A Study of Filter-Bank Processing of ECG Signal for Diagnostic Applications

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Abstract—**Processing and classification of electrocardiogram (ECG) recordings are some of the most challenging fields of biomedical signal processing owing to the fact that ECG signals commonly exhibit complex temporal morphology and contain numerous artifacts of data collection process. This paper presents study of filter bank based processing of ECG signal for the purpose of atrial fibrillation diagnostics. The examined diagnostic system relies on the statistical description of signal's energy distribution in the individual filter banks as a feature vector for the considered classification algorithms, namely Support Vector Machines (SVM) and Artificial Neural Networks (ANN). The considered statistical measures include mean, variance, skewness and kurtosis. The effect of filter bank number on the ability to differentiate between atrial fibrillation and healthy ECG signals is examined and the diagnostic relevance of each statistical parameter is also ascertained. A systematic study of diagnostic accuracy is imposed on the choice of feature vector, whereby various combinations of filter banks and the statistical measures are evaluated. The results show that the an optimal selection of sub-bands in conjunction with the appropriate choice of statistical descriptors can lead to a considerable reduction in the feature vector size without adverse effects on classification accuracy levels.**

Keywords-biomedical signal processing; ECG diagnostics; filter-bank processing; support vector machines; artificial neural networks;

I. INTRODUCTION

Electrocardiogram (ECG) is a time-varying signal corresponding to the electrical activity of cardiac muscle and is readily obtained as a measurement of the potential difference between electrodes placed on a surface of the skin.

Even after decades of research, ECG signal analysis remains one of the most challenging undertakings in modern biomedical signal processing. ECG signal can be described as a non-stationary, quasi-periodic waveform that frequently exhibits complex non-linear temporal morphology and to various degrees, contains artifacts of data collection process, such as baseline wander (caused by respiration) and high-frequency electromyography noise that arises from muscle activity.

Over the years, a number of approaches to ECG signal analysis have been proposed, including Gauss curve modeling via nonlinear optimization algorithms [1], Hilbert Transform based modeling [2], Characteristic Waveform modeling [3], Mealy and Moore automata model [4], threshold methods [5], wavelet transform and principal component analysis [6], Archetypical Analysis [7], Hidden Markov modeling [8], [9], and Filter Bank approach [10], [11].

By decomposing signal into various frequency sub-bands, Filter Bank (FB) approach enables independent processing of temporal and spectral domains. Filter Bank signal processing methods have been successfully employed on range of ECG applications, including beat detection; beat classification, ECG enhancement and noise alert [10-13].

This paper presents a study of filter bank based processing of ECG signal for the purpose of atrial fibrillation diagnostics. The proposed system employs the filter bank approach to decompose an ECG signal into a number uniformly distributed frequencies intervals to derive the energy distribution in the individual filter banks. A set of statistical measures are used to describe the energy distribution in each sub-band and to form a feature vector for the ECG signal. Support Vector Machines (SVM) and Artificial Neural Networks (ANN) are considered as the principal algorithms for the classification of the ECG feature vectors. In this paper, the effect of filter bank number on the diagnostic accuracy is examined. The diagnostic relevance of each statistical parameter and individual sub-bands is studied. Furthermore, in a systematic study of feature vector candidates, an attempt is made to reduce the feature vector size without significantly sacrificing diagnostic accuracy. The feature vector size reduction is based on the optimal selection of sub-bands and statistical descriptors.

The remainder of this paper is organized as follows. In Sections II, a review of considered classification algorithms, namely ANN and SVM is provided. The proposed system for classification of the ECG signal is presented in Section III. Section IV presents and discusses the experimental results. Section V concludes the paper.

