Analytical and Simulation Study on Different Magnetic Inclusion Structures

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I. INTRODUCTION

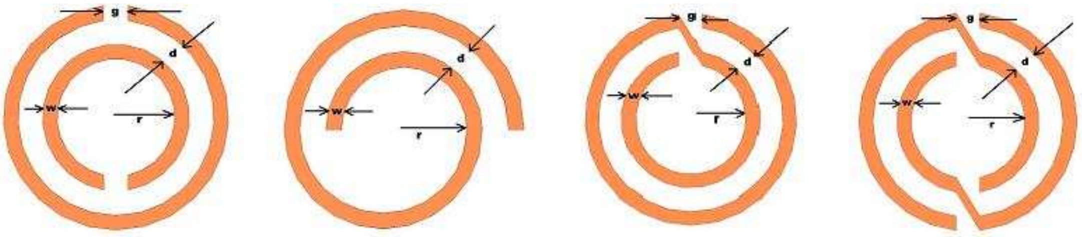
Left Handed Material (LHM) is an artificial material whose properties are determined by the structural arrangement of unit cell in a periodic array rather than the chemistry of the material it is made of. The possibility of negative effective permeability of a magnetic structure was first reported by Pendry et al [1]. To enhance the value of negative permeability, a number of inclusion was suggested by Roy et al[2] that resulted in the multiple split ring resonators (MSRR). They also made a comparative study of MSRR, SR(Spiral resonator), LR(Labyrinth resonator). Following the basic resonator structure J.D.Baena et al [3] proposed two structures: the so called two turn spiral-resonator (TTSR) and, non bi-anisotropic-spiral-resonator (NBSR). We derived the analytical expressions for the negative permeability of TTSR and NBSR structures which is not available in published literature. Our analytical results have been compared with commercially available FEM based simulation software(Ansys-HFSS). The approach is to establish the e.m.f. equation of the structure and from that we derive the effective relative permeability (μ_{reff}) of the particular magnetic inclusion structure and then do parameter extraction studies via simulation. The discussion shows that NBSR and TTSR eliminate the cross polarization effect and is thus superior candidate.

II. ANALYTICAL MODELING OF MAGNETIC INCLUSION STRUCTURE

Assume that magnetic field vector is parallel to the axes of the unit cell. The unit cell structures of MSRR, SR, TTSR, NBSR shown in Figure 1.The induced magnetic field in the structure is depend on incident magnetic field strength **H**, the induce current density **j**, volume fill factor $F = (\pi r^2/h^2)$ of the periodic structure, where r is the radius of the inner most ring and h, v, are the horizontal and vertical lattice constant. The intensity of time-varying magnetic field is constant (quasi-static case); the magnetic flux through the metallic ring does not change, resulting in zero induced electromotive force (*e.m.f*), for the MSRR [2] which is given by:

$$-\pi r^2 \left(N-1\right)^2 \mu_0 \frac{\partial}{\partial t}\left[H + j - \frac{\pi r^2}{h^2} j\right] - 2\pi r\left(N-1\right) j R_c v + \frac{v\int j dt}{(2\pi r/6)C_1} = 0 \qquad (1)$$

Where μ_0 is free space permeability, N is the number of metallic inclusions, R_c and C_1 are the resistance of the metallic strips and distributed capacitance between the strips per unit circumferential length given by, $C_1 = (\varepsilon_0/\pi)\ln[wN/d(N-1)]$, with ε_0 is the free space permittivity ,w is the strip width and d is the separation between the strips of the adjacent ring. Equation (1) has been derived assuming that length of the splits, gap length g; for all rings equals 60^{o} in the MSRR we are neglecting the gap capacitance. The effective relative permeability μ_{reff} in terms of the average induced magnetic field strength H_{av} and the average induced magnetic flux density B_{av} over a unit volume cell can be expressed



(a) **(b)** **(c)** **(d)**

Figure.1. Scematic representation of (a) MSRR (b) SR (c) TTSR (d) NBSR

as :

$$\mu_{reff} = B_{av} / \mu_0 H_{av} = H / H_{av} \tag{2}$$

Therefore, the expression for effective relative permeability of MSRR[2], with $\mathrm{i} = \sqrt{-1}$, is:

$$\mu_{reff} = 1 - \frac{\pi r^2 / h^2}{\left\{1 + [\mathrm{i}(2R_c v) / \omega r \mu_o (N-1)] - (3v) / [\pi^2 r^3 \mu_o \omega^2 C_1 (N-1)^2]\right\}} \tag{3}$$

