Generalized Behavioral Steady State Model of Class E Switching Circuits

Hiroto Kamei and Yuichi Tanji

Department of Electronics and Information Engineering, Kagawa University
2217–20 Hayashi-cho, Takamatsu, Kagawa 761–0396, Japan
Email: tanji@eng.kagawa-u.ac.jp

Abstract—The steady-state behavioral model of class E amplifier is derived by using generalized eigenvalue decomposition. The circuit equations obtained after replacing MOSFET with an ideal switch are expressed in a Weierstrass canonical form. By treating the initial conditions rigorously, the steady-state conditions are obtained, even if an impulse mode takes place in the circuit. Therefore, the proposed method provides a generalized behavioral steady-state model.

1. Introduction

The class E switching-mode circuits have become increasingly valuable components in many applications, e.g., radio transmitters, switching mode-dc power supplies, devices of medical applications, and so on. Due to the class E switching, namely, both zero voltage switching (ZVS) and zero derivative switching (ZDS), the class E switching circuits can achieve high power conversion efficiency at high frequencies. To design the class E amplifier, the steady-state behavioral model is necessary in order to analyze the behavior approximately. However, when there exists an impulse mode in the behavioral model, we may not analyze the circuit. Therefore, we need to improve the behavioral modeling [1] for the case with impulse mode.

In this study, we provide a generalized behavioral steady state model of class E switching circuits. First, MOSFET in the class E switching circuit is replaced with an ideal switch. Then, the circuit equations depending on on and off states of the switch are separately obtained by MNA formulation. The resultant circuit equations are called a descriptor system in control community. By treating the initial conditions rigorously, the initial conditions that provide the steady state responses are derived, if an impulse mode happens in the circuits. Therefore, the proposed method is a generalized behavioral modeling that would apply to modeling of other resonant power converts.

2. Class E Amplifier

A basic circuit topology of the class E amplifier is shown in Fig. 1(a). The class E amplifier consists of dc-supply voltage V_D, dc-feed inductor L_C, n-channel MOSFET S, shunt capacitor C_S, and series resonant circuit composed of inductor L_0, capacitor C_0, and output resistor R. For high power conversion efficiency, the zero voltage switching (ZVS) and zero derivative switching (ZDS) must be achieved simultaneously at the turn-on instant of switch. These conditions are called the class E ZVS/ZDS conditions that are written by

$$v_s|_{t=T} = 0, \tag{1}$$

$$\left.\frac{dv_s}{dt}\right|_{t=T} = 0, \tag{2}$$

where v_S is the voltage of shunt capacitor C_S and T is the switching period. It is assumed that the switch turns on at t = kT for an integer k. In this paper, n-channel MOSFET of Fig. 1(a) is replaced with an ideal switch, AC generator as gate-to-source voltage and drain-to-source parasitic capacitance[2]. Then, the circuit of Fig. 1(a) is analyzed, separated into on ($0 \le t \le T/2$) and off ($T/2 \le t \le T$) states of switch S [4],.

3. Behavioral Modeling

3.1. Formulation

The circuit equations of the simplified circuit shown in Fig. 1(b) are written by MNA formulation. Where, AC generator V_g have two states, ON or OFF.

The MNA equations are expressed by

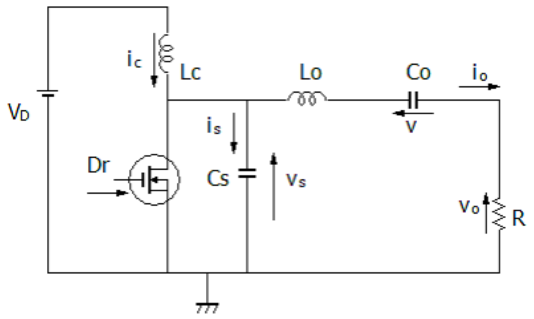
$$\begin{bmatrix} G & A^T \\ -A & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ i(t) \end{bmatrix} + \begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix} \frac{d}{dt} \begin{bmatrix} v(t) \\ i(t) \end{bmatrix} = \begin{bmatrix} J \\ E \end{bmatrix} u(t), \tag{3}$$

where G, A, C, and L are conductance, incident, capacitance, inductance matrices, respectively. J and E are vectors of independent current and voltages sources, respectively. $v(t)$ and $i(t)$ are node voltages and branch currents, respectively, and $u(t)$ is unit step function.

