# A Hybrid 2D/3D Multilevel Green's Function Interpolation Method for Microstrip Antenna Array

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*Abstract* - A new implementation of the multilevel Green's function interpolation method (MLGFIM) is proposed to simulate the microstrip antenna array. A hybrid 2D/3D (H2D/3D) multilevel partitioning is devised for the specific problem, from which a quad-tree will be constructed. The radial basis function (RBF) interpolation method is adopted to perform the peer-level and lower-to-upper-level interpolation of the multilayered Green's function. The fast Fourier transform (FFT) is applied to accelerate the translation procedure at proper levels. With the proposed method, the full-wave simulation of the printed antenna array with hundreds of elements is realized on a personal computer.

*Index Terms* — microstrip antenna array, multilayered Green's function interpolation method, FFT.

#### I. INTRODUCTION

The microstrip antenna array has been used in a wide variety of applications [1]. But the full-wave simulation of very large microstrip structures is still a challenging problem. Application of the moment method (MoM) to the mixed potential electric field integral equation (MPIE) is a typical method to analyze the microstrip structures [2]-[3]. But in this MPIE-MoM method, a dense matrix equation will be generated. When this matrix equation is solved by an iterative method, the complexity will be  $O(N^2)$  for conventional matrix-vector multiplication, which is prohibitively high for large-scale problems. Recently, a fast algorithm, viz. the multilevel Green's function interpolation method [4]-[5], has been proposed to efficiently solve the electromagnetic problems. Different to other methods, the MLGFIM relies on the interpolation of the Green's function, which makes the algorithm a kernel-independent method. Theoretically the MLGFIM is more suitable to solve the multilayered problem. The actual performance will be investigated in this paper.

#### II. H2D/3D MLGFIM-FFT ALGORITHM

A typical microstrip antenna array is shown in Fig. 1, which contains both 2D objects (patch) and 3D objects (via). At first the 2D multilevel partitioning is applied according to the 2D boundary of the antenna array. A quad-tree is constructed as shown in Fig. 2. The microstrip structure may have more than one dielectric layer. The leafy node (node at the lowest level) will contain the patches printed at different interface and the vias drilled at different layer dividedly. The



Fig. 2. Quad-tree constructed from the multilevel partition and the crosslevel and peer-level translation.

patch

translation

via

2D interpolation points will be assigned to the patch while the 3D interpolation points to the via. The interactions between the patches are easily processed by the traditional 2D interpolation. Usually there are not so many vias, it is not quite efficient to perform the lower-to-upper-level interpolation for them. The interpolation for the via is just performed at the lowest level. To calculate the interaction between the via and the patch, a cross-level translation procedure can be devised. For example, in Fig. 2, the interaction of the patch in the node at level 2 is translated to the via in the node at level 3 directly.

The interpolation is mainly performed to the Green's function in the far field, which is usually dominated by the surface wave. The surface wave has an asymptotic form as  $e^{-jk_p\rho} / \sqrt{\rho}$ , where  $k_p$  denotes the location of the poles. For the general multilayered problem, the Green's function will be different depending on the vertical locations of the field and source points. And the multilayered Green's function includes a dyadic Green's function plus a scalar Green's function. The far field of these Green's functions may be dominated by the surface wave modes with different wavenumbers. Usually the interpolation parameters can be

In the algorithm, the translation procedure occupies most of the computation time. When the interpolation points are uniformly distributed, the translation matrix will be a Toeplitz matrix and then the procedure can be accelerated by the FFT. The process is mathematically described as

$$\overline{B}_{m_l,k,l} = \sum_{N_{m_l,l} \in N_{m_l,l}} \overline{\overline{G}}_{m_l,l;n_l,l} \cdot \overline{S}_{n_l,l} = \sum_{N_{m_l,l} \in N_{m_l,l}} \overline{G}_{m_l,l;n_l,l} \otimes \overline{S}_{n_l,l} 
= FFF^{-1} \left\{ \sum_{N_{m_l,l} \in N_{m_l,l}} FFT\left\{\overline{G}_{m_l,l;n_l,l}\right\} \cdot FFT\left\{\overline{S}_{n_l,l}\right\} \right\}$$
(1)

where  $\overline{\overline{G}}_{m_i,l:n_i,l}$  is the translation matrix and the definition can be found from [4] and [5]. Before the FFT is applied to the vector  $\overline{S}_{n_i,l}$ , it should be properly padded with zeroes according to the relative position of the node and its interaction node.

## **III. NUMERICAL RESULTS**

In this section, a 32×16 printed dipole antenna array is simulated with the proposed method. In Fig. 3(a), the detailed parameters are W=8.45mm, L=7.25mm, a=16mm, b=23mm. Two patches are printed on the two sides of the substrate to form a dipole antenna. A metal plate is added as a reflector. The frequency is set to 12.5GHz. Then the antenna array is extended to 32×16. The simulated radiation patterns in both E and H-plane are compared with the measured results in Fig. 4. From the data, reasonable agreement can be observed, especially at the main beam and the first few side lobes. The number of unknowns is 217,875. A 7-level tree is constructed with the number of interpolation points to be  $K_6=5$ ,  $K_5=6$ ,  $K_4=8$ ,  $K_3=14$ , and  $K_2=19$ . The FFT is applied to level 3 and 2. The computation time per iteration is about 1.98 seconds. The total solution time is no more than 2 hours on a personal computer. To accelerate the convergence rate of the iteration method, the sparse approximate inverse (SAI) preconditioner is also applied.

## IV. CONCLUSION

The MLGFIM is first implemented to solve the multilayered problem. Some new techniques are applied to improve the efficiency of the algorithm further, i.e. the H2D/3D partitioning, the cross-level translation and the FFT. The printed antenna array with hundreds of elements is efficiently solved and the simulated results achieved reasonable agreement with the measured results.

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Fig. 3. (a) Geometric of a  $4\times 2$  printed dipole array. (b) Multilayered Structure. (c)  $32\times 16$  array. (d) Antenna under test.



Fig. 4. The radiation patterns in (a) E-plane and (b) H-Plane.

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