

# Particle Energy Momentum Transport for Negative Refractive Index Material (NRM) - Anomalous Concepts

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## I. Introduction

The Left-handed metamaterial (LHM) is found to exhibit counter-intuitive electromagnetic properties [1] and its phase velocity ( $v_p$ ) and group velocity ( $v_g$ ) are  $180^\circ$  apart [1, 2], thus they show negative refractive index [3]. We have been experimenting on the structured meta-materials for negative refractive index at microwave frequencies. The paper is not regarding the experiment, but nature of energy momentum transport in NRM. The negative refraction is under the experimental research for past decade [3, 5]. If a pulse of EM radiation is launched inside a negative refractive index material (NRM), it gets squeezed sharpened, and similarly a spherical wave front in positive indexed media gets flattened as it propagates inside the NRM [4], yet it is interesting if in the meta-material parlance particle-wave theory be founded. Here we give possible classical explanations to these counterintuitive phenomena and also a new explanations regarding energy momentum if applied to this negative indexed material; new definitions.

## II. Phase and group refractive index in NRM

Let us demarcate the two refractive indices, and this demarcation is essential in explaining the NRM theory. Take the refractive index dispersive that is a function of frequency call it phase refractive index ( $n_p(\omega)$ ). Expansion of Taylor series for this dispersive phase refractive index (only to its first derivative expansion), is termed as group refractive index ( $n_g(\omega)$ ). Meaning that

$$n_g(\omega) = n_p(\omega) + \omega \frac{dn_p(\omega)}{d\omega} = \frac{d}{d\omega} \{ \omega n_p(\omega) \} \quad (1)$$

We clarify that the, observed negative refraction is for as;  $n_p(\omega_0) < 0$  but always  $n_g(\omega_0) > 0$  as per causality principle. Equation (1) should be read at a particular  $\omega_0$  resonant frequency of interest, where we are observing a negative refractive index. The (2) gives model of NRM.

$$n_p(\omega) = 1 - (\omega_0^2)/(\omega^2) \quad (2)$$

Differentiation (2) with respect to  $\omega$  gives  $dn_p(\omega)/d\omega = 2(\omega_0^2)/\omega^3$  putting this and (2) in (1) gives

$$n_g(\omega) = 1 + (\omega_0^2)/(\omega^2) \quad (3)$$

For plasmonic system to achieve NRM we need  $n_p = -1$  where  $\epsilon_{r-} = -1$  and  $\mu_{r-} = -1$ , one can have electric plasma frequency overlapped, as  $\omega_{ep} \approx \omega_{mp} \approx \omega_0$  below which  $\epsilon_{r-}$  and  $\mu_{r-}$  are negative, so we get NRM as (2). Also from (2) we find that  $n_p(\omega) = -1$  when  $\omega_0^2/\omega^2 = 2$ . Putting this value of frequency, we obtain that  $n_g = 3$  when  $n_p = -1$  at the frequency of operation Surface Mode Resonances. Thus we say that the phase refractive index is negative in NRM and the group refractive index is positive in NRM. Well  $n_p$  is related to phase and phase velocity and  $n_g$  is related to group or packets or energy travel, thus group velocity.

### III. Particle inside in NRM media

Let a photon pulse of monochromatic frequency  $\omega_0$  be travelling in free space. Observer sitting on the crest and another observer sitting on the envelope, will find the relative motion as zero as in free space  $v_p = v_g = c$ . While the photon enters the NRM the two observers will be travelling in opposite directions, as inside NRM  $v_p = -v_g$ . This is the nature of canonical momentum that is generator of infinitesimal translations, and the infinitesimal translations of the 'waves' corresponds to motion of its crests and troughs (the phase), and in NRM 'opposes' the direction of motion of radiation (the group). It is for this reason the canonical momentum points in the opposite direction to the mechanical momentum inside NRM. Well can we write canonical momentum as  $p_c \stackrel{\text{def}}{=} (\text{sgn } n_p / \sqrt{|n_p n_g|}) \hbar \omega_0 / c$ ? Let us postulate this new definition of canonical momentum; the proof will come naturally as we proceed.