Similarly for SR structure the effective relative permeability [2] is:

$$\mu_{reff} = 1 - \frac{\pi r^2 / h^2}{\left\{1 - [i(2R_c v) / \omega r \mu_0 (N-1)] - v / [2\pi^2 r^3 \mu_0 \omega^2 C_1 (N-1)^2]\right\}} \tag{4}$$

TTSR, NBSR structures, are combination of above two resonators MSRR and SR. The net *e.m.f* equation for TTSR has been derived by us as:

$$-\pi r^2 (N-1)^2 \mu_0 \frac{\partial}{\partial t}\left[H + j - \frac{\pi r^2}{h^2} j\right] - 2\pi r (N-1) j R_c v + \frac{v \int j dt}{2\pi r C_1 + N C_2} = 0 \tag{5}$$

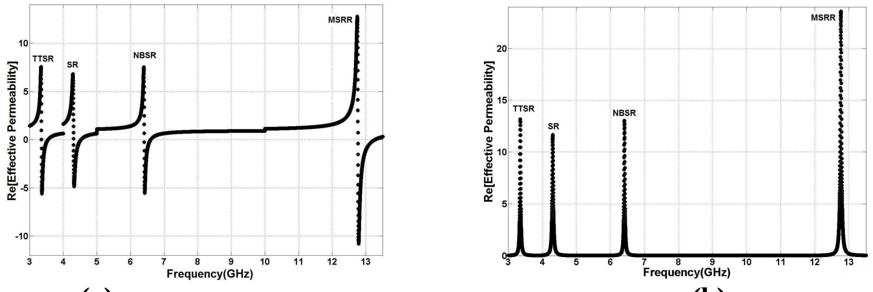
which leads to, effective relative permeability μ_{reff} for TTSR as;

$$\mu_{reff} = 1 - \frac{\pi r^2 / h^2}{\left\{1 + \mathrm{i}(2R_c v) / \omega r \mu_o (N-1) - (v) / \pi r^2 \mu_o \omega^2 (N-1)\left(2\pi r C_1 (N-1) + N C_2\right)\right\}} \tag{6}$$

Taking the account of equation (5) we can write the final expression of effective relative permeability (μ_{reff}) for NBSR as:

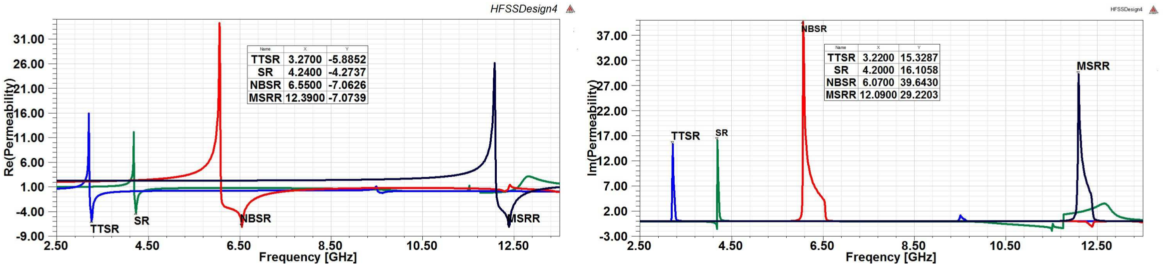
$$\mu_{reff} = 1 - \frac{\pi r^2 / h^2}{\left\{1 + \mathrm{i}(2R_c v) / \omega r \mu_o (N-1) - (v) / \pi r^2 \mu_o \omega^2 (N-1)\left(2\pi r C_1 (N-1) + \{N/2\} C_2\right)\right\}} \tag{7}$$

Where $C_2 = \varepsilon_0 w t_m / g$, is the gap capacitance. The gap capacitance C_2 (=1 for TTSR) and (= 2 for NBSR). The analytical results for all four MNG structure are plotted in the Figure -2 ((a) and (b) real and imaginary part of MSRR,SR,TTSR,NBSR). The variation μ_{reff} and the frequencies of interests are thus function of shape. Figure-3 show simulation results. The Figure-4 show the current distribution in resonance. Real part of μ_{reff} for single sided and double sided SRR are also given in Figure 5.