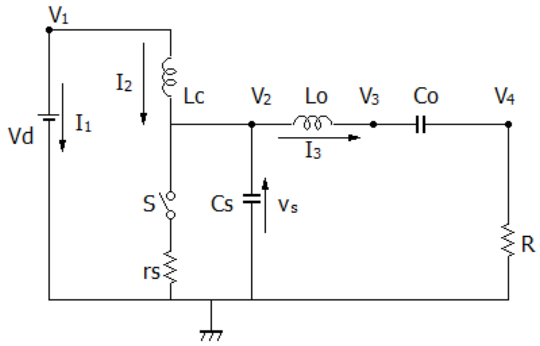
We rewrite (3) into

$$E\frac{dx(t)}{dt} = Ax(t) + Bu(t), \tag{4}$$

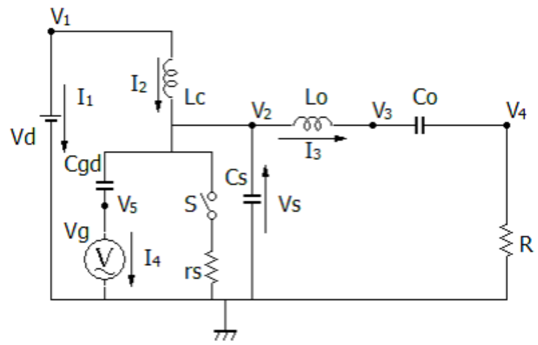
The system represented by (4) is called a descriptor system, where the matrix E is generally singular [6]. When (E, A) is regular, $\det(sE - A) \not\equiv 0$. If E is nonsingular, (4) is reduced to a state equation. In this paper, the circuit of Fig. 1(b) is expressed as (4), separated into on and off states of the switch S.



(a)



(b)



(c)

Figure 1: Class E amplifier. (a)Circuit model with MOS-FET. (b)Circuit model with an ideal switch. (c)Circuit model with an ideal switch and Drain-to-Source parasitic capacitance.

Applying the generalized eigenvalue decomposition to (E, A), we obtain

$$SET = \begin{bmatrix} I & 0 \\ 0 & N \end{bmatrix}, \quad SAT = \begin{bmatrix} \Lambda & 0 \\ 0 & I \end{bmatrix}, \tag{5}$$

where S and T are transfer matrices. Λ is a Jordan form composed of finite eigenvalues and N is nilpotent. These expressions are called a Weierstrass canonical form. Using these expressions, we can rewrite (4) into

$$\dot{x}_s(t) = \Lambda x_s(t) + B_s u(t), \tag{6}$$

$$N\dot{x}_f(t) = x_f(t) + B_f u(t), \tag{7}$$

where

$$T^{-1}x(t) = \begin{bmatrix} x_s(t) \\ x_f(t) \end{bmatrix}, \quad SB = \begin{bmatrix} B_s \\ B_f \end{bmatrix}. \tag{8}$$

Equations (6) and (7) are corresponding to finite eigenvalues of (E, A) and infinite ones, respectively.

3.2. Finding Initial Conditions

The general solutions of (6) and (7) are respectively assumed by

$$x_s(t) = \gamma(t)x_s(0_-) + \phi(t)B_s, \tag{9}$$

$$x_f(t) = -\sum_{i=1}^{\mu-1} \big(N^i \delta^{(i-1)}(t) x_f(0_-) + B_f N^i u^{(i-1)}(t)\big), \tag{10}$$