A slab of NRM of length  $d$ , is placed at,  $z = d/2$ . Call  $v_p$  phase velocity and  $v_g$  as group velocity of monochromatic EM signal travelling in the region  $0 < z < (d/2)$ , where the  $n_p(\omega) = +1$  with relative permeability  $\mu_{r+} = 1$ , and permittivity  $\epsilon_{r+} = 1$ . Conventionally, we can write for the dispersion less ideal region ( $z < (d/2)$ ) that;  $v_p v_g = c^2$ . Here we are assuming that  $v_p = (\omega/k) = c$ ;  $v_g = (d\omega/dk) = c$  in a vacuum where EM waves are travelling in ideal condition. Now for meta-material (lossless and ideal case, with  $n_p = -1$ ) we can write, an approximate relation, for region of NRM ( $(d/2) < z < (3d/2)$ ) where  $\mu_{r-} = \epsilon_{r-} = -1$ ; with refractive index as  $n_p(\omega_0) = -1$ , and  $n_g(\omega_0) \cong +1$  this enables the propagating modes inside the LHM slab. While inside NRM we assume.  $v_g \approx c$ , we get approximately  $v_p v_g \cong -c^2$ . Total energy mass momentum expression from relativistic approach is (4).

$$\mathbf{E}^2 = p^2 c^2 + m^2 c^4 = p^2 (v_p v_g) + m^2 (v_p v_g)^2 \quad (4)$$

Where  $\mathbf{E}$  is total energy  $p$  is canonical (wave) momentum of the particle;  $m$  is mass of the particle. Now put  $v_p v_g = -c^2$  in (4), to get (5), that is inside NRM (which is ideal).

$$\mathbf{E}^2 = p^2 (-c^2) + m^2 (-c^2)^2 = m^2 c^4 + (-p^2 c^2) \quad (5)$$

What we are observing is that corpuscular energy pertaining to group velocity is retained by the particle but the energy due to waves (phase velocity) or due to carrier or due to canonical momentum inside NRM is becoming imaginary. That is positive root of the second term of (5)

we get  $ipc$ . We can also say that canonical momentum inside the NRM where  $n_p = -1$  and  $n_g = +1$  is  $p_c = -p = -\hbar\omega_0/c$ , opposite of the canonical momentum of the free space. In this case no corpuscular (mechanical) energy is transferred to the NRM medium.

We take our practical NRM with  $n_p = -1$  and  $n_g = +3$  Here we retard the group velocity to  $c/3$ , and have phase reversal with phase velocity inside NRM as  $-c$  then  $v_p v_g = -c^2/3$ , and put the same in (4) to get (6)

$$\mathbf{E}^2 = p^2(v_p v_g) + m^2(v_p v_g)^2 = p^2(-c^2/3) + m^2(-c^2/3)^2 = (m^2 c^2/3)^2 + (-p^2/3)c^2 \quad (6)$$

Here the particle inside the NRM has less total energy; the difference of energy has been absorbed by the media itself. Expression (6) suggests one third of the energy  $(1/3)mc^2$  is retained by the ‘photon’ inside the NRM slab, and the two thirds of its energy are given to the NRM slab. Here too the wave-energy is imaginary as  $ipc/\sqrt{3}$ , and corresponding canonical momentum of the photon inside NRM is  $p_c = -(1/\sqrt{3})p = -(1/\sqrt{3})\hbar\omega_0/c$ . Let us call this particular ‘imaginary’ component of energy

#### IV. A thought experiment and discussions

Well let us consider the length of NRM slab, as  $Z$  with  $n_p = -1$  and  $n_g = 3$ . The photon is retarded in comparison to its position in absence of medium by distance;  $z = (c - v_g)Z/v_g = (n_g - 1)Z$ .

The relativistic form of Newton’s first law of motion requires that the centre-of-mass energy of a system not subjected to any external force should be stationary or in uniform motion. Our medium is isolated from such external influence then the relevant total energy is sum of photon energy  $\hbar\omega_0$  and the rest mass energy of the medium  $Mc^2$ , where  $M$  is mass of medium. The fact that photon has been retarded by the medium means the centre-of-mass-energy can only have been in uniform motion if the medium has itself moved to the right by a distance  $\Delta z$ , then the moments are  $(\Delta z)(Mc^2) = (z)(\hbar\omega_0)$ . Substituting  $z$ , we obtain  $\Delta z = (\hbar\omega_0 Z)(n_g - 1)/Mc^2$ . This motion can only take place if energy transfer takes place from photon whilst inside the medium. The required velocity of the medium is  $v_g(\Delta z)/Z$  from which we can readily obtain momentum  $p^{\text{medium}} = Mv_g \Delta z/Z = (\hbar\omega_0/c)[1 - (v_g/c)] = (2/3)p_0$ . Where  $p_0 = \hbar\omega_0/c$  is the initial momentum of the photon in free space. Momentum conservation suggests that we ascribe the difference between the initial momentum and this medium momentum to the photon momentum inside the medium. The mechanical momentum of photon in this NRM would be via classical definition as  $p_m = n_p^2 \hbar\omega_0/n_g c = v_g n_p^2 \hbar\omega_0/c^2 = \hbar\omega_0/3c = (1/3)p_0$  and the canonical momentum of the photon inside the NRM slab is

