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Design Curve Tracing of Class E Amplifiers by IPSO

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Abstract—This paper presents a method for finding the design characteristics of class E amplifiers using a particle swarm optimization (PSO) algorithm. To obtain the whole characteristics, the independent-minded PSO (IPSO) is introduced and the design curves are traced by restarting the IPSO. Accuracy of the proposed method is demonstrated for some examples.

1. Introduction

The class E switching-mode circuits have become increasingly valuable components in many applications, e.g., radio transmitters, switching mode-dc power supplies, devices of medical applications, and so on. Since the class E switching, namely, both zero voltage switching (ZVS) and zero derivative switching (ZDS), the class E switching circuits can achieve high power conversion efficiency at high frequencies. However, the design of the class E amplifiers is quite difficult because two switching conditions should be satisfied simultaneously on the steady-state of the circuits.

Since invention of the class E amplifier, many analytical descriptions of this circuit have been presented [1]-[3]. Although these treatments give useful guidance for the designs of class E amplifiers, it is impossible for these design methods to consider all the circuit effects. To improve the weakness, the initial solutions and class E ZVS and ZDS switching conditions are simultaneously determined by the Newton method [4]. Moreover, this method is combined with circuit simulator [5]. However, it is well-known that Newton method requires a good initial solution to ensure the convergence. This means that this method relies on a user's knowledge of class E amplifiers.

In this paper, a method for finding the design characteristics using a PSO, which is an extension of optimization by PSO [6], is presented. First, IPSO is introduced to find a design value, where with a different objective function, the IPSO is restarted to find the global best potion in a wide parameter region. Next, increasing or decreasing a parameter, we trace the design characteristics.

In the illustrative examples, it is shown that accuracy of the proposed method is comparable to the Newton method that is known to be very accurate [4].

2. Class E Amplifier

A circuit topology of the class E amplifier is shown in Fig. 1(a). The class E amplifier consists of dc-supply voltage V_D, dc-feed inductor L_C, n-channel MOSFET S, shunt capacitor C_S, and series resonant circuit composed of inductor L_0, capacitor C_0, and output resistor R. For achieving high power conversion efficiency, the zero voltage switching (ZVS) and zero derivative switching (ZDS) should be satisfied simultaneously at the turn-on instant of switch. These conditions are called the class E ZVS/ZDS conditions which are written by

$$v_s(T) = 0, \tag{1}$$

$$\left.\frac{dv_s}{dt}\right|_{t=T} = 0, \tag{2}$$

where T is the switching period and we assume that the switch turns on at $t = kT$ for an integer k. In oder to impose the conditions (1) and (2), design parameters such as values of passive elements and device parameters of the MOSFET S should be adjusted optimally. Moreover, the class E switching conditions should be satisfied on the steady-state of the circuit, which makes design of class E amplifier difficult.

The resonant filters in the class E amplifier usually have a high Q value, which means that the transition time is long until it reaches the steady-state and the transient analysis of class E amplifier requires a large computational cost. It is uncertain how many cycles of input voltage is necessary until the circuit reaches on the steady-state. Therefore, a method for finding the steady-state solution directly should be used for analyzing the class E amplifier. By using a simulator as HSPICERF, we can know the steady-state response. However, the computational cost is still large for the optimization. Therefore, we express approximately the steady-state responses in a closed form, where the MOSFET is replaced with an ideal switch as shown in Fig. 1(b).

Since the circuit equation of Fig. 1(b) is classified into on the 'on' and 'off' states of the ideal switch S. The circuit equation at the on state is written by

$$\frac{d\boldsymbol{x}_{on}(t)}{dt} = \boldsymbol{\alpha_1}\boldsymbol{x}_{on}(t) + \boldsymbol{\beta}, \quad (0 \le t \le T/2) \tag{3}$$

where $\boldsymbol{x}_{on}(t) = [i_C(t), v_S(t), i_O(t), v(t)]_{on}$ is the state vector at the on state. $\boldsymbol{\alpha}_1$ and $\boldsymbol{\beta}$ are coefficient matrix and a constant vector related with DC voltage supply, respectively. As (3), the circuit equation at the off state is expressed by

$$\frac{d\boldsymbol{x}_{off}(t)}{dt} = \boldsymbol{\alpha}_2\boldsymbol{x}_{off}(t) + \boldsymbol{\beta}, \quad (T/2 \le t \le T) \quad (4)$$

where $\boldsymbol{x}_{off}(t) = [i_C(t), v_S(t), i_O(t), v(t)]_{off}$ is the state vector at the off state.