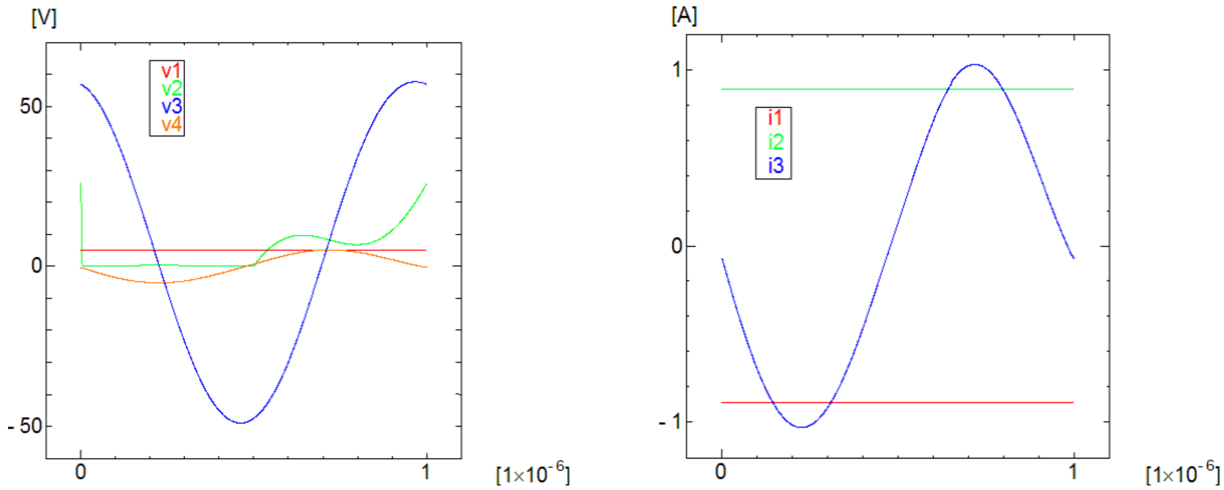
where μ satisfies $N^\mu = 0$. Although the circuit of Fig. 1(b) is separated into on and off states of the switch S, both the circuits are linear and passive, then, μ is at most 2 is known [7]. Therefore, the solutions (9) and (10) are rewritten by

$$\begin{bmatrix} x_s(t) \\ x_f(t) \end{bmatrix} = \begin{bmatrix} \gamma(t) & 0 \\ 0 & -N\delta(t) \end{bmatrix} \begin{bmatrix} x_s(0_-) \\ x_f(0_-) \end{bmatrix} + \begin{bmatrix} \phi(t) & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} B_s \\ B_f \end{bmatrix} u(t) + \begin{bmatrix} 0 & 0 \\ 0 & -N \end{bmatrix} \begin{bmatrix} B_s \\ B_f \end{bmatrix} \delta(t). \tag{11}$$

As a result, the behavior of circuit shown in Fig. 1(b) is determined by the initial values $x_s(0_-)$ and $x_f(0_-)$. Since the solutions include dependence of impulse function $\delta(t)$, an impulse mode takes place in the circuit. This means that conventional numerical integration formulae cannot be used to analyze the circuit necessarily and even the transient responses may not be known. Hence, the initial values are rigorously treated in order to find the steady state behavior.

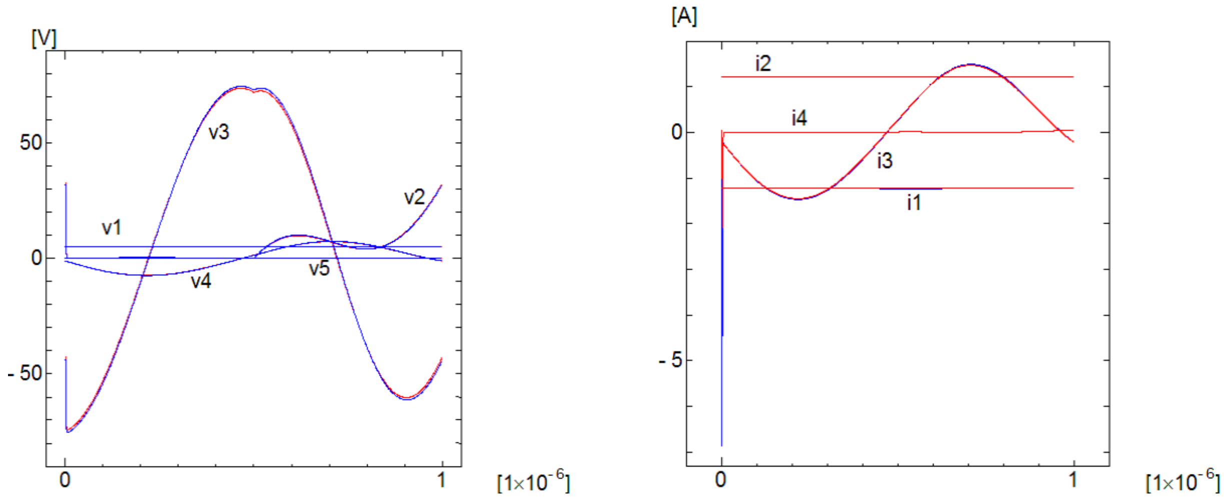
From the fact that $\delta(t) = 0$ in $0_+ \le t$, the solutions of the descriptor system can be written by

$$x(t) = \alpha(t)x(0_-) + \beta(t)Bu(t), \tag{12}$$



(a) (b)

Figure 2: Steady state responses of class E amplifier without impulse mode. (a)Voltage responses. (b)Current responses.