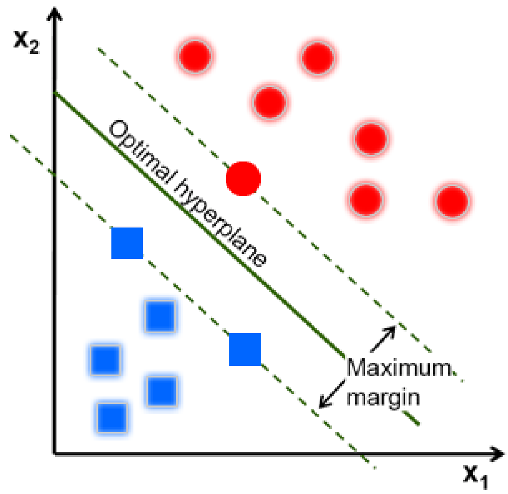

Figure 1. SVM hyperplane

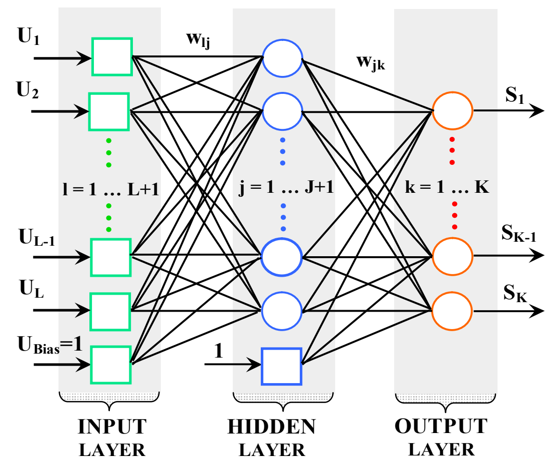

Figure 2. ANN Structure

II. Classification Algorithms

In this section, a review of the considered classification algorithms is presented.

A. Support Vector Machine

The basic principle behind SVM is to map the input vector to a higher dimensional space and to construct a hyperplane to classify the training data [14] [15], as illustrated in Fig. 1. In order to maximize the distance between the two classes and to get the optimal hyperplane, two parallel hyperplanes are constructed on each side of the separating hyperplane. The larger distance between these two hyperplanes means the better classification. Based on the sample class, parallel hyperplanes are constructed in form of:

$$\mathbf{wx} + b = \pm 1 \tag{1}$$

Where $\mathbf{w}$ represents a p-dimensional vector and b is the offset parameter that enables increasing the separating hyperplane margin. All the points $\boldsymbol{x}$ along the hyperplane, represent the supporting vectors of the hyperplane. The optimal separating hyperplane can be found by using Lagrange multipliers, as in:

$$L_{(w,b,a)} = \frac{1}{2}\|\mathbf{w}\| - \sum_{i}^{n} a_i (y_i(\mathbf{w}^T \mathbf{x}_i + b) - 1) \tag{2}$$

Here, a_i represents a Lagrange's multiplier. Lagrange multiplier are minimized with respect to $\boldsymbol{w}$ and b and maximized with respect to a_i for (a_i>0). Primal or dual form can be used for solving this problem. Under the SVM framework, nonlinearity of classification process is addressed via kernel functions. In this paper, it is assumed that linear separation of features is not possible and thus, SVM is employed in conjunction with the Radial Basis Function 'kernel' method, with the following mapping being applied on given feature space:

$$\mathbf{K}(\mathbf{x}_i, \mathbf{x}_j) = \exp(-\gamma \|\mathbf{x}_i, \mathbf{x}_j\|^2) \tag{3}$$

Here, γ denotes the kernel parameter. The RBF kernel maps samples nonlinearly and compared to polynomial kernel it has less hyperparameters and numerical difficulties.

B. Artificial Neural Networks

Fig. 2 presents a schematic diagram of a general artificial neural network architecture consisting of L inputs, J neurons in a hidden layer and K outputs. Each neuron computes the weighted sum of its inputs and subsequently, passes the output through a activation function to obtain a neuron response. The most important features of neural network classifier are related to its architecture, the choice of activation function and the choice of training method. The informal experiments have shown that a 6-neuron hidden layer structure in conjunction with a sigmoid activation function and the Levenberg-Marquardt training algorithm constitute the optimal ANN design under the proposed classification scheme.