Equations (3) and (4) are both linear and time invariant. Thus, the solutions are expressed via the eigen value decomposition of the coefficient matrix $\boldsymbol{\alpha}$: $\boldsymbol{\alpha}\boldsymbol{S} = \boldsymbol{S}\mathrm{diag}\{\lambda_1, \ldots, \lambda_4\}$, where $\lambda_1, \ldots, \lambda_4$ are the eigen values. The solutions are then written by

$$\boldsymbol{x}_{on}(t) = \boldsymbol{\gamma}_1\boldsymbol{x}(0) + \boldsymbol{\phi}_1, \quad (5)$$

$$\boldsymbol{x}_{off}(t) = \boldsymbol{\gamma}_2\boldsymbol{x}(T/2) + \boldsymbol{\phi}_2, \quad (6)$$

where

$$\boldsymbol{\gamma}_1 = \boldsymbol{S}\mathrm{diag}\{e^{\lambda_1 t}, \ldots, e^{\lambda_4 t}\}\boldsymbol{S}^{-1},$$

$$\boldsymbol{\phi}_1 = \boldsymbol{S}\mathrm{diag}\{\frac{e^{\lambda_1 t}-1}{\lambda_1}, \ldots, \frac{e^{\lambda_4 t}-1}{\lambda_4}\}\boldsymbol{S}^{-1}\boldsymbol{\beta}.$$

$\boldsymbol{\gamma}_2$ and $\boldsymbol{\phi}_2$ can be written similarly to $\boldsymbol{\gamma}_1$ and $\boldsymbol{\phi}_1$. From the steady-state condition; $\boldsymbol{x}_{on}(0) = \boldsymbol{x}_{off}(T)$, the initial conditions that give the steady-state responses are obtained from

$$\boldsymbol{x}(0) = (\boldsymbol{I} - \boldsymbol{\gamma}_2\boldsymbol{\gamma}_1)^{-1}(\boldsymbol{\gamma}_2\boldsymbol{\phi}_1 + \boldsymbol{\phi}_2)\boldsymbol{\beta}, \quad (7)$$

where $\boldsymbol{I}$ is the identity matrix.

In order to impose the class E ZVS/ZDS conditions (1) and (2), the design of the class E amplifier is defined as an optimization problem. The objective function is given by

$$f(\xi_1, \ldots, \xi_n) = \sqrt{|v_S(T)|^2 + |i_S(T)|^2}, \quad (8)$$

where $\xi_1, \ldots, \xi_n$ are design parameters. $v_S(T)$ and i_S are respectively the voltage of the shunt capacitor C_S and current flowing though it on the steady-state. It should be noted that the capacitance of C_S is used as a scaling factor of the time derivative $dv_S/dt|_{t=T}$ of (2).

3. Independent-minded Particle Swarm Optimization (IPSO)

In PSO, multiple potential solutions called "particles" coexist. At each time step, each particle flies toward its own past best position (*pbest*) and the best position among all the particles (*gbest*). Namely, All the particles always influence each other in PSO. On the other hand, the particles of IPSO have independency, thus, the connections are stochastically decided at every step. Therefore, all the particles are not affected by *gbest*, and a *pbest* does not always affect the swarm at a step.

Each particle has two kinds of information; position and velocity. The position of each particle i and its velocity are represented by $\boldsymbol{X}_i = (x_{i1}, \cdots, x_{id}, \cdots, x_{iD})$ and $\boldsymbol{V}_i = (v_{i1}, \cdots, v_{id}, \cdots, v_{iD})$, respectively, where $(d = 1, 2, \cdots, D)$, $(i = 1, 2, \cdots, M)$. The steps of IPSO are summarized as follows.