(a) (b)

Figure 3: Steady state and transient responses of class E amplifier.(a)Voltage responses. (b)Current responses.

where

$$\alpha(t) = T \begin{bmatrix} \gamma(t) & 0 \\ 0 & 0 \end{bmatrix} T^{-1},$$

$$\beta(t) = T \begin{bmatrix} \phi(t) & 0 \\ 0 & -I \end{bmatrix} S.$$

For the on state of the switch S, the solution of (12) $(0_+ \le t \le T/2_{-0})$ is written by

$$x_1(t) = \alpha_1(t)x_1(0_-) + \beta_1(t)B_1u(t). \quad (13)$$

For the off state of the switch S, the solution of (12) $(T/2_{+0} \le t \le T_{-0})$ is written by

$$x_2(t) = \alpha_2(t)x_2(T/2_{-0}) + \beta_2(t)B_2u(t). \quad (14)$$

Since $x_1(t)$ and $x_2(t)$ are continuous at $t = T/2_{-0}$. The boundary condition from on state to off one is obtained from

$$x_2(T/2_{-0}) = x_1(T/2_{-0}). \quad (15)$$

On the other hand, the condition from off to on, actually, the steady-state condition is expressed as

$$x_1(0_-) = x_2(T_{-0}). \quad (16)$$

It should be noted that the conditions (15) and (16) do not deny possibility that an impulse takes places or the responses becomes discontinuous at $t = 0$ or $t = T/2$. Namely, this says that although the solutions fully satisfy (11) and depend the impulse function $\delta(t)$ and unit step function $u(t)$, the boundary conditions are determined independently of these functions.

Using (15) and (16), we describe the initial values as follows:

$$x_1(0_-) = \Big(I - \alpha_2(T)\alpha_1(T/2)\Big)^{-1} \times \Big(\alpha_2(T)\beta_1(T/2)B_1 + \beta_2(T)B_2\Big), \quad (17)$$

$$x_2(T/2_{-0}) = \Big(I - \alpha_1(T/2)\alpha_2(T)\Big)^{-1} \times \Big(\alpha_1(T/2)\beta_2(T)B_2 + \beta_1(T/2)B_1\Big). \quad (18)$$

We can calculate the steady state responses by using (13) and (14) with the initial conditions (17) and (18).

4. Simulation

We calculated the steady state responses of Fig. 1(b), where the parameters were set as $V_d = 5$ V, $L_C = 7.96$ mH, $L_O = 7.96\mu$H, $C_O = 3$ nF, $C_S = 5$ nF, $R = 5\ \Omega$, $rs = 0.16\ \Omega$, and $T = 1.0 \times 10^{-6}$s. Figures 2(a) and 2(b) shows the steady state responses. In this case, impulse mode does not happen. Thus, we can apply the SVD-based behavioral modeling [1]. The responses 2(a) and 2(b) are identical to those obtained from [1].

The second example is the circuit as shown in Fig. 1(c), where the parameters were set as $V_d = 5$ V, $L_C = 7.96$ mH, $L_O = 7.96\mu$H, $C_O = 3$ nF, $C_S = 5$ nF, $R = 5\ \Omega$, $rs = 0.16\ \Omega$, $C_g d = 128$pF, $V_g = 5$V, and $T = 1.0 \times 10^{-6}$s. Figures 3(a) and 3(b) shows the steady state responses. For a comparison, the transient responses were calculated over 10,000 cycles associated with the input pulse. In Figs. 3(a) and 3(b), the blue lines are the responses obtained by constraining the steady state conditions, and the red lines are the transient responses. Both responses are not completely fitted, even though the transient responses are calculated over 10,000 cycles. To calculate the transient responses, 38,431 seconds are necessary, whereas using the steady-state behavioral, we calculated the responses in 1.1 seconds. Therefore, the behavioral model is remarkably useful for the analysis of class E amplifier. It should be noted that the impulses happen at 0 and T in Figs. 3(a) and 3(b).

5. Conclusions

We have presented a method for calculating the steady state responses of class E amplifiers with impulse mode. That is calculated by using the generalized eigenvalue decomposition, where the circuit equations are expressed in a descriptor system. We showed the feasibility of the steady state responses which compare the obtained results of the steady state responses and transient responses.

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