The *Levenberg-Marquardt* method is, effectively, a hybrid algorithm that combines the advantages of the second order Gauss-Newton method and the first order Steepest Descent [16]. The *Levenberg-Marquardt* learning method adopts a general second order learning formulation:

$$\mathbf{w}_{m+1} = \mathbf{w}_m + \Delta\mathbf{w}_m = \mathbf{w}_m - \mathbf{H}^{-1}\mathbf{g}_m, \quad m \geq 0 \tag{4}$$

Here, $\mathbf{w_m}$, $\mathbf{g}_m$ denote the ANN weight values and the error derivative at iteration step m, respectively. However, in *Levenberg-Marquardt* learning method, Hessian matrix $\mathbf{H}$, is modified to include a conditioning term $e^{\lambda}\mathbf{I}$, which ensures that the approximated Hessian matrix is readily invertible. The *Levenberg-Marquardt* weight update takes the following form.

$$\mathbf{w}_{m+1} = \mathbf{w}_m - \left(\mathbf{J}_m^{\mathbf{T}}\mathbf{J}_m + e^{\lambda}\mathbf{I}\right)^{-1}\mathbf{J}_m\mathbf{e}_m, \quad m \geq 0 \tag{5}$$

Here, **I** represents the identity matrix, e denotes a natural exponential, while λ corresponds to an automatically evaluated constant that ensures the stability of solution. The Jacobian matrix **J** represents the first order derivative and can be expressed as:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial e_{11}}{\partial w_1} & \frac{\partial e_{12}}{\partial w_1} & \cdots & \frac{\partial e_{1M}}{\partial w_1} & \cdots & \frac{\partial e_{p1}}{\partial w_1} & \frac{\partial e_{p2}}{\partial w_1} & \cdots & \frac{\partial e_{pM}}{\partial w_1} \\ \frac{\partial e_{11}}{\partial w_2} & \frac{\partial e_{12}}{\partial w_2} & \cdots & \frac{\partial e_{1M}}{\partial w_2} & \cdots & \frac{\partial e_{p1}}{\partial w_2} & \frac{\partial e_{p2}}{\partial w_2} & \cdots & \frac{\partial e_{pM}}{\partial w_2} \\ \vdots & \vdots & \cdots & \vdots & \cdots\vdots & \vdots & \vdots & \cdots & \vdots \\ \frac{\partial e_{11}}{\partial w_N} & \frac{\partial e_{12}}{\partial w_N} & \cdots & \frac{\partial e_{1M}}{\partial w_N} & \cdots & \frac{\partial e_{p1}}{\partial w_N} & \frac{\partial e_{p2}}{\partial w_N} & \cdots & \frac{\partial e_{pM}}{\partial w_N} \end{bmatrix}^T \tag{6}$$

Here, $e_{p,i}$ represents training error for i^{th} output for the p^{th} training pattern.

III. ECG Classification System

Fig. 3 presents a block diagram of the proposed system for atrial fibrillation diagnostics based on classification of features derived from spectral analysis of ECG signal. The classification feature vectors are derived in the following manner. An FIR Filter Bank, is used to decompose the ECG signal into N uniformly distributed frequencies intervals in the range from 0Hz to 62.5 Hz. The upper limit of the considered frequency range is selected as the value that is only marginally lower than the maximum frequency represented by a signal with the sampling rate of 128Hz. This sampling frequency value corresponds to the lowest sampling frequency in the considered evaluation database. The low-pass, high-pass and the band-pass filters in the filter bank are designed as finite impulse response filters with 60 coefficients. All filters in the filter-bank exhibit linear phase property and have the identical group delay. For each sub-band, energy of the ECG signal is evaluated over the duration of 5 seconds, every 5 seconds. Subsequently, the energy in each sub-band is scaled to represent the percentage of total energy found in the range of 0-62.5 Hz. The energy values are collected for each sub-band over the preset time duration. Subsequently, four statistical parameters, namely mean, variance, skewness and kurtosis are used to independently describe the energy distribution in each sub-band. These parameters are concatenated to form a 4xN long ECG feature vector. When a database of feature vectors representing the healthy and pathological ECG signals is formed, the feature vectors are normalized using z-score normalization method. The normalized feature vectors are employed for training of testing of a classification algorithm. In this paper, support vector machine and artificial neural network are considered as candidates for ECG classification.

