYOUT DESCRIPTION

Method	V_{max}	SE_V	Km	SE_Km
Lineweaver-Burk	-94.625	1.297	0.863	0.103
Hanes or Woolf	-95.594	1.066	0.967	0.136
Eadie-Hofstee	-94.180	1.198	0.804	0.109
Hyperbolic	-94.980	1.198	0.887	0.116
Eisenthal-C.Bowden	-94.819	nan	0.799	nan

With the calculated values of V_{max} and K_m five variations of Michaelis–Menten equation are used in further research as it is shown in Eq. (9). The concentration of a substrate *[S]* parameter from the original formula is replaced with the distance.

$$V = \frac{V_{max} \times d}{K_m + d} \tag{9}$$

V. RESULTS

The comparison of the five variants of the MM model, ITU, and Log-Distance models is made with the Root Mean Square Error (RMSE). RMSE for seven models is given in Table III. According to the results, the most accurate is the Log-Distance model with the RMSE value 2.9883. Very close to the Log-Distance are four MM models with RMSE value ranging from 3.01 to 3.04, with the Hyperbolic as the most accurate MM model variant.

TABLE III
PAGE LAYOUT DESCRIPTION

Model	RMSE [dBm]
MM Lineweaver-Burk	3.0240
MM Hanes or Woolf	3.0397
MM Eadie-Hofstee	3.0403
MM Hyperbolic	3.0197
MM Eisenthal-C.Bowden	3.1008
ITU	6.5730
Log-Distance	2.9883

The graphical fitting of five Michaelis–Menten models is shown in Fig. 1, where model calculated curves, are presented with the measurement results during the experiment. In the following figure (Fig. 2) the MM models are presented together with Log-Distance (red line), and ITU propagation loss models (blue line). The similarity between Log-Distance and MM models is visible in this figure.

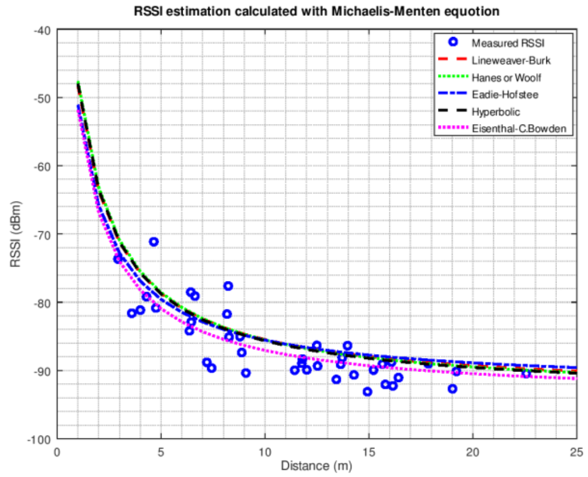

Fig. 1. Comparison of Michaelis–Menten models

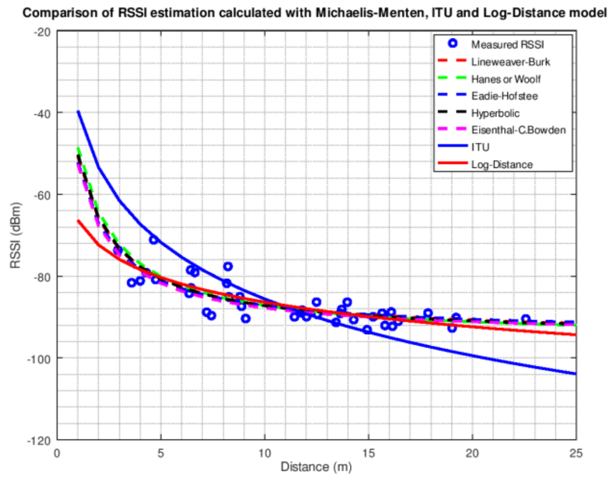

Fig. 2. Comparison of Michaelis–Menten with ITU and Log-Distance models

From the results, it can be concluded that although the Log-Distance model is the most accurate for the given data set, the Michaelis–Menten enzyme kinetics model is also very applicable for the path loss calculation or RSSI estimation. All five calculated variants of the Michaelis–Menten model are only slightly less accurate than the Log-Distance propagation loss model.

VI. CONCLUSION

In this paper, the applicability of the Michaelis–Menten model for RSSI estimation is analyzed. Michaelis–Menten (MM) equation is a commonly used model for enzymatic reactions and can be used in pharmacokinetics, vitro drug elimination or drug-drug interaction experiments, etc. The experiment is based on data set collected in indoor environments with Bluetooth Low Energy devices. Although the standard Log-Distance model has the greatest accuracy, the MM model shows satisfying RSSI estimation capabilities.

The presented results and accuracy of the Michaelis–Menten models justify further research in its applicability for RSSI estimation. This research should be expanded to other indoor wireless technologies such as ZigBee and Wi-Fi, or outdoor technologies such as LoRa, NB-IoT, etc.

The potential application of the Michaelis–Menten model can be very interesting for long-range wireless technologies in urban environments in the scenarios where numerous existing path loss propagations models are not able to give accurate calculations. These scenarios can include fixed location wireless node, as well as the mobile ones.

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