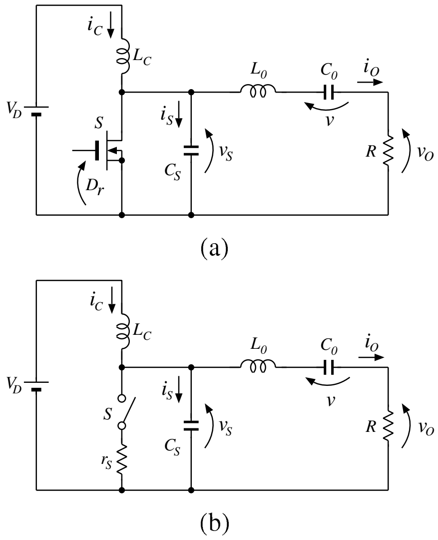

Figure 1: Class E amplifier. (a)Circuit model with MOSFET. (b)Circuit model with an ideal switch.

(Step1) (Initialization) Let a generation step t be 0. Initialize the particle position $\boldsymbol{X}_i$ ($x_{id} \in [x_{\min}, x_{\max}]$) randomly, its velocity $\boldsymbol{V}_i$ to zero, and $\boldsymbol{P}_i = (p_{i1}, p_{i2}, \cdots, p_{iD})$ with a copy of $\boldsymbol{X}_i$. Evaluate the objective function $f(\boldsymbol{X}_i)$ for each particle i and find $\boldsymbol{P_g} = (p_{g_1}, p_{g_2}, \cdots, p_{g_D})$ with the best function value among all the particles.

(Step2) Decide whether each particle i is connected to the others according to $\boldsymbol{r_3}_i = (r_{3i1}, r_{3i2}, \cdots, r_{3iD})$, an element of which is a ramdom number $\in (0, 1)$ for i. If a component $\boldsymbol{r_3}_i$ satisfies $r_{3id} \le K$, the particle i is connected to the other particles; $i \in S_c$, where S_c is a set of particles connected to the swarm. If not, the particle i is isolated from the swarm, then, the particle i and the others does not interact. K is a constant cooperative coefficient which is the independent probability of the particles, and a value $\in [0.01, 0.1]$ can be used for most problems.

(Step3) Evaluate the fitness $f(\boldsymbol{X}_i)$ for each particle i. Update the personal best position (*pbest*) as $\boldsymbol{P}_i = \boldsymbol{X}_i$ if $f(\boldsymbol{X}_i) < f(\boldsymbol{P}_i)$.

(Step4) Let $\boldsymbol{P}_l$ represent the best position *lbest* with the best *pbest* among particles being connected to others. Update *lbest* $\boldsymbol{P}_l = (p_{l1}, p_{l2}, \ldots, p_{lD})$ according to

$$l = \arg\min_i f(\boldsymbol{P}_i), \quad i \in S_c. \quad (9)$$

It should be noted that even if a $f(\boldsymbol{P}_i)$ is the minimum *pbest* among all the particles, *lbest* is not affected if i is not connected to the others.

(Step5) Update $\boldsymbol{V}_i$ and $\boldsymbol{X}_i$ of each particle i according to

$$v_{id}(t+1) = \begin{cases} wv_{id}(t) + c_1r_1(p_{id} - x_{id}(t)) \\ \quad +c_2r_2(p_{ld} - x_{id}(t)), \\ \qquad\qquad r_{3id} \leq K \\ wv_{id}(t) + c_1r_1(p_{id} - x_{id}(t)), \\ \qquad\qquad r_{3id} > K \end{cases} \quad (10)$$

$$x_{id}(t+1) = x_{id}(t) + v_{id}(t+1), \quad (11)$$

where w is the inertia weight, c_1 and c_2 are two positive acceleration coefficients, $c_1 = c_2$ in general, and r_1 and r_2 are uniform random numbers $U(0,1)$. The equation (10) with the cooperativeness K stochastically determines whether each particle is affected by *lbest* or not. When $K = 0$, all the particles move depending only on *pbest*, and when $K = 1$, the algorithm is the same with the standard PSO.

(Step6) Let $t = t+1$ and go back to (Step2).