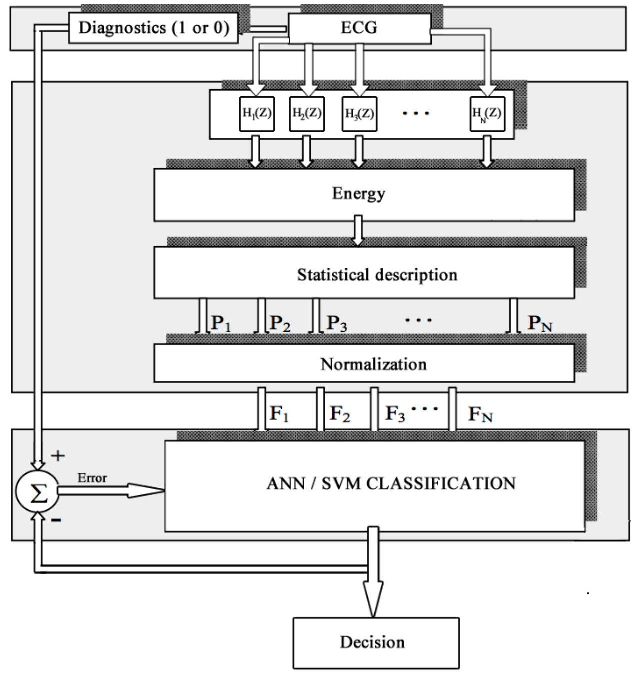

Figure 3. A proposed system for ECG diagnostics

IV. Results and Discussion

The performance of the proposed ECG classification method is evaluated on a database of 76 one-hour-long examples of ECG signal waveforms [23], including 38 examples of healthy control subjects from the Fantasia database, sampled at 250 Hz, 38 examples of Atrial Fibrillation from the Long Term AF Database, sampled at 128 Hz. Prior to the experiment all the data are re-sampled at 128 Hz and the baseline wander is removed from the signal with the linear phase, high-pass with the cutoff at 0.8 Hz.

The dataset is randomly divided into training (60% of available data) and testing data (40% of available data). The classification performance is evaluated in terms of sensitivity (Se), specificity (Sp) and accuracy (Acc), defined in (7), (8) and (9), respectively.

$$Se = TP/(TP + FN) \tag{7}$$

$$Sp = TN/(TN + FP) \tag{8}$$

$$Acc = TP + TN/(TP + TN + FP + FN) \tag{9}$$

Here, TP and TN denote the number of true positive and true negative cases, respectively, while FP and FN denote the number of false positive and false negative cases, respectively. The classification accuracy results are reported as the median of classification accuracy values collected from an ensemble of 50 independent training and testing scenarios.

In the first experiment, while using the same set of statistical measures for characterization of sub-band energy distribution, the effect of filter bank number on the ability to differentiate between atrial fibrillation and healthy ECG signals is examined. The number of filter banks is varied in the range between 4 to 12 filters. In each case, the statistical description of a sub-band energy distribution is characterized by all four of the considered statistical measures, namely mean, variance, skewness and kurtosis. The results are reported in Table I. The results show that SVM outperforms ANN classification algorithm in terms of overall classification accuracy. Thus, for SVM, the classification performance is examined in more detail and the sensitivity and specificity results are also reported. The results from Table I indicate that the classification performance is strongly affected by the choice of filter bank number. The highest SVM and ANN classification accuracy levels of 87.1% and 84.6%, respectively, are attained when the ECG signal is decomposed into 6 uniformly distributed frequencies intervals in the range from 0-62.5 Hz. As the number of filter banks is increased or lowered from the optimal value, the quality of classification performance clearly diminishes. The sensitivity and specificity results closely follow the classification accuracy levels and are similarly affected by the filter bank number. When all four statistical measures are employed, the size of the feature vector corresponding to the optimal number of filter banks is 24.

In the next section, the relevance of individual sub-bands for the quality of diagnostic performance is evaluated using a 6 filter-bank system. Feature vectors are derived using the information from a single sub-band only. Again, all four statistical measures are employed to characterize the energy distribution in individual sub-bands. The classification accuracy results are reported in Table II, for each of 6 sub-bands. In this experiment, SVM and ANN offered a very similar performance. The results show that compared to the remaining sub bands, the features based on the lowest frequency band offer the smallest capacity to successfully differentiate between atrial fibrillation and healthy ECG signals. The results would indicate that the 3rd and 2nd sub-bands have the highest diagnostic relevance, closely followed by 5th, 6th and 4th band.