$$p_c = (\text{sgn } n_p) / \sqrt{|n_p n_g|} \hbar\omega_0/c = -(1/\sqrt{3})\hbar\omega_0/c = -(1/\sqrt{3})p_0 \quad (7)$$

The above expressions, state that 1/3 of the mechanical momentum, is retained by the ‘photon’ inside this NRM. This is well equating as if 1/3 of ‘particular’ photon corpuscular energy is retained, the photon mass as though becoming 1/3; on contrary to our classical theory that photon does not have mass. Let us leave this argument at this stage. The contradiction is embedded in principle of theory of ‘wave-particle’ duality. Really if we consider photon as particle that its

linear momentum will  $p = mv$  be and this mechanical momentum is proportional to velocity. But while considering photon or radiation as wave then its linear momentum is  $p = h/\lambda = \hbar k = \hbar \omega/v$  here, is inversely proportional to velocity. This contradiction is unessential if the medium is free space dispersion less vacuum (where  $v=c$ ), but certain problem if the photon is inside a media (positive refractive indexed or negative refractive indexed). Here in we could balance the retardation effect stating that the corpuscular energy of photon is transferred to the medium thereby inside NRM the retardation of photon takes place. What was intriguing was unaccounted imaginary energy of the photon inside the NRM. There are phenomena in NRM slab as ‘growing nature of evanescent waves’. Also, there are cusps formations at the boundary as phase reversal inside NRM gives those cusps, which travel in transverse direction of the wave packet direction (in free space). The activation of these phenomena does require energy, perhaps comes from reactive energy as discussed above that is the ‘imaginary’ energy inside NRM. Therefore this reactive energy (imaginary component due to canonical momentum) perhaps excites surface modes, helps in growing evanescent waves, and forms oscillating travelling wave cusp of charges and surface currents at the NRM boundary, and carries the crests and troughs in opposite direction inside NRM! Could we reframe the canonical momentum inside a media as, was postulated in previous section too that is  $p_c = (\text{sgn } n_p / \sqrt{|n_p n_g|}) \hbar \omega_0 / c$ , well energy balance says so. This is a new way to define canonical momentum inside slab, be it positive refractive indexed or negative refractive indexed system.

## V. Conclusion

The new concepts regarding particle energy momentum inside slab a NRM slab and new concept of reactive energy inside Negative Indexed Material is proposed, to meet the future theoretical advances on these realized negative indexed materials.

## VI. Acknowledgement

This work is supported fully by Board of Research in Nuclear Science (BRNS) Department of Atomic Energy (DAE) as a project named ‘Left Handed Maxwell Systems’.

## VII. References

- [1] V. Veselago, “The electrodynamics of substances with simultaneously negative values of  $\epsilon$  and  $\mu$ ”, Soviet Physics USPEKHI Vol. 10, No. 4, pp. 509 - 514, 1968.
- [2] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat Nasser, and S.Schultz, “Composite Medium with simultaneously negative permeability and permittivity”, Physical Review Letters, Vol.84, No.18, pp. 4184 - 4187, 2000.
- [3] D. R .Smith and N. Kroll, “Negative refractive index in left handed materials”, Physical Review Letters, Vol. 85, No.14, pp. 2933 - 2936, 2000.
- [4] R. W. Ziolkowski, E. Heyman, “Wave propagation in media having negative permittivity and permeability”, Physical review E, Vol. 64, 056625, 2001.
- [5] R. Shelby, D. R. Smith, and S. Schultz, “Experimental verification of a negative index of refraction”, Science, Vol. 292, pp. 77 - 79, 2001.