4. Design Curve Tracing

4.1. Design Parameters

The design parameters of class E amplifier in this paper are given as follows [4]. 1)$\omega = 2\pi f$; the operating (switching) angular frequency, 2)$\omega_0 = 2\pi f_0 = 1/\sqrt{L_0C_0}$; the resonant angular frequency, 3)$Q = \omega L_0/R$; the loaded quality factor, 4)$A = f_0/f = \omega_0/\omega$; the ratio of the resonant frequency to the operating frequency, 5)$B = C_0/C_S$; the ratio of the capacitance of a resonant circuit capacitor to that of a shunt capacitor, 6)$H = L_0/L_C$; The ratio of the inductance of resonant circuit inductor to that of a dc-feed inductor, and 7)D; the duty ratio of the switch, $0 < D < 1$.

In our design, the parameters $[f, Q, R, H]$ are given in advance and the two capacitors C_0 and C_S are determined by IPSO. Then, A and B are obtained as design characteristics.

4.2. Finding Initial Parameters

IPSO provided in Sect. 3 is sufficient to find a best position even though the objective function has multimodality. However, if the searching area is too wide, the algorithm fails. The class E switching conditions (1) and (2) are also satisfied for a class E amplifier with integer multiple of the operating switching frequency $f = 1/T$. This means that there exist multiple solutions for the problem with (8). Hence, we need to select a solution with the operating switching frequency. To find such a solution, the total harmonic distortion (THD) is defined as

$$\text{THD} = \frac{1}{|V_1|}\sum_{k>2}^{O+1}|V_k|, \quad (12)$$

where V_k is the k-th frequency component of the output voltage v_o. The frequency components are calculated by the fast Fourier transform, where $2O$ sampled points are taken into account. THD is a criterion how the output waveform includes the fundamental frequency only. Thus, we use the THD as a objective function at early stage of optimization and the IPSO is restarted with initial vector that is placed around the global best position.

IPSO is superior to PSO for a problem with multimodal function. However, the convergence for unimodal case is slower that that of PSO. To find the class E switching conditions, a point with small fitness value (typically less than 10^{-10}) has to be found. Hence, IPSO is switched to PSO after the fitness value reaches a small value to ensure the convergence.

The above procedures are almost successful, but the algorithm sometimes fails. One of the reasons of the failure is the inertia coefficient w in (10). As steps of IPSO increase, the coefficient w prevents the particles from moving their positions largely, and the swarm cannot reach the global best position. Therefore, if steps are beyond the defined maximum steps, the IPSO should be restarted.

To restart the IPSO, we define some parameters. The particles are constrained as

$$x_{ik} = U(0,1)(x_{max} - x_{min}) + x_{min}. \quad (13)$$

The parameters x_{max} and x_{min} are determined as

$$x_{max} = \eta \max\{p_{g1}, p_{g2}\}, \quad (14)$$

$$x_{min} = \frac{1}{\eta}\min\{p_{g1}, p_{g2}\}, \quad (15)$$

where $\{p_{g1}, p_{g2}\}$ is the global best potion before restart. The initial positions of particles are assigned as

$$x_{ik} = 2\xi U(0,1)p_{gd} + (1-\xi)p_{gd}, d = 1, 2. \quad (16)$$

Namely, the initial positions are placed around the global best portion. The global best position should be included in the initial portions, which enhances the convergence of restarted IPSO.

4.3. Curve Tracing

The goal of this paper is to find dependence of A and B on the load quality factor Q. First, by using the IPSO provided in the previous subsection, a value of A or B is found. Next, increasing or decreasing Q, the valve of A or B is traced, where the initial particle positions are set as (16). Since PSO is a stochastic method that depends on random number, it cannot be guaranteed that the best position is surely found. Therefore, if the IPSO cannot find a best position with fitness value that is less than a user defined value, the IPSO should be restarted.