The next experiment aims to address the effect of using information from multiple sub-bands in the feature vector derivation on the quality of atrial fibrillation diagnostics. In fact, we aim to ascertain the specific combination of sub-bands that would minimize the feature vector size, without the undesired decrease in the classification accuracy level. Although, exhaustive evaluation of sub-band combinations is performed, only the most relevant results are reported in Table III. The results show that a direct combination of sub-bands with the highest diagnostic relevance, as evaluated in the previous experiment, does not necessarily result in the expected increase in the accuracy rate. Individually, 2nd and 3rd bands have the highest diagnostic relevance, but the combined set of features from the two bands does not lead to an accuracy level higher than that attained on the 3rd sub-band alone (SVM classification accuracy = 83.9%). A similar observation was made when the features from spectrally adjacent sub bands 5th and 6th were combined. This observable fact indicates that there exists a significant amount of information redundancy associated with the identification of atrial fibrillation between the individual sub-bands and especially, between the adjacent sub-bands. The sub-band pairing that produced the highest classification accuracy of 85.4%, involved the 3rd and 4th sub-bands and was associated with SVM classifier. In consideration of the minimum number of sub-bands required to reach the same accuracy level as when all the bands were used, it was found that a 12 parameter long feature vector, derived exclusively from the 3rd, 4th and 6th band, attains a maximum attainable accuracy rate of 87.1%. Thus, it can be concluded that through a careful selection of sub-bands, a 50% reduction in feature vector size can be attained, without sacrificing the accuracy rate of a classifier.

This study also considered the diagnostic relevance of each statistical parameter. When individual statistical measures are considered in isolation in the experiment involving all 6 sub-bands, the mean and variance of sub-band energy distribution are demonstrated to be the most effective in representing the heart condition relevant information, Table IV. When using mean value alone as a feature vector parameter a classification accuracy level of 83.9% is achieved for both SVM and ANN classification algorithm. Nevertheless, the results also establish that the skewness and kurtosis values are very relevant measures for the ECG diagnostics. When used in combination on all six sub-bands, the mean and variance of sub-band energy distribution form a 12 parameter long feature vector that can result in a maximum attainable accuracy level of 87.1%. Thus, it can be concluded that the optimal selection of statistical measures for the sub-band energy distribution can yield up to 50% reduction in the feature vector size without any detrimental effects on the classification performance.

In the final experiment, an attempt is made to reduce the number of statistical measures characterizing the energy distributions in the 3rd, 4th and the 6th band, while still attaining the accuracy level of 87.1%. The results are reported in Table V. It was found that at least 3 statistical measures are necessary, namely mean, variance and kurtosis for the classifier accuracy level to remain unchanged. This corresponds to a 9 parameter long feature vector and the SVM classifier. The specificity and sensitivity results for the SVM classifier are also reported. It can be observed that they closely related to classification accuracy results.

The results show that through an optimal selection of sub-bands and statistical descriptors a 62.5% reduction in the feature vector size can be achieved, without reduction in the classification accuracy level - 87.1%. The ability to reduce the size of feature vector without detrimental effects on the classification performance is attributed to the redundancy in diagnostically relevant information among different sub-bands as well as among different statistical measures. The decrease in the dimensionality of pattern representation has implications in the classification speed increase and lowering of computational cost.

TABLE I. CLASSIFICATION ACCURACY VS FILTER BANK NUMBER

	Number of Filter Banks in the range 0-62.5 Hz						
	N=4	**N=5**	**N=6**	**N=7**	**N=8**	**N=10**	**N=12**
No. Features	16	20	24	28	32	40	48
ANN accuracy	81.9%	81.2%	84.6%	77.4%	77.5%	77.4%	77.4%
SVM accuracy	83.9%	83.9%	87.1%	80.6%	80.6%	83.9%	80.6%
SVM sensitivity	82.4%	84.2%	86.7%	84.6%	80.%	78.6%	80.0%
SVM specificity	85.7%	83.3%	87.5%	77.7%	81.3%	88.2%	81.2%

TABLE II. CLASSIFICATION ACCURACY FOR INDIVIDUAL SUBBANDS

6 Filter Banks	Sub-bands					
	S1	**S2**	**S3**	**S4**	**S5**	**S6**
Frequency range (Hz)	0-10.4	10.4-20.8	20.8-31.3	31.3-41.7	41.7-52.1	52.1-62.5
No. Features	4	4	4	4	4	4
ANN accuracy	61.3%	80.6%	83.9%	61.3%	74.2%	74.2%
SVM accuracy	54.8%	80.6%	83.9%	64.5%	74.2%	71.0%