(a) **(b)**

Figure.2. (a)Re [μ_{reff}] vs. frequency (b) Im [μ_{reff}] vs. frequency for different magnetic inclusion



(a) **(b)**

Figure. 3. (a) Re [μ_{reff}] vs. frequency (b) Im [μ_{reff}] vs. frequency for different magnetic inclusion

III. SIMULATION OF MAGNETIC INCLUSION STRUCTURES

The simulation is based on transmission coefficient (S_{21}) and reflection coefficient (S_{11}) where we have used the parameter extraction method given by D.R.Smith[4]. We obtain the refractive- index (n) and the impedance (Z) of the mue negative(MNG) structures MSRR,SR,TTSR,NBSR and then extract μ_{reff} , as noted in below Table-I

Table-I: Showing Analytical & Simulated Magnetic Parameters

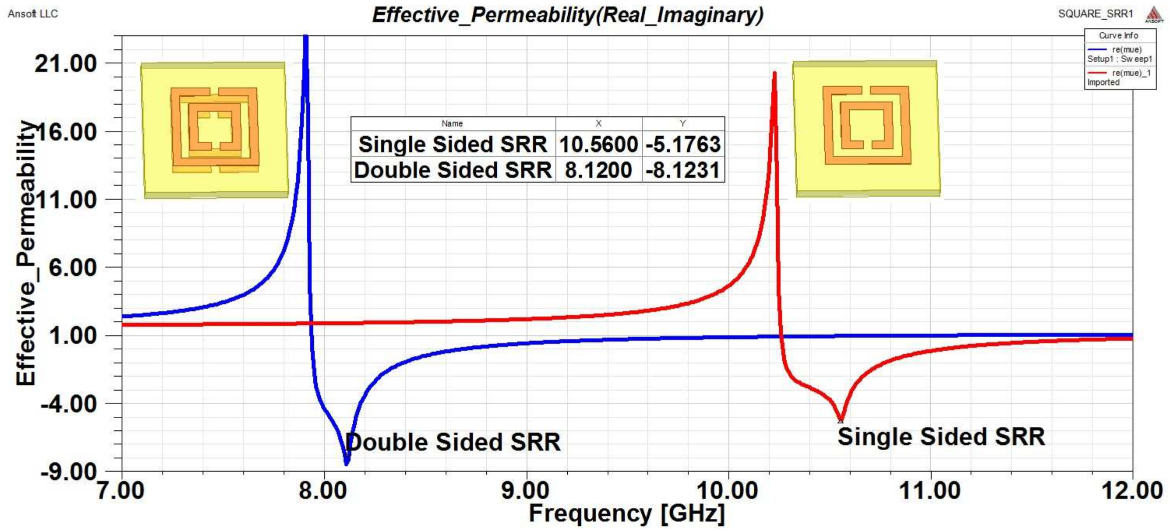
Different MNG Structure	***RESULTS***							
	Analytical Results				***Simulation Results***			
	f_{m0}(GHz)	f_{mp}(GHz)	$\Delta f = f_{mp} - f_{mp}$(GHz)	μ_{reff}	f_{m0}(GHz)	f_{mp}(GHz)	$\Delta f = f_{mp} - f_{mp}$(GHz)	μ_{reff}
*Multiple Split Ring Resonator(**MSRR**)*	12.7	13.2	0.51	-11.2	12.39	12.84	0.45	-7.06
*Spiral Ring(**SR**)*	4.31	4.50	0.19	-5.0	4.24	4.35	0.11	-4.27
*Two Turn Spiral Resonator(**TTSR**)*	3.36	3.66	0.30	-5.4	3.27	3.63	0.36	-5.88
*Non Bi-anisotropic Spiral Resonator(**NBSR**)*	6.41	6.8	0.39	-5.5	6.56	7.01	0.45	-7.07

IV. RESULTS AND DISCUSSION

A comparison of MSRR,SR,TTSR, and NBSR performance (analytically) Figure:- 2 and (simulation) Figure:- 3 reveals that every other parameter remaining same then magnetic resonance frequency(f_{mo}) magnetic plasma frequency(f_{mp}) and μ_{reff} from analytical and simulated results are as follow: MSRR>TTSR>SR>NBSR. The large difference in analytical and simulated results for μ_{reff} for MSRR is due to cross-polarization effect or magneto electric coupling. Also, printing square SRR pattern on both sides of the dielectric substrate creates coupling between neighboring unit cell, which causes degradation in magnetic performance; although having opposing current flows in the same unit cell. On the other hand NBSR and TTSR where SRR(both side) are modified by 180^{o} rotation of symmetry in plane element, as a consequence of this symmetry, cross polarization effects are eliminated for NBSR and TTSR. Therefore, for designing of MNG material NBSR and TTSR are superior than SRR.

MSRR ***SR*** ***TTSR*** ***NBSR***

Figure. 4. Induced surface current density for different MNG structure at resonance



Name	X	Y
Single Sided SRR	10.5600	-5.1763
Double Sided SRR	8.1200	-8.1231

Figure. 5. Effective Permeability (Real) for Double sided and single sided Square SRR

V. ACKNOWLEDGEMENT:

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VI. References

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