5. Results

Two examples are given in this section. In the IPSO, the parameters were set as follows; the population size

Table 1: Dependence of output power, C_S, and C_0 on loaded quality factor Q

Q	PR/V_D^2		$C_S \omega R$		$C_0 \omega R$	
	ref. [2]	IPSO	ref. [2]	IPSO	ref. [2]	IPSO
∞	0.57680	0.57680	0.18360	0.18360	0	0.00002
20	0.56402	0.56397	0.19111	0.19111	0.05313	0.05313
10	0.54974	0.54963	0.19790	0.19790	0.11375	0.11375
5	0.51659	0.51662	0.20907	0.20907	0.26924	0.26924
3	0.46453	0.46452	0.21834	0.21834	0.63467	0.63467
2.5	0.43550	0.43550	0.22036	0.22036	1.01219	1.01219
2	0.38888	0.38888	0.21994	0.21994	3.05212	3.05212
1.7879	0.35969	0.35970	0.21770	0.21770	∞	147596

In IPSO, $Q = 5 \times 10^4$ was used instead of ∞ to calculate each factor.

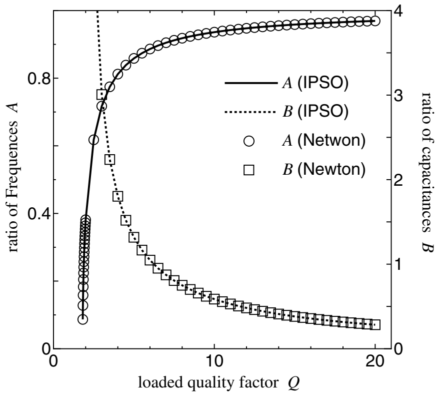

Figure 2: The design characteristics for Example 2.

$M = 24$, the inertia weight $w = 0.729$, and the acceleration coefficients $c_1 = c_2 = 1.494$. The cooperative coefficient K of IPSO was 0.1. The maximum generation of each simulation was 2000. The initial values x_{max} and x_{min} in (13) were 10^{-7} and 10^{-10}, respectively. The parameter in (14) and (15) was set to $\eta = 5.0$, and $\xi = 0.15$ in (16) for restart of IPSO. After the objective function with THD was used until 30 iterations, IPSO was restarted with (8). Moreover, the IPSO was switched to PSO after the fitness value reaches 0.01.

5.1. Example1

The first example is one tabulated in [2]. In this example, $H = 0$, $r_S = 0$, and $D = 0$ are assumed. The dc-feed inductance then is $L_C = \infty$. However, we cannot assign infinity for solving the circuit equations numerically and $H = 1.0 \times 10^{-10}$ was used.

To calculate dependence of A and B, the loaded quality factor Q is divided in three regions, $[100, 5 \times 10^5]$, $[2, 20]$, and $[1.7879, 2]$. After the values at 100, 2, and 2 were calculated for respective region, the design curves were traced, where 229, 37, and 52 points were respectively taken into account. The stopping condition (8) for IPSO was set to 1.0×10^{-10}. Table 1 gives a comparison between the proposed method and [2]. For $Q = 1.7879$, IPSO minimized the objective function until 4.3×10^{-6}. We used Octave, which is a reliable numerical analysis software, to calculate the design characteristics. For $Q = 1.7879$, Octave reported bad condition number of matrices associated with calculation of the objective function. This means that $Q = 1.7879$ is the limit that the design characteristics are obtained. In Table 1, the proposed method is almost in accordance with [2].

5.2. Example2

As the second example, the following specifications were given; the operating frequency 1 MHz, the input voltage 5 V, the output resistor $R = 5\Omega$, and $H = 0.001$. Figure 2 shows dependence of A and B. To obtain the design characteristics, the values at $Q = 5$ were calculated. the design curve was traced to both directions. Increasing Q, 36 points were taken into account. Oppositely, decreasing Q, 25 points were taken. For a comparison, the Newton method [4] was used to find the dependence. In Fig. 2, the result of IPSO is identical to that of the Newton method. At $Q = 1.81$, both methods did not converge.

6. Conclusions

The design characteristics of class E amplifiers are found by IPSO. First, a value of dependence on the loaded quality factor is found by IPSO. To enhance the performance of IPSO, THD is introduced as an objective function. Moreover, IPSO is switches to PSO to ensure the convergence. The whole characteristics are found, increasing or decreasing Q. The proposed method gives identical results of the Newton method [4]. In the Newton method, knowledge associated with class E amplifiers is necessary. Such knowledge is not necessary for the proposed method, which makes it very powerful.

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