TABLE III. OTIMIZING SUBBAND SELECTION FOR CLASSIFICATION

6 Filter Banks	Sub-bands					
	S3+S2	**S3+ S4**	**S3+S5**	**S3+S6**	**S2+S4**	**S3+S4+S6**
No. Features	8	8	8	8	8	12
ANN accuracy	80.6%	83.8%	70.9%	80.6%	80.6%	80.6%
SVM accuracy	83.9%	85.4%	80.6%	83.9%	83.9%	87.1%

TABLE IV. CLASSIFICATION ACCURACY VS. SIGNAL MEASSURES

6 Filter Banks	Measures constituting a feature vector						
	mean	**variance**	**skewness**	**Kurtosis**	**mean & variance**	**Mean & variance & skewness**	**all**
No. Features	6	6	6	6	12	18	24
ANN accuracy	83.9%	79.0%	77.4%	77.4%	83.8%	80.6%	84.6%
SVM accuracy	83.9%	80.6%	77.4%	77.4%	87.1%	87.1%	87.1%

V. CONCLUSION

This paper presents a systematic study on the use of feature vectors derived from the filter bank processing of ECG signal for the diagnostic applications of heart conditions, and in particular atrial fibrillation. Here, the considered ECG features are derived as a set of statistical descriptors (mean, variance, skewness and kurtosis) of the energy distribution in the individual filter banks.

TABLE V. OPTIMIZING THE SUBBAND SELECTION AND THE CHOICE OF FEATURE VECTOR MEASSURES

6 Filter Banks	Sub-bands			
	S3+S4+S6	**S3+S4+S6**	**S3+S4+S6**	**S3+S4+S6**
Meassures	**mean & variance**	**mean & variance & kurtosis**	**Mean & variance & skewness**	**all**
No. Features	6	9	9	12
ANN accuracy	83.9%	83.9%	80.6%	80.6%
SVM accuracy	83.9%	87.1%	83.9%	87.1%
SVM sensitivity	76.5%	86.7%	83.3%	81.3%
SVM specificity	92.9%	87.5%	84.6%	93.3%

In addition, both the Artificial Neural Networks and Support Vector Machines are considered as the ECG classification algorithms and the optimal choice is evaluated. The results have shown that the classification accuracy is strongly affected by the choice of filter bank number. When all four statistical measures were employed, the highest classification accuracy rate of 87.1% is attained when the ECG signal is decomposed into 6 uniformly distributed frequencies intervals in the range from 0-62.5 Hz. This accuracy level corresponds to a 24 parameter long feature vector and the SVM classifier.

The diagnostic relevance of individual sub-bands is evaluated for a 6-filterbank system. The results show that 3rd and 2nd band, followed by 5th, 6th and 4th band are the most relevant bands for atrial fibrillation diagnostics. The results of a more in-depth study show that a direct combination of sub-bands with the highest diagnostic relevance does not necessarily result in the expected increase in the accuracy rate.

In fact, the sub-band pairing that produced the highest classification accuracy of 85.4% (SVM classifier) is associated with the 3rd and 4th sub-bands. In an attempt to minimize the number of required sub-bands, while keeping the classification accuracy of level of 87.1%, a 12 parameter long feature vector is derived exclusively from the 3rd, 4th and 6th band.

In addition, the diagnostic relevance of each statistical parameter is ascertained. In the experiment involving all 6 sub-bands, when individual statistical measures are considered in isolation, the mean and variance of sub-band energy distribution are demonstrated to be the most effective in discriminating between atrial fibrillation and healthy ECG signals. In a study involving various combinations of statistical descriptors and the choice of sub-bands, it was found that a classification accuracy level of 87.1% can be reached using a 9 parameter long feature vector based on only three sub-bands and three statistical measures. The results show that through the careful selection of sub-bands and statistical descriptors, a 62.5% reduction in the feature vector size can be attained without any detrimental effects on the classification performance. This observable fact can be attributed to the redundancy in diagnostically relevant information among the different sub-bands as well as among the different statistical measures. The decrease in the dimensionality of pattern representation has implications in the classification speed increase and lowering of computational cost.

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