Estimation of Far-field Emission of Magnetic Coupled Resonant Wireless Power Transmission Using Equivalent Circuit Model

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Abstract

Equivalent circuit of magnetic-coupled wireless power transmission (WPT) is discussed. Both direct-fed type and indirect-fed type are considered. Parameters of the equivalent circuit are obtained from mechanical parameters of WPT structure. S parameters and far-field radiation power can be calculated correctly, which is validated by MoM calculation.

Keywords : WPT, Magnetic Resonance Equivalent Circuit, Far-field Emission

1. Introduction

Recently, wireless power transmission (WPT) using magnetic resonance is discussed actively for practical use. It is necessary to maximize efficiency of transmitting power and reducing far-field emission. Electromagnetic field simulation is widely used. However, equivalent circuit is useful for its small calculation cost.

We have studied equivalent circuit of the direct-fed and indirect-fed WPT [1] [2]. However, radiation power could not be calculated by summation of the power dissipated in the radiation resistance, because radiation powers caused by transmitting (TX) and receiving (RX) coils are added in voltage, not in power.

In this paper, we consider equivalent circuit of magnetic-resonant WPT, in which radiation power can be calculated directly as a power dissipated in the radiation resistance. Method of moment (MoM) simulation shows that the equivalent circuit has a capability to estimate S parameters and far-field emission correctly.

2. Analysis Model

One turn loop models of direct-fed and indirect-fed type are shown in Fig. 1. Voltage source with output impedance Z_s are connected to the TX coil with resonant capacitance C_0. Load impedance Z_L is connected to the RX coil with resonant capacitance C_0.

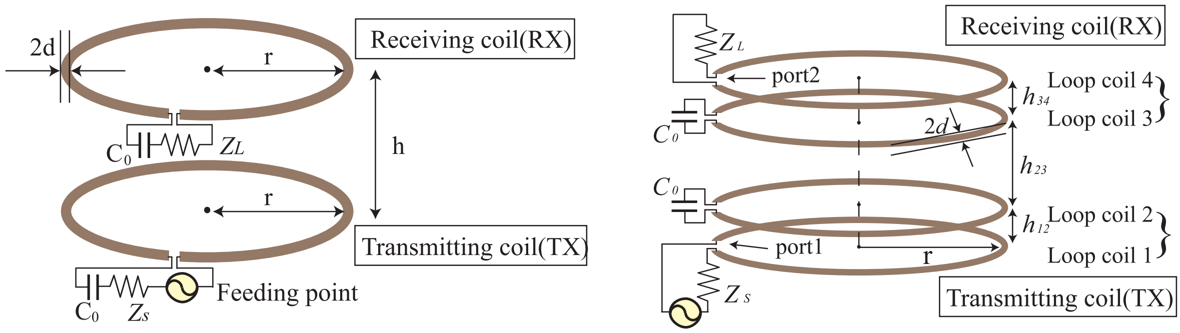
In the Fig. 1(a), one turn loop model of direct-fed is characterized by radius r, spacing h between TX and RX and section radius d of the conducting wire. In the Fig. 1(b), one turn loop model of indirect-fed is characterized by radius r, spacing h_{12} , h_{23}, h_{34}, between each loop coils and section radius d of the conducting wire.

Conducting wire is made of copper ($\sigma = 5.8 \times 10^7 [1/m \cdot \Omega]$). Resonant frequency f_0 is around 15MHz, where loop length $2\pi r$ is very short in comparison with the wavelength λ_0 at resonant frequency.

3. Equivalent Circuit

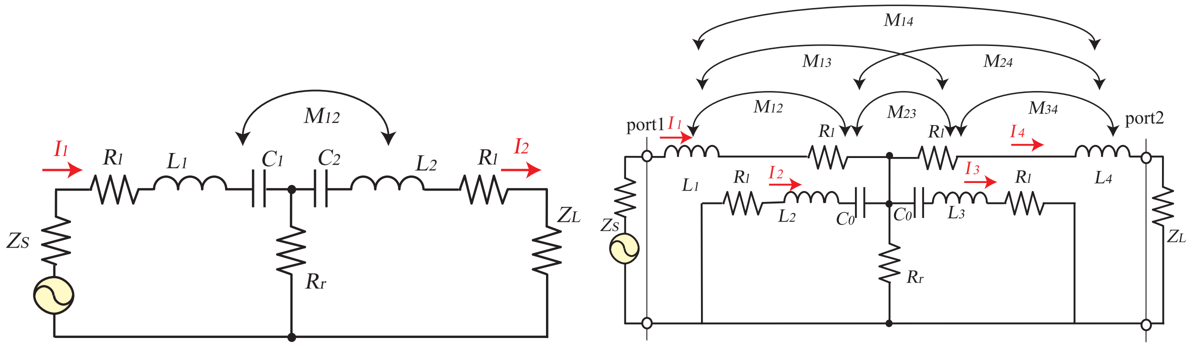
3.1 Equivalent Circuits Expression of One Turn Loop Models

Equivalent circuit models are shown in Fig. 2. (a) is direct-fed model, (b) is indirect-fed model. L is self inductance, M is mutual inductance, R_r is radiation resistance, R_l is conductor loss resistance, Z_S



(a) Direct-fed model (b) Indirect-fed model

Figure 1: Consideration models



(a) Direct-fed model (b) Indirect-fed model

Figure 2: Equivalent circuits of one turn loop coils

is output impedance, Z_L is load impedance.

3. 2 Calculation of Circuit Parameter

3. 2.1 Mutual Inductance

Mutual inductance M is computed from Neumann's formula.

$$M = \frac{\mu_0}{4\pi} \oint_1 \oint_2 \frac{d\vec{l_1} \cdot d\vec{l_2}}{r_{12}} \tag{1}$$

Where μ_0 is magnetic permeability of vacuum, $d\vec{l_1}$, $d\vec{l_2}$ is line element of the coil, r_{12} is spacing between $d\vec{l_1}$ and $d\vec{l_2}$. Especially, mutual inductance between the coaxial circular coils with radii r_1 and r_2 is computed from Neumann's formula (1).

$$\begin{aligned} M &= \mu_0 \sqrt{r_1 r_2} \left\{ \left(\frac{2}{k} - k \right) K(k) - \frac{2}{k} E(k) \right\} \qquad (2) \\ k &= \frac{4 r_1 r_2}{(r_1 + r_2)^2 + h^2} \end{aligned}$$

Where $K(k)$ and $E(k)$ are the first kind and second kind of elliptic integral.

$$K(k) = \int_0^{\pi/2} \frac{1}{\sqrt{1 - k^2 \sin^2 \phi}} d\phi \tag{3}$$

$$E(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \phi d\phi} \tag{4}$$

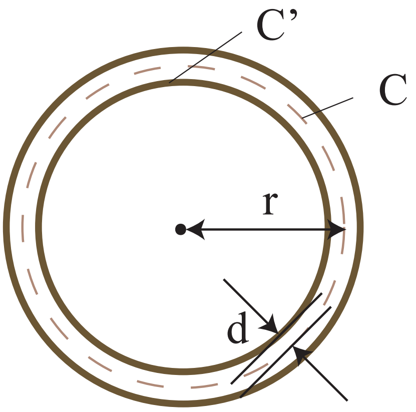


Figure 3: Loop coil

3. 2.2 Self-Inductance

Self inductance L is sum of internal inductance L_i and external inductance L_e ($L = L_i + L_e$). Internal inductance L_i is calculated as follows. A magnetic field exists in the domain of the conducting wire because current flow in skin depth $1/\sqrt{\omega\mu\sigma}$. If inter linkage magnetic flux in the domain in the conducting wire is Φ_i, magnetic flux density is B_i, magnetic flux number is $N(=\{r'^2-(d-\delta)^2\}/\{d^2-(d-\delta)^2\})$, internal inductance L_i is shown as Eq.(5).

$$L_i = \frac{\Phi_i}{I} = 2\pi r \frac{\int_{d-\delta}^{d} N B_i dr'}{I} \tag{5}$$

External inductance L_e is calculated as follows. External inductance is caused by magnetic flux at external area of conducting wire. External inductance L_e is mutual inductance of center line C and inside loop C' (Fig. 3) because magnetic flux produced by current I on center line C and magnetic flux on inside loop C' is equivalent. Then external inductance L_e is computed from Eq.(2) substituting $r_1 = r$, $r_2 = r - d$, $h = 0$. Especially, when $r_1 \approx r_2$, $h \ll r_1, r_2$, external inductanceL_e is shown as Eq.(6).

$$L_e \approx \mu_0 r(\ln \frac{8r}{d} - 2) \tag{6}$$

3. 2.3 Radiation Resistance

Radiation resistance R_r is expressed by Eq.(7), because the loop coil length $2\pi r$ is very short in comparison with the wavelength λ.

$$R_r = 20\pi^2(\beta r)^4 \tag{7}$$

Where $\beta(= 2\pi/\lambda)$ is phase constant, $\beta = 2\pi/\lambda$.

3. 2.4 Ohmic Loss

Ohmic loss $R_l[\Omega]$ is expressed by Eq.(8).

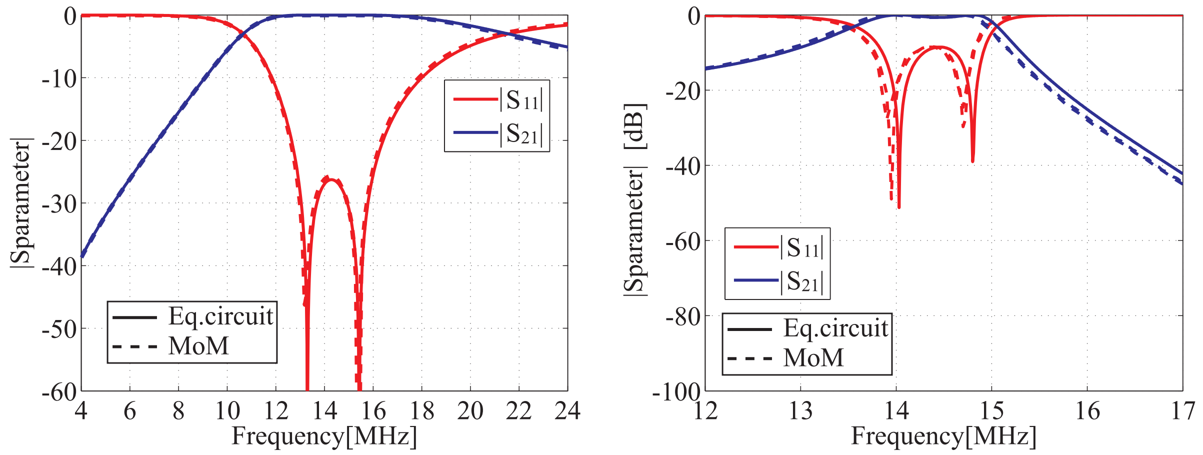
$$R_l = \frac{2\pi r}{\sigma \cdot S} \tag{8}$$

Where S is current flowing area. Considering skin effect, S is expressed by Eq.(9).

$$S = \pi\{d^2 - (d-\delta)^2\} \tag{9}$$

4. Result of Simulation

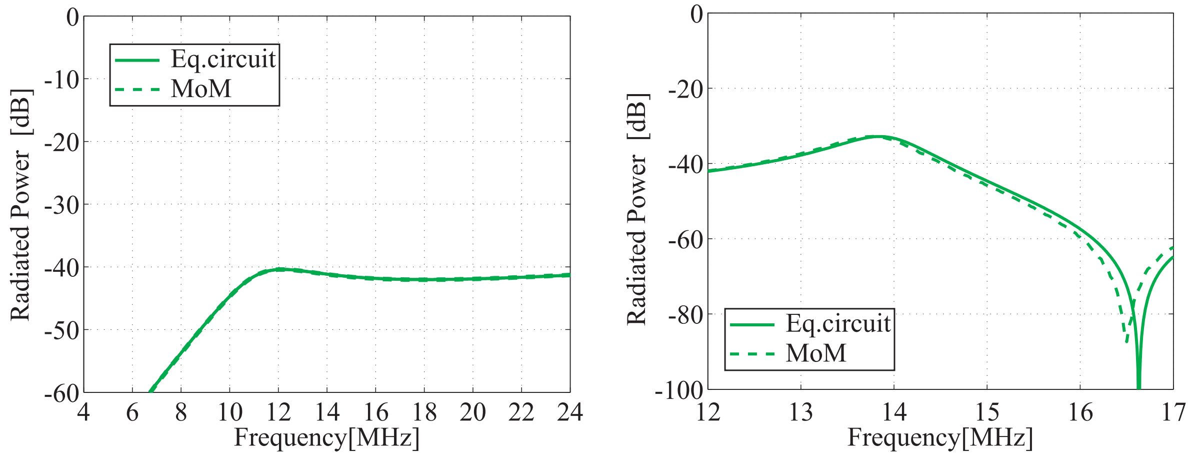
Calculation result of S parameters by the equivalent circuit and MoM is shown in Fig. 4. Far-field emission characteristic (P_r) by equivalent circuit and MoM is shown in Fig. 5. It can be said that the equivalent circuit has a capability to calculate S parameters and far-field emission correctly.



(a) Direct-fed model

(b) Indirect-fed model

Figure 4: S parameters



(a) Direct-fed model

(b) Indirect-fed model

Figure 5: Far-field emission characteristic

5. Conclusion

Equivalent circuit of magnetic-resonant WPT is discussed. Radiation power can be calculated directly as a power dissipated in the radiation resistance. Calculation results of S parameters far-field emission power is validated by MoM simulation